When teachers know how to effectively use the unique features of computer applications, they can address the varying cognitive strengths and needs of different students.
How can we leverage the power of technology to teach mathematics and engage diverse learners in critical thinking and problem solving? “Cognitive” technology tools have been described as “technologies that help transcend the limitation of the mind . . . in thinking, learning and problem solving activities” (Pea 1985, p. 168). These tools also respond to a user’s commands and make mathematical actions more overtly apparent (Zbiek et al. 2007).

By definition, these cognitive mathematical tech tools are not simply the traditional remedial drill-and-practice computer programs. Users of these tools have the capability to graph, model, compute, visualize, simulate, and manipulate, which amplify mathematical properties and concepts. These cognitive technology tools have the potential to broaden the representational tools that are available to teachers and students.

To leverage cognitive tech tools for mathematics teaching and learning, teachers must consider the needs of diverse learners and be equipped to support their learning difficulties by taking advantage of this technology. Most important, teachers must have “tech-knowledgy”: the knowledge necessary to use cognitive tech tools effectively to construct mathematical knowledge, evaluate the mathematical opportunities presented, and design learning tasks with these tools that amplify the mathematics.

Ways to leverage technology by integrating cognitive tech tools to promote critical thinking and problem solving will be described as well as examples of how they have been used. These tools play a pivotal role in the classroom by enhancing math for special needs learners and other diverse learners.

THE NEEDS OF DIVERSE LEARNERS

Before discussing how to employ tech tools to meet the needs of diverse learners, one must first understand the challenges that these students face. With English language learners (ELLs), the challenge is both the conversational and academic language that they need to use to participate in class discussions. Although developing and sustaining important “mathematical talk” is not easy in a classroom with a high population of ELLs, they are some of the most critical skills to develop.

Traditionally, special education students have difficulty with the metacognitive aspect of problem solving, such as identifying and selecting appropriate strategies, organizing information, monitoring problem-solving processes, and generalizing strategies to new
situations (Sliva 2004). For many of these students, a poor academic self-concept coupled with a negative experience in mathematics can contribute to learned helplessness (Elliot and Dweck 2005; Weiner 2005).

These special needs students often receive more direct instruction practices applied to discrete mathematics learning objectives without opportunities to extend to the critical-thinking level. However, research shows that students with special needs can develop mathematical understandings beyond simple rote repetition of algorithms and procedures (Woodward and Baxter 1997).

Physical manipulatives generally have been effective tools that can be used to turn an abstract concept into a concrete configuration. They have also been attributed to improved student learning. However, when students work with manipulatives, one major challenge is that handling multiple pieces can create an excessive cognitive load for learners (Kaput 1989).

Essentially, students are unable to track all their actions when using the manipulatives. They also struggle to connect multiple actions with mathematical abstraction and symbol manipulation. Recent research seems to indicate that the built-in constraints in virtual manipulatives can help overcome some of the limitations of their physical counterparts. For example, research has shown that using virtual manipulatives can benefit both ELLs (Moyer, Niezgoda, and Stanley 2005) and lower ability learners (Suh and Moyer 2008). Having the research grounding for teaching diverse learners can help teachers better understand how they might integrate technology to support student learning.

DEVELOPING TECH-KNOWLEDGY FOR DIVERSE LEARNERS
Teachers need to have knowledge of tech tools so that they can design tasks that will amplify the mathematics when teaching diverse learners. Kaput (1992) noted that the impact of tech tools in mathematics learning and teaching is the ability to off-load routine tasks, such as computations, to compact information and provide greater efficiency in learning. Suh, Johnston, and Doud (2008) highlight a few of the abilities of tech tools when they are used in mathematics:

1. Linked representations can provide connections and visualization between numeric and visual representations.
2. Immediate feedback allows students to check their understanding throughout the learning process, which prevents misconceptions.
3. Interactive and dynamic objects take mathematics from a noun to a verb (from mathematics to mathematize).
4. Opportunities are possible to teach and represent mathematical ideas in nontraditional ways.
5. Ease of differentiation and scaffolding help meet the needs of diverse learners and others.

