Alternating Current Circuits

AC Circuit

An AC circuit consists of a combination of circuit elements and an AC generator or source. The output of an AC generator is sinusoidal and varies with time according to the following equation:

$$\Delta \upsilon = \Delta V_{\max} \sin 2\pi f t = \Delta V_{\max} \sin \omega t$$

$\Delta \upsilon$ is the instantaneous voltage..

$\Delta V_{\max}$ is the maximum voltage of the generator.

f is the frequency at which the voltage changes, in Hz.

rms Current and Voltage

The *rms* ("root-mean-square") *current* is the <u>*direct current*</u> that would dissipate the same amount of energy in a resistor as is actually dissipated by the AC current.

$$I_{\text{rms}} = \frac{I_{\max}}{\sqrt{2}} = .707 I_{\max}$$

Alternating voltages can also be discussed in terms of rms values.

$$\Delta \upsilon = \frac{\Delta V_{\max}}{\sqrt{2}} = .707 \Delta V_{\max}$$

Ohm's Law in an AC Circuit

rms values are used when discussing AC currents and voltages.

- **AC ammeters and voltmeters are designed to read rms values.**
- **Many of the equations will be in the same form as in DC circuits.**

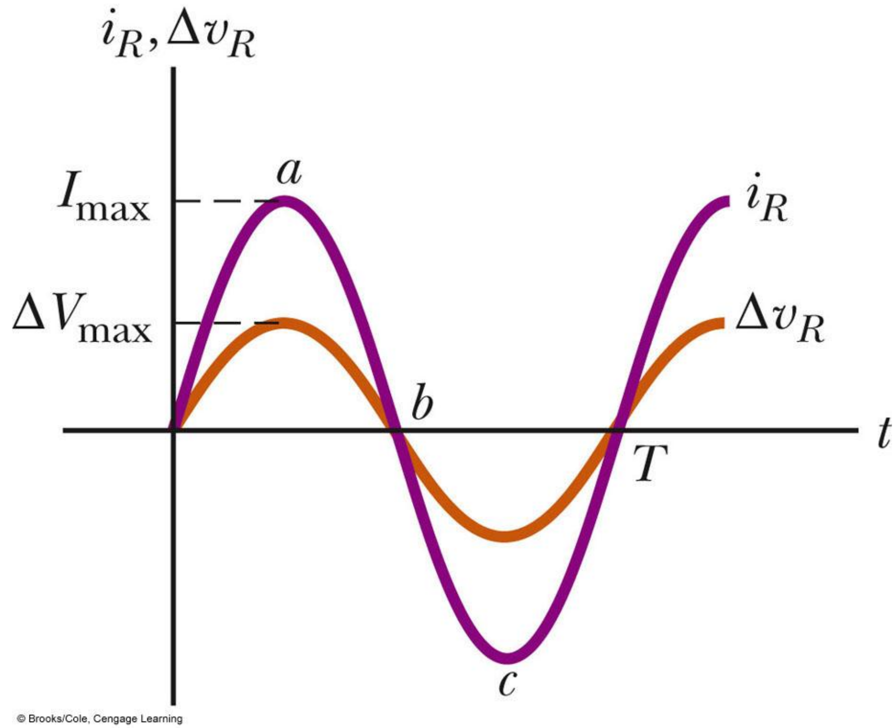
Ohm's Law for a resistor, *R*, in an AC circuit

$$\Delta V_{R,\mathrm{rms}} = I_{\mathrm{rms}} R$$

(Also applies to the maximum values of *v* and *i*.)

Resistor in an AC Circuit

Consider a circuit consisting of an AC source and a resistor. The graph shows the current through and the voltage across the resistor.



The current and the voltage reach their maximum values at the same time. The current and the voltage are said to be *in phase*.

Resistors in an AC Circuit

The direction of the current has no effect on the behavior of the resistor.
The rate at which electrical energy is dissipated in the circuit is given by

$$\mathscr{P} = i^2 R$$

where i is the instantaneous current.
The heating effect produced by an AC current with a maximum value of I_{max} is not the same as that of a DC current of the same value.
The maximum current occurs for a small amount of time.

Power in an AC Circuit

The average power dissipated in resistor in an AC circuit carrying a current I_{rms} is

$$\mathscr{P}_{\text{ave}} = I_{\text{rms}}^2 R$$

Notation Note

TABLE 21.1

Notation Used in This Chapter

	Voltage	**Current**
Instantaneous value	Δv	i
Maximum value	ΔV_{max}	I_{max}
rms value	ΔV_{rms}	I_{rms}

Capacitors in an AC Circuit

Consider a circuit containing a capacitor and an AC source.
The current starts out at a large value and charges the plates of the capacitor.

