Central limit theorems for Markov chains

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Abstract

Let $\{X_n\}$ be a stationary ergodic Markov chain on the state space (S, Σ, m) with transition probability P, Markov operator $Pg(x) = \int g(y)P(x, dy)$, and P-invariant probability m (the invariant initial distribution). We assume that the chain is defined on the space of trajectories $(\Omega, \mathcal{A})$, where $\Omega := \Pi_{n=0}^{\infty} S$ and $\mathcal{A} := \otimes_{n=0}^{\infty} \Sigma$. We denote by $\mathbb{P}_x$ the probability on $\mathcal{A}$ corresponding to the initial probability δ_x (the chain starts from x), and put $\mathbb{P}_m = \int \mathbb{P}_x dm$ for the probability of the chain with initial distribution m. The P-invariance of m is equivalent to the shift-invariance (in Ω) of $\mathbb{P}_m$.

Given $f \in L_2(S, m)$ with $\int f\, dm = 0$ we obtain an additive functional on the chain defined by $S_n(f) := \sum_{k=1}^{n} f(X_k)$. The original problem is to find conditions on f such that $\frac{1}{\sqrt{n}} \sum_{k=1}^{n} f(X_k)$ converges in distribution, in $(\Omega, \mathbb{P}_m)$, to a centered normal distribution (possibly degenerate). This is the *annealed* Central Limit Theorem (CLT) for f.

The second problem is to find conditions on f such that for a.e. $x \in S$, the CLT holds for $S_n(f)$ in $(\Omega, \mathbb{P}_x)$; i.e., for a.e. $x \in S$, $\frac{1}{\sqrt{n}} \sum_{k=1}^{n} f(X_k)$ converges in distribution, in $(\Omega, \mathbb{P}_x)$, to a centered normal distribution. This is the *quenched* CLT. If the quenched CLT holds and the variance of the limiting normal distribution is a.s. a constant σ_f, then the annealed CLT holds, and the variance of the limiting normal distribution is also σ_f.

In this talk I will survey the developments on both problems, and present some of the recent results. This talk is based on several joint papers with Yves Derriennic.