To help teachers evaluate how to get the most out of technology to meet diverse learning needs, I created a resource called Tech-Knowledgy: Planning with Technology to Meet Diverse Learning Needs (see table 1). This planning guide described the variety of cognitive tech tools available and how their specific attributes can mediate individual learning needs. Using this guide with middle school teachers through lesson study (Lewis 2002), teachers were better able to select the most effective tech tools to plan and support the learning of specific mathematics concepts for those students who experience learning or language difficulties.

In addition, to define what mathematics tools are effective, I used the criteria set by the National Research Council (2001) for choosing representations. The criteria included evaluating and balancing characteristics such as transparency, efficiency, generality, clarity, and precision when selecting representations.

Placing importance on the mathematical representations within these tools helped teachers move beyond merely looking for engaging items to becoming critical consumers of technology who could provide mathematical power. For example, teachers evaluated whether the mathematical ideas could be seen easily through the representations (transparency), were easy to use (clarity), and if the tools could be applied to multiple concepts (generality).

EXAMPLES OF COGNITIVE TECH TOOLS IN USE
The classroom examples that follow were the result of a collaboration with middle-grades teachers on lesson study where we planned, taught,
observed, and then debriefed. The lessons focused on promoting algebraic connections. The topic of algebra was chosen because it is an important prerequisite to understanding more abstract mathematics. Research has shown the greatest impact of technology on algebra activities with middle school and secondary-level students.

The use of graphing calculators, computer algebra systems, and other software programs helped students understand and visualize graphical representations of symbolic forms (Kieran 2007).

Although developing visual models for algebra is crucial, conventional instruction on algebra tends to be rule based. In particular, special needs learners often receive direct instruction on how to perform algorithmic procedures using mnemonic devices or steps to follow without having opportunities to construct conceptual understandings of mathematical processes. The following examples will show how technology can be

<table>
<thead>
<tr>
<th><strong>Affordances Gained by Technology</strong></th>
<th><strong>Addressing Diverse Needs</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Linked representations</td>
<td>Makes connections more explicit through multiple representations. (See <a href="http://nlvm.usu.edu%E2%80%99s">http://nlvm.usu.edu’s</a> fraction-comparing tool, which allows numeric and pictorial representations—region and line models—to appear on screen simultaneously.)</td>
</tr>
<tr>
<td>Capability to graph, compute, visualize, simulate, and manipulate</td>
<td>Provides visuals and interactive modeling of problems; allows for experiments and conjectures (i.e., a spreadsheet allows the user to manipulate data and graph in multiple ways). (See the interactive applets at <a href="http://brainingcamp.com">http://brainingcamp.com</a>, which show simulation of tree growth and tabular and graphic representations.)</td>
</tr>
<tr>
<td>Collaborative learning through computer games and electronic work</td>
<td>Allows users to collaborate and interact with other users by using a computer. NCTM’s Illuminations site contains games that students can play with a friend and learn math strategies. (See <a href="http://calculationnation.nctm.org/">http://calculationnation.nctm.org/</a> and <a href="http://illuminations.nctm.org/">http://illuminations.nctm.org/</a>.)</td>
</tr>
<tr>
<td>Linked vocabulary</td>
<td>Develops vocabulary and cognitive academic language skills by linking vocabulary terms to definition and visuals. (See <a href="http://www.teachers.ash.org.au/jeather/maths/dictionary.html">http://www.teachers.ash.org.au/jeather/maths/dictionary.html</a>.)</td>
</tr>
<tr>
<td>Guided exploration and constraint and support systems</td>
<td>Relieves cognitive load (step by step and more structured) and attention issues. The built-in constraint and support system allows users to focus on important mathematical concepts and behaviors. (See the algebra balance scale on <a href="http://nlvm.usu.edu">http://nlvm.usu.edu</a>.)</td>
</tr>
<tr>
<td>Immediate feedback</td>
<td>Eliminates error patterns and misconceptions and provides formative feedback for students to self-assess their learning.</td>
</tr>
<tr>
<td>Online calculator/computation aide</td>
<td>Off-loads computation so that students can focus on analyzing relationships and patterns. (See the displaying number patterns at <a href="http://standards.nctm.org/document/eexamples/chap4/4.5">http://standards.nctm.org/document/eexamples/chap4/4.5</a>.)</td>
</tr>
<tr>
<td>Dynamic actions and animation</td>
<td>Provides enough engagement, but must not be too distracting for learners. (See <a href="http://explorelearning.com">http://explorelearning.com</a>.)</td>
</tr>
<tr>
<td>Spoken words and directions</td>
<td>Supports language processing difficulty. According to the theory of multimedia learning (Mayer 2001), one learns better when presented with visual and verbal codes (spoken narrative). (See <a href="http://brainpop.com">http://brainpop.com</a>.)</td>
</tr>
<tr>
<td>Tiered lessons and tasks</td>
<td>Eases differentiation and allows for self-paced learning.</td>
</tr>
<tr>
<td>Ability to switch languages</td>
<td>Facilitates cognitive academic language skills. (See <a href="http://nlvm.usu.edu">http://nlvm.usu.edu</a>, which allows users to switch to French, Spanish, and English.)</td>
</tr>
<tr>
<td>Customizable and replicable</td>
<td>Allows users to adapt it to create their own problems or models. Users can revisit the site, try again, and review.</td>
</tr>
</tbody>
</table>