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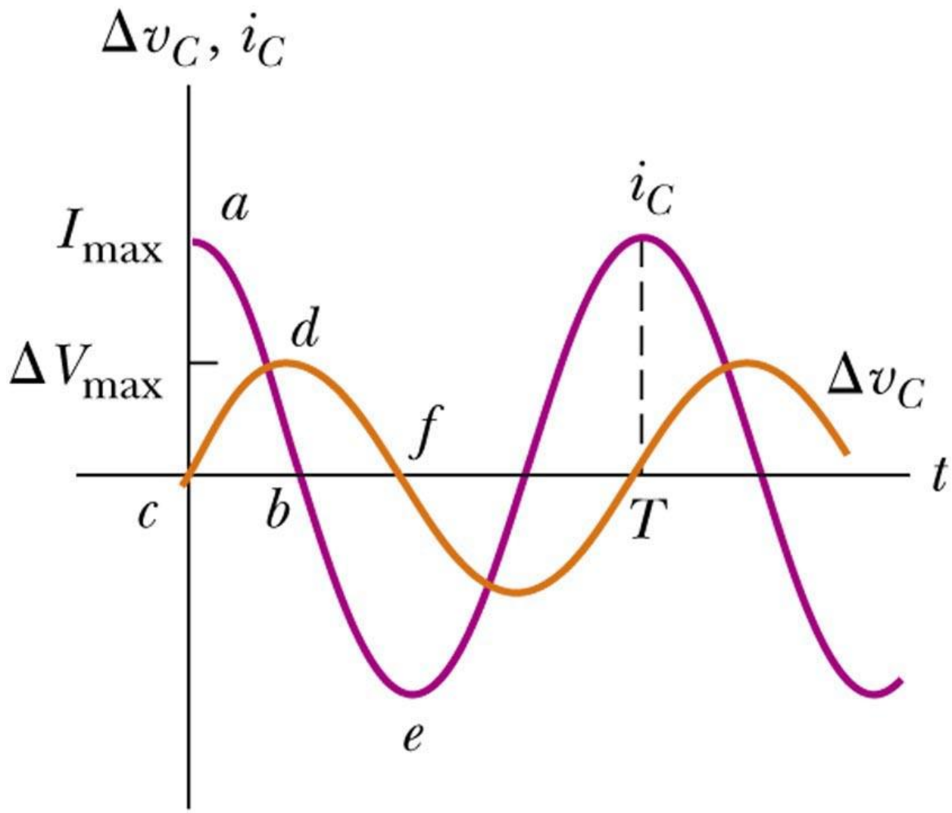
There is initially no resistance to hinder the flow of charge (current) while the plates are not charged.

Capacitors in an AC Circuit

Consider a circuit containing a capacitor and an AC source;
The current starts out at a large value and charges the plates of the capacitor.
There is initially no resistance to hinder the flow of charge (current) while the plates are not charged.
As the charge on the plates increases, the voltage across the plates increases and the current in the circuit decreases.

Capacitors in an AC Circuit

The current reverses direction.
The voltage across the plates decreases as the plates lose the charge they had accumulated.



The voltage across the capacitor lags behind the current by 90°.

Capacitive Reactance and Ohm's Law

The *impeding effect* of a capacitor on the current in an AC circuit is called the *capacitive reactance* and is given by

$$X_{\mathrm{C}} = \frac{1}{2\pi fC}$$

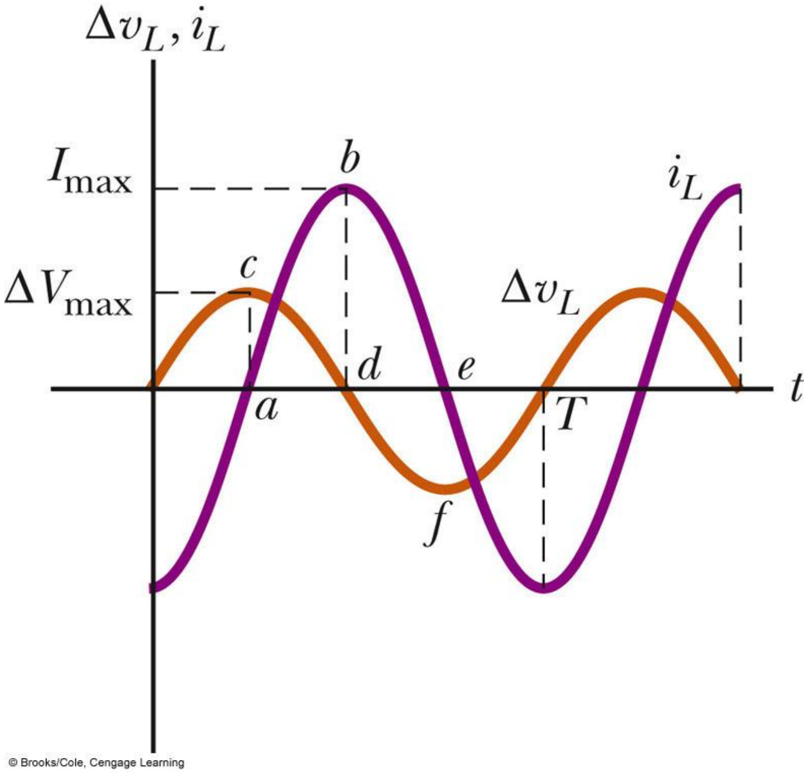
When f is in Hz and C is in F, X_{C} will be in ohms.

