CSC 400 Discrete Structures
Midterm Review

Exam Schedule and Format

- Date: 2/14/2019 (Thu) for Section 802, and any day between 2/14 (Thu) and 2/16 (Sat) for Section 810.
- Location: Regular classroom for Section 802; individually arranged location for Section 810.
- Time: 2.5 hours -- 5:45 – 8:15 pm for Section 802; Proctored 2.5-hour exam for Section 810.

- Format: Pencil-and-paper. You write answers on the exam paper.
- No computer is allowed. But a ONE page note is allowed (both sides). No other materials.
- A calculator is allowed (although I don't think you will need it).
- Topics: From the beginning till the end of Set Theory.
  - §1.2
  - §2.1, §2.2, §2.3 (light)
  - §3.1, §3.2, §3.3, §3.4 (light)
  - §4.1, §4.2 (light), §4.3, §4.6
  - §5.1, §5.2, §5.3, §5.4

Review Questions

1. For propositions P and Q below, state whether or not P ≡ Q. Write a truth table to justify your answer.

   P: $p \vee q \rightarrow r$
   Q: $(p \rightarrow r) \wedge (q \rightarrow r)$

2. [§2.2, Exercise #20e (*)] Write the negation of the following statement.

   e. If x is nonnegative, then x is positive or x is 0.

3. [§2.2, Exercise #48 (*)] Use the logical equivalences $p \rightarrow q \equiv \sim p \vee q$ and $p \leftrightarrow q \equiv (\sim p \vee q) \wedge (\sim q \vee p)$ to rewrite the given statement forms without using the symbol → or ↔, and (b) use the logical equivalence $p \vee q \equiv \sim(\sim p \wedge \sim q)$ to rewrite each statement form using only ∧ and ~.

   $p \vee \sim q \rightarrow r \vee q$

4. [§3.2, Exercise #18] Write the negation of the following statement.

   $\forall x \in R$, if $x(x + 1) > 0$ then $x > 0$ or $x < -1$.

5. [§3.3, Exercise #15b] Write the negation of the following statement.

   ∀ odd integers n, ∃ an integer k such that $n = 2k + 1$.

6. Determine the truth value of the following proposition. If it is true, prove it using direct proof. If it is false, state the negation explicitly and give a counterexample.

   <u>Proposition</u>: The sum of any three consecutive integers is divisible by 3.

7. Determine the truth value of the following proposition. If it is true, prove it using direct proof. If it is false, state the negation explicitly and give a counterexample.

   Proposition: ∀a∀b such that a, b are real numbers, $\sqrt{\boldsymbol{a}+\boldsymbol{b}} = \sqrt{\boldsymbol{a}} + \sqrt{\boldsymbol{b}}$

8. Prove the following proposition by using proof by contradiction.

   Proposition: If a product of two positive real numbers is greater than 100, then at least one of the numbers is greater than 10.

9. [§5.1, Exercise #3] Write the first four terms of the sequence defined by the formula

$$\boldsymbol{c_i} = \frac{(-\mathbf{1})^{i}}{\mathbf{3}^{i}}, \text{ for all integers i} \geq 0$$

10. [§5.1, Exercise #12] Find explicit formula for the following sequence

$$\frac{1}{4}, \frac{2}{9}, \frac{3}{16}, \frac{4}{25}, \frac{5}{36}, \frac{6}{49}$$

11. [§5.2, Exercise #7 (*)] Prove the following statement using mathematical induction. For all integers n >= 1,

$$1 + 6 + 11 + 16 + .. + (5n - 4) = \frac{n \cdot (5n - 3)}{2}$$

12. Use mathematical induction to prove that for all integers n >= 3;

$$3 + +4 + 5 + .. + n = \frac{(n-2)(n+3)}{2}$$

13. [§5.2, Exercise #14 (*)] Prove the following statement using mathematical induction. For all integers n >= 0,

$$\sum_{i=1}^{n+1} i \cdot 2^{i} = n \cdot 2^{n+2} + 2$$

14. Prove the following statement using mathematical induction. For all integers n >= 2,

$$2^{n} \leq (n+1)!$$

15. Use mathematical induction to prove that for all integers n >= 0; $\mathbf{8^n - 1}$ is divisible by 7.

16. [§5.4, Exercise #1] Suppose a sequence a_1, a_2, .. is defined as follows. Prove (using Strong Mathematical Induction) that a_n is odd for all integers n >= 1.

$$a_1 = 1, a_2 = 3, a_k = a_{k-2} + 2a_{k-1}$$