For more cognitive tech tools, go to http://mason.gmu.edu/~jsuh4/mathbridges/index.html.

Table 1 Information found in *Tech-Knowledgy: Planning with Technology to Meet Diverse Learning Needs* explores what features of technology can help which diverse need.
leverage to enhance instruction using algebraic applets and spreadsheets.

**Example 1: Analyzing Rates of Change**

One applet that teachers analyzed for teaching algebraic concepts was called the Healthy Forest, which can be found at the brainingcamp.com site. Interesting and complex problem-solving contexts use real-world simulations and rich three-dimensional interactive animation. The context for this Web site is a forest being planted to offset a city’s increasing carbon dioxide emissions. This particular unit focuses on five major topics: rate of change, graphs, functions, problem solving, and connections. In addition to the applet shown in figure 1, students are introduced to the problem with the following scenario:

Three different trees are shown. Each grows at a different speed and has a different maximum height. Move the slider below the graph to see how each tree’s height changes with age. Which tree is the tallest at 10 years? Which tree is tallest at 50 years? Can you explain why the tallest tree at 10 years is not the tallest at 50 years? What do you notice happening to the Pine tree at 25 years?

Manipulating this applet gave students opportunities to explore and develop a sense of the mathematics. Moving the slider on the graph corresponded to the simulated growth of the trees on the right, which allowed students to make a direct connection to the significance of the slope on the graph. This is often a source of confusion for many middle-grades students. When used in the classroom, students noticed that the three different graphs meant something unique for the growth rate of each tree.

A special education teacher who used this applet with a diverse group of students shared this account of her experiences:

Oftentimes we get so caught up collecting and organizing data and graphing data in multiple ways without taking the opportunity to really spend time interpreting what the graphs mean. When I used this in class, all my kids wanted to make sense of the graphs so they could determine which tree would use the most carbon dioxide to grow and as a result remove this harmful gas from the air. They felt like environmentalist working on this problem and did not even realize the complex math they were engaged in.

The unique design features encouraged students to make conjectures by interacting with the software and learning by trying what-if scenarios. Meanwhile, its tiered levels of difficulty and vocabulary links can support students at their individual levels and in learning academic vocabulary.

After working on the computer with the scenarios, students were asked to describe what helped them transfer their learning of interpreting a graph. One student wrote:

In today’s math, looking at the graphs, I noticed the White Pine tree grew at a fast rate but then after 25 years it did not grow, whereas the Sequoia tree grew slower than the White Pine at first but it continued to grow to be 200 ft. in 50 years. That is one tall tree!

**Example 2: Building Rules for Linear Functions**

The following example is from an eighth-grade lesson on building rules to represent linear functions. The lesson-study team planned an algebraic task by posing this question:

You have decided to use your allowance to buy an mp3 purchase plan. Your friend Alex is a member of i-sound and pays $1.00 for each download. Another friend, Taylor, belongs to Rhaps and pays $13.00 for each download. Which plan is the best for you? How do you know?
a month for an unlimited number of downloads. A third friend, Chris, belongs to e-musical and pays a $4.00 monthly membership fee and $0.40 a month per download. Each friend is trying to convince you to join their membership plan. Under what circumstances would you choose each of these plans, and why?