Ohm's Law for a capacitor in an AC circuit:

$$\Delta V_{C,\mathrm{rms}} = I_{\mathrm{rms}} X_C$$

Inductors in an AC Circuit

Consider an AC circuit with a source and an inductor.
The current in the circuit is impeded by the back emf of the inductor.



The voltage across the inductor leads the current by 90°.

Inductive Reactance and Ohm's Law

The effective resistance of a coil in an AC circuit is called its *inductive reactance* and is given by

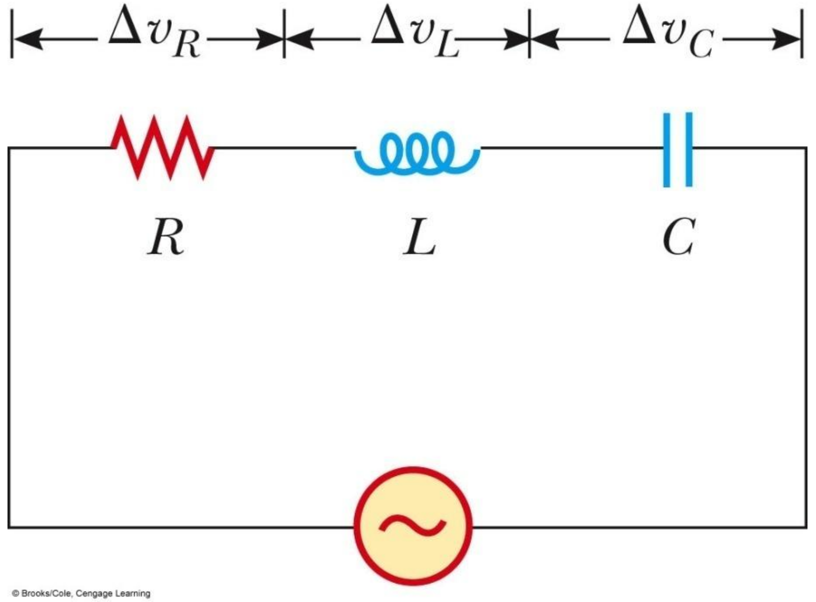
$$X_L = 2\pi fL$$

When f is in Hz and L is in H, X_L is in ohms.

Ohm's Law for the inductor:

$$\Delta V_{L,\mathrm{rms}} = I_{\mathrm{rms}} X_L$$

The RLC Series Circuit



The resistor, inductor, and capacitor can be combined in a circuit.

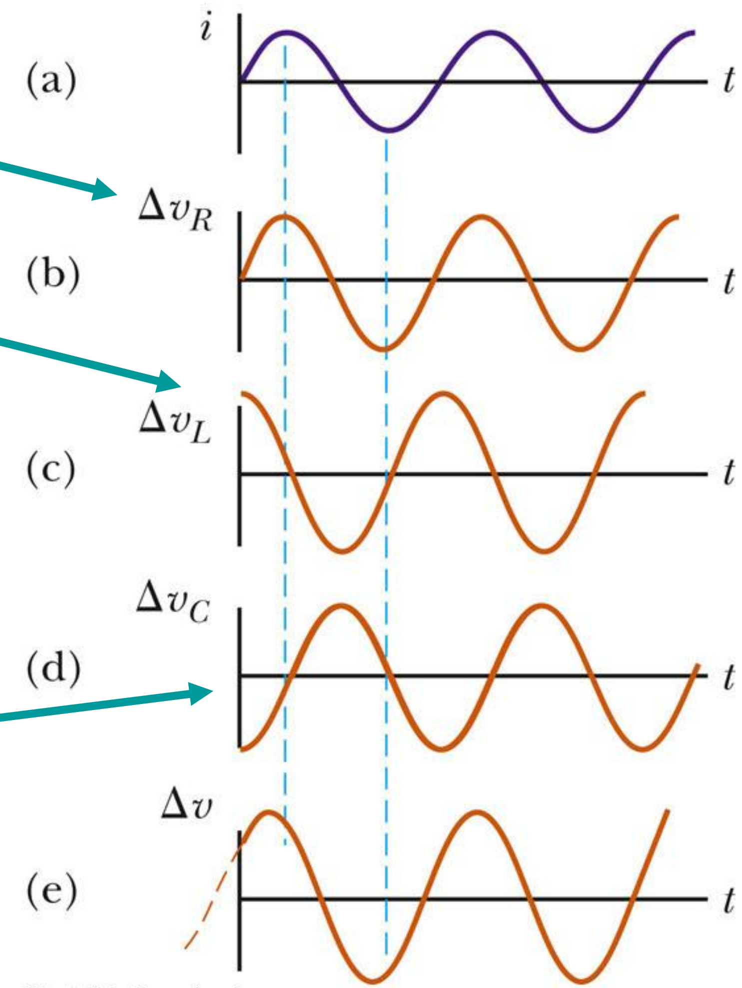
At any instant, the current in the circuit is the same everywhere.
The current varies sinusoidally with time.