Special needs students used a spreadsheet (see fig. 2) to graph the three plans. It allowed them more time to analyze the different plans instead of spending their time plotting points. In this example, technology amplified the opportunity available for students to interact with the problem, make conjectures, and test those conjectures to confirm or reject their hypothesis. Generating representations quickly allowed time for experimentations and deeper analysis and discussion about mathematics with higher complexity than merely creating line graphs.

One special education teacher who taught this lesson to students with special learning needs was ecstatic to share this thought after using the spreadsheet and graph:

Next I asked if they thought there was any significance where lines intersected each other. They knew this right away, with no prompting. Then I asked about one point where the lines intersected, but there wasn’t a whole number. One student volunteered that you couldn’t get half a download. Another said you could, but they would charge you for the whole thing. They were looking at the problem and deciding if it was reasonable! I work with the kids every day on fractions and decimals, and I wonder if they will be able to access the general education curriculum in high school. If their teachers modify things for them, algebra should be a no brainer.

As Kaput (1992) noted, the impact of technological tools in mathematics learning and teaching is the ability to off-load some routine task, in this case, creating a line graph from scratch, which provided learning efficiency in terms of compacting and enriching experiences. This example is not meant to discount the importance of students’ ability to translate data into different graphical forms. Many times, students have few opportunities for this type of critical thinking in which they use their interpretative skills, because the curriculum is so densely packed and because time is not allocated to exploring the depth and complexity of mathematical ideas.

Example 3: Balancing Equations Using the Virtual Balance Scale
In the following lesson study, students in the sixth grade explored the concept of balancing a linear equation using a dynamic algebra balance. Students were introduced to the dynamic algebra scales from the National Library of Virtual Manipulatives (http://nlvm.usu.edu). They viewed balanced boxes that represented the unknown \(x\) and blocks that represented numbers (see fig. 3). One feature of the virtual balance scale was that it explicitly linked a dynamic balance scale with the symbolic representation of the algebraic equations that were presented on the scale. When students typed in a symbolic command such as “subtract 3x from both sides,” the dynamic feature of the applet removed three of the \(x\) boxes from both sides of the balance scale and simultaneously displayed a new equation on the screen. The equation window tracked moves made by the student, thereby scaffolding the process of solving for \(x\), the doing and undoing algorithmic process, and explicitly providing the connection between the equations and the actions of the balance scale.

During class sessions, when the teacher asked students to explain their solution processes, students used the equation window, which is where these processes had been recorded by the virtual applet. In addition to the special features of the linked representations, the dynamic capability of the tilting balance scale reinforced the concept of the equal sign representing the idea of “same as” instead of the common misconception of the equal sign meaning that the answer is or is not doing something (NCTM 2000).
Final thoughts

Students benefited from the unique aspects of cognitive tech tools, which can provide a variety of learning supports for special needs students and ELLs in math class. The use of the graphs, tables, simulations, and applets allowed students to think and reason about rates of change, linear functions, and balancing equations. Opportunities to work with student partners encouraged mathematical discourse.

For students with limited English, teachers felt that technology provided them with a visual tool that gave them access to the mathematical language and the academic vocabulary (Marzano and Pickering 2005) necessary for them to participate in class discussions.

Unique features of the virtual tools enabled special needs students to off-load the task of maintaining both pictorial images and symbolic notations as the images and notations changed in response to the students’ input. This allowed students to focus more on the mathematical processes and relationships and make sense of the mathematics. Kaput (1992) stated that constraint-support structures built into computer-based learning environments “free the student to focus on the connections between the actions on the two systems [notation and visuals], actions, which otherwise have a tendency to consume all of the students’ cognitive resources even before translation can be carried out” (p. 529).

The potential of these tools, used in lessons where teachers and students are engaged in meaningful math discussions, is important to explore for special needs learners. The linked representations in the virtual algebra balance environment offer metacognitive support, such as keeping a record of the user’s actions and numeric notations. Learners are then able to use their cognitive capacity to observe and reflect on connections and relationships among the representations.

Teachers are inundated with new curriculum, new best practices, and new mandates and tests, not to mention the challenges from our ever-changing society. Making sense of mathematics teaching in the twenty-first century will require teachers to consider the best available resources and technology, based on research, and discriminate and evaluate these resources so that they can be leveraged to yield the most effective and meaningful learning for students.

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**ONLINE RESOURCES**


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