Current and Voltage Relationships in an RLC Circuit

The instantaneous voltage across the resistor is in phase with the current

The instantaneous voltage across the inductor leads the current by 90°

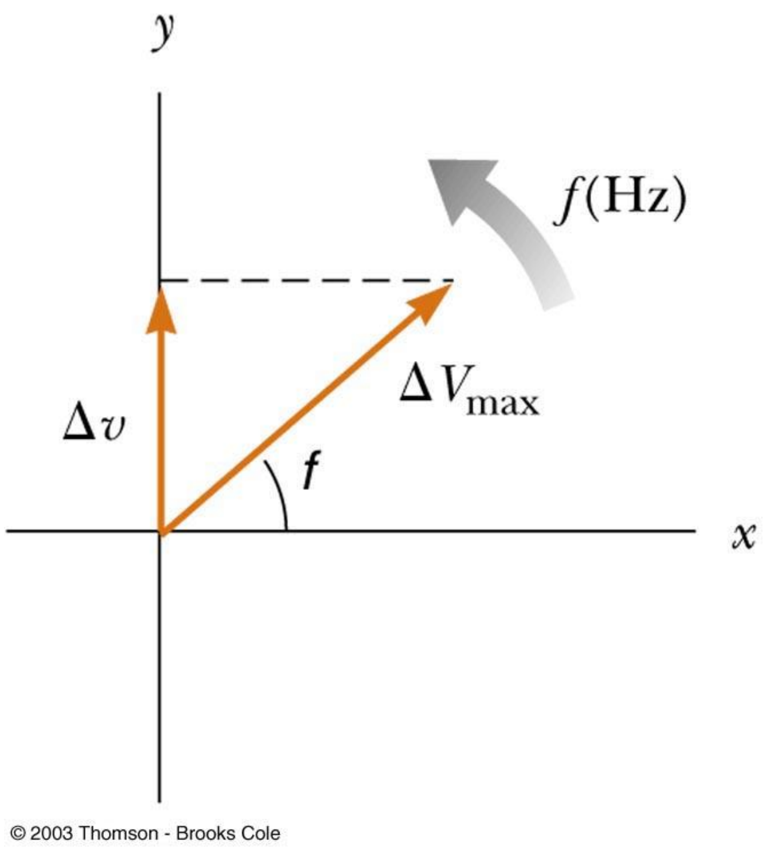
The instantaneous voltage across the capacitor lags the current by 90°



Phasor Diagrams

To account for the different phases of the voltage drops, vector techniques are used.

Represent the voltage across each element as a rotating vector, called a *phasor.*
The diagram is called a *phasor diagram.*

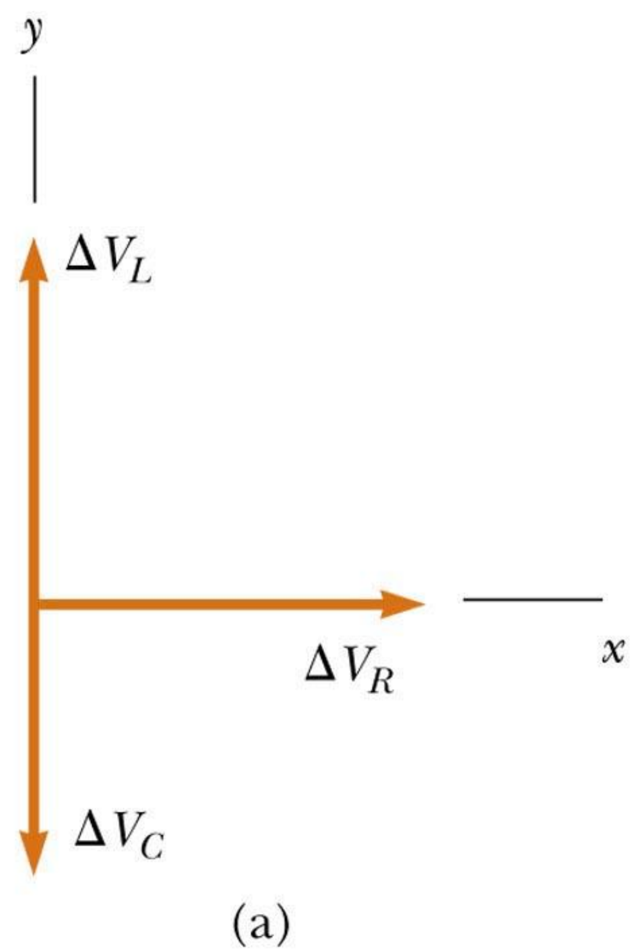


Phasor Diagram for RLC Series Circuit

The voltage across the resistor is on the +*x* axis since it is in phase with the current.

The voltage across the inductor is on the +*y* axis since it leads the current by 90°.

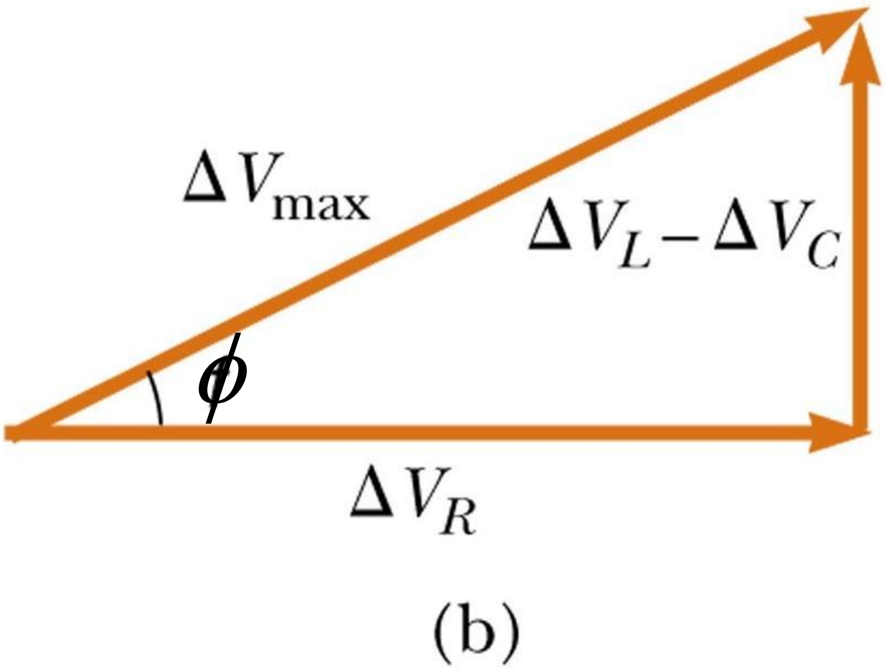
The voltage across the capacitor is on the –*y* axis since it lags behind the current by 90°.



Phasor Diagram

Phasors are added as vectors to account for the phase differences in the voltages.

ΔV_L and ΔV_C are on the same line and so the net *y* component is $\Delta V_L - \Delta V_C$.



ΔV_{max} From the Phasor Diagram

The voltages are not in phase, so they cannot simply be added to get the voltage across the combination of the elements or the voltage source

$$\Delta V_{max} = \sqrt{\Delta V_R^2 + \left(\Delta V_L - \Delta V_C\right)^2}$$

$$\tan\phi = \frac{\Delta V_L - \Delta V_C}{\Delta V_R}$$

ϕ **is the *phase angle* between the current and the maximum voltage.**

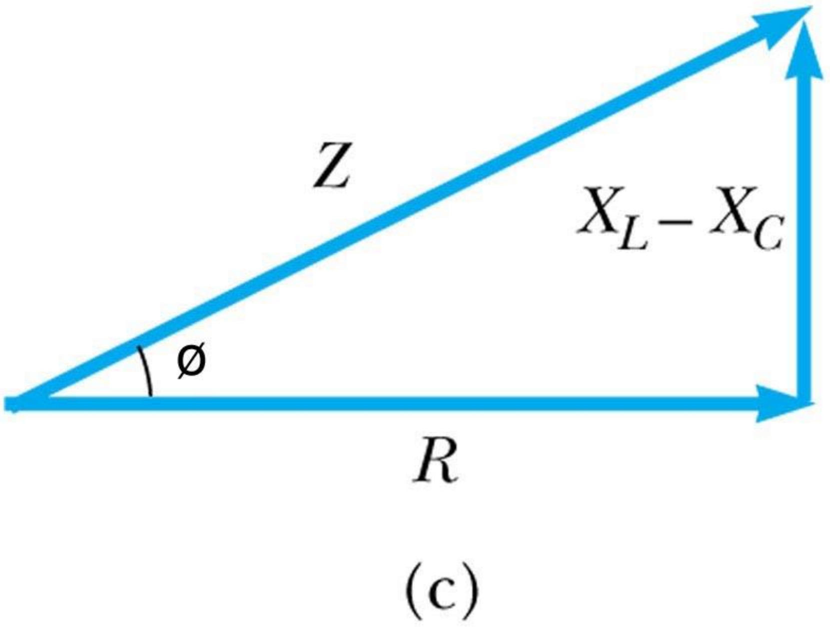
The equations also apply to rms values.

Impedance of a Circuit

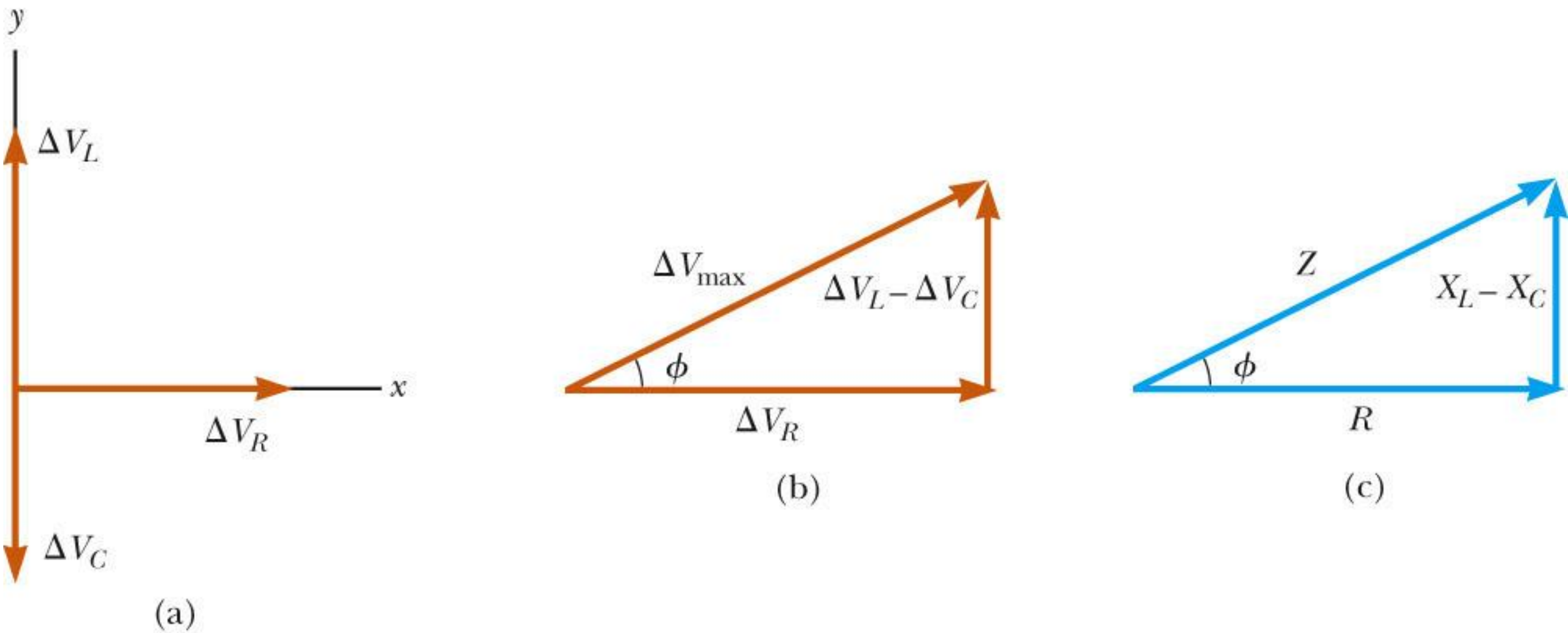
The impedance, *Z*, can also be represented in a phasor diagram.

$$Z = \sqrt{R^2 + \left(X_L - X_C\right)^2}$$

$$\tan\phi = \frac{X_L - X_C}{R}$$



Phasor Applets



http://www.walter-fendt.de/ph14e/accircuit.htm

http://www.magnet.fsu.edu/education/tutorials/java/ac/index.html

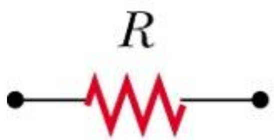
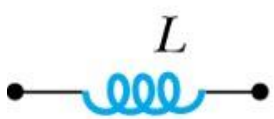
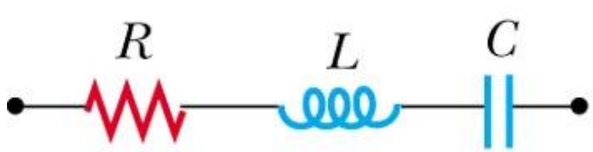
Impedance and Ohm's Law

Ohm's Law can be applied to the impedance.

$$\Delta V_{\max} = I_{\max} Z$$

This can be regarded as a generalized form of Ohm's Law applied to a series AC circuit.

Summary of Circuit Elements, Impedance and Phase Angles

	R	0°
	X_C	–90°
	X_L	+90°
	$\sqrt{R^2 + X_C^2}$	Negative, between –90° and 0°
	$\sqrt{R^2 + X_L^2}$	Positive, between 0° and 90°
	$\sqrt{R^2 + (X_L - X_C)^2}$	Negative if $X_C > X_L$ Positive if $X_C < X_L$

Problem Solving for AC Circuits - 1

Calculate the inductive an capacitive reactances, X_L and X_C.

Be careful of units – use F, H, Ω

Use X_L and X_C with R to find Z.

Find the maximum current or maximum voltage drop using Ohm's Law,

$$\Delta V_{\max} = I_{\max} Z$$

Problem Solving for AC Circuits - 2

Calculate the voltage drops across the individual elements using the appropriate form of Ohm's Law.

Obtain the phase angle.

Power in an AC Circuit

The average power delivered by the generator is converted to internal energy in the resistor:

$$\mathcal{P}_{\text{av}} = I_{\text{rms}} \Delta V_R = I_{\text{rms}} \Delta V_{\text{rms}} \cos\phi$$

$\cos\phi$ is called the *power factor* of the circuit.

Phase shifts can be used to maximize power outputs.

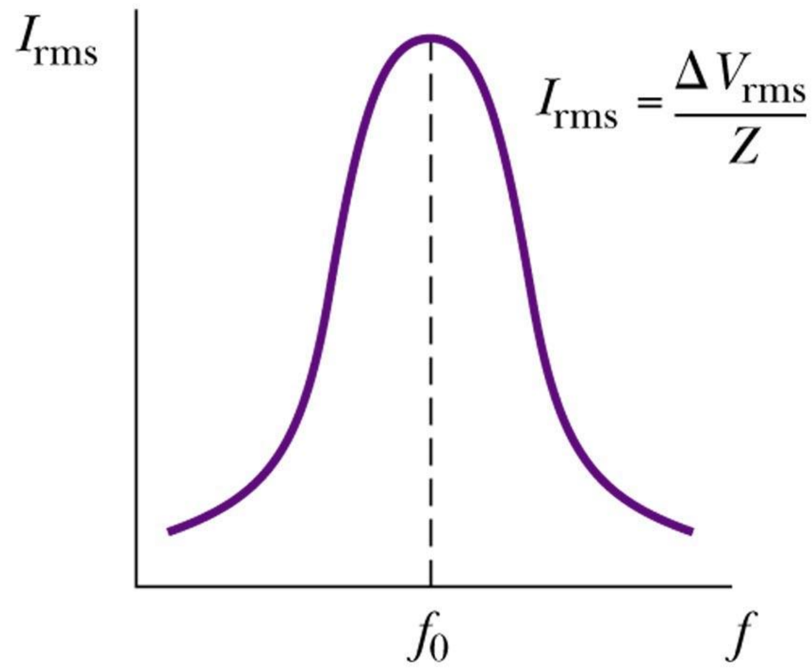
Resonance in an AC Circuit

Resonance **occurs at the frequency, f_o, where the current has its maximum value.**

To achieve maximum current, the impedance must have a minimum value. This occurs when $X_L = X_C$

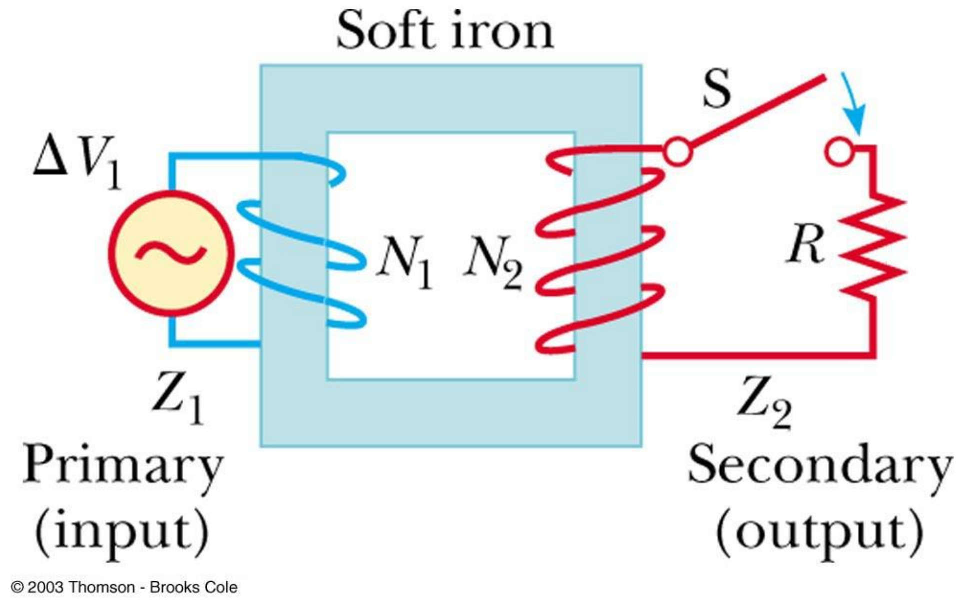
Then,

$$f_o = \frac{1}{2\pi\sqrt{LC}}$$



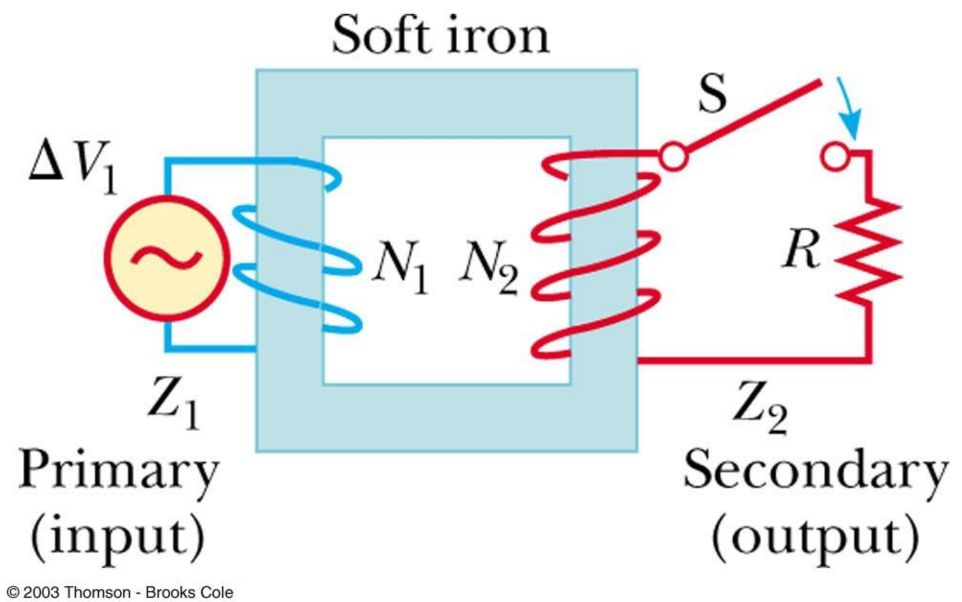
Transformers

An AC transformer consists of two coils of wire wound around a core of soft iron,
The side connected to the input AC voltage source is called the *primary* and has N_1 turns.



Transformers

The other side, called the *secondary*, is connected to a resistor and has N_2 turns. The core is used to increase the magnetic flux and to provide a medium for the flux to pass from one coil to the other.

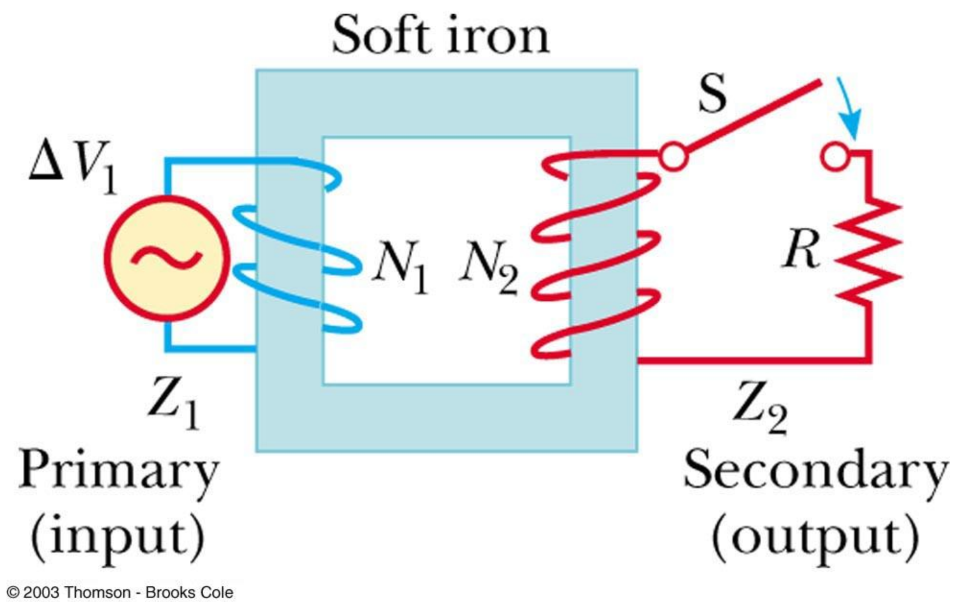


The rate of change of the flux is the same for both coils.

Transformers

The voltages are related by

$$\Delta V_2 = \frac{N_2}{N_1} = \Delta V_1$$



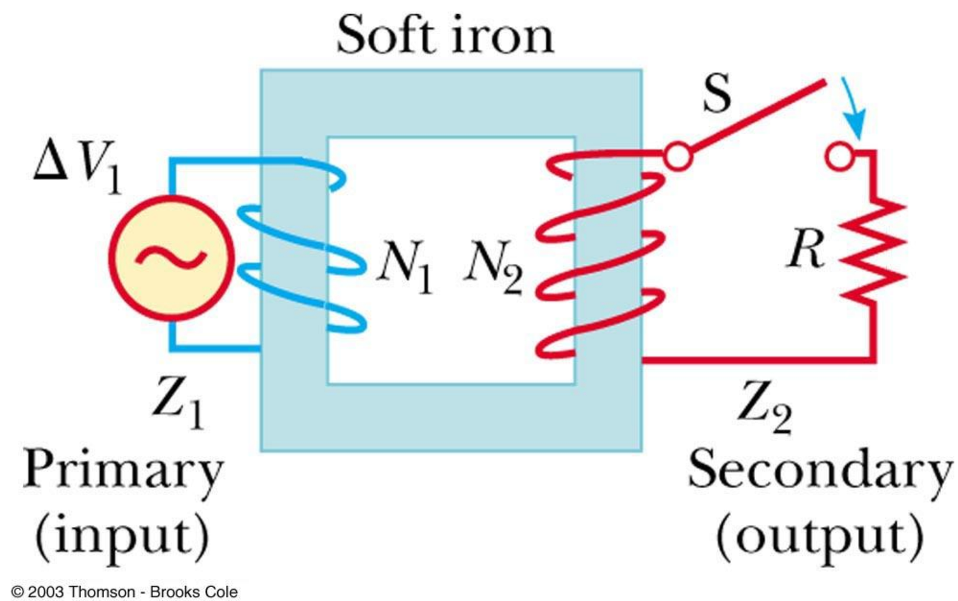
When $N_2 > N_1$, the transformer is referred to as a *step up* transformer.
When $N_2 < N_1$, the transformer is referred to as a *step down* transformer.

Transformers

The power input into the primary equals the power output at the secondary:

$I_1 \Delta V_1 = I_2 \Delta V_2$

(Conservation of energy)



This assumes an ideal transformer.

In real transformers, power efficiencies typically range from 90% to 99%.