CSC 400 Discrete Structures
Final Review

Exam Schedule and Format

- Date: 3/21 (Thu) for Section 802, and any day between 3/21 (Thu) and 3/23 (Sat) for Section 810.
- Location: Regular classroom (Lewis 1007); Individually arranged location for Section 810.
- Time: 2.5 hours -- 5:45 – 8:15 pm for Section 802; Proctored 2.5-hour exam for Section 810.

- Format: Pencil-and-paper. You write answers on the exam paper.
- No computer is allowed. But a ONE page note is allowed (max. both sides).
- A calculator is also allowed (including a smartphone, but if used only for calculation).
- Topics: Comprehensive from the beginning (Logic) till Counting Methods. But focus will be on the topics/sections after the midterm, which are:
  - Set Theory -- §6.1, §6.2, §6.3
  - Functions -- §7.1, §7.2
  - Relations -- §8.1, §8.2, §8.3
  - Algorithms -- §4.8
  - Recurrence Relations -- §5.6, §5.7
  - Counting Methods ---§9.1, §9.2, §9.3, §9.5

Review Questions

1. [§6.3, Exercise #6, p. 372] Prove or disprove: For all sets A and B, $\boldsymbol{A} \cap (\boldsymbol{A} \cup \boldsymbol{B}) = \boldsymbol{A}$.

2. [§7.2, Exercise #26, p. 415] Let D be the set of all finite subsets of positive integers, and define T : $Z^+ \to D$ by the rule: For all integers n, T (n) = the sum of the positive divisors of n.
   a) Is T one-to-one? Prove or give a counterexample.
   b) Is T onto? Prove or give a counterexample.

3. Determine which of the properties are true for all functions *f* from a set X and which are false for some function *f* . Justify your answers.
   a) For all subsets A and B of X, if $A \subseteq B$, then $f(A) \subseteq f(B)$.
   b) For all subsets A and B of X, $f(A \cup B) = f(A) \cup f(B)$.

4. [§8.1, Exercise #11, p. 448 (*)] Let A = {3, 4, 5} and B = {4, 5, 6} and let S be the "divides" relation. That is, for all $(x, y) \in A \times B$, $x \, S \, y \Leftrightarrow x \mid y$. State explicitly which ordered pairs are in S and S^{-1}.

5. Determine whether the relation R on the set of all real numbers, but excluding 0, is reflexive, symmetric, anti-symmetric, and/or transitive, where $(x,y) \in R$ if and only if
   a) $x + y = 0$
   b) $x = \pm\ y$
   c) $x = 2y$
   d) $xy \geq 0$

6. Let A be set of non-zero integers and let R be the relation on A×A defined by

   (a,b) R (c,d) whenever ad = bc.

   Prove that R is an equivalence relation.

7. [§8.3, Exercise #9, p. 475 (*)] The following relation R is an equivalence relation on the set A. Find the distinct equivalence classes of R.

   X = {−1, 0, 1} and A = $\wp$(X).

   R is defined on $\wp$(X) as follows: For all sets S and T in $\wp$(X), S **R** T ⇔ the sum of the elements in S equals the sum of the elements in T.

8. Use the Euclidean algorithm (Algorithm 4.8.2, p. 224) to find the greatest common divisor of 527 and 62. Show the values of variables a, b and r in each iteration.

9. [§5.7, Exercise #7, p. 315 (*)] Use iteration to guess an explicit formula.

   $e_k = 4e_{k-1} + 5$

   $e_0 = 2$

10. Use strong mathematical induction to verify the correctness of the formula you obtained in the previous question.

11. Let F_n be Fibonacci sequence where $F_0 = 0$, $F_1 = 1$ and $F_n = F_{n-1} + F_{n-2}$. Prove that $\sum_{k=1}^{n} F_k = F_{n+2} - 1 \quad \forall n \geq 1$.

12. How many positive integers between 5 and 31 which are:

    a) divisible by 3? Which integers are these?
    b) divisible by 4? Which integers are these?
    c) divisible by 3 and by 4? Which integers are these?

13. How many different functions are there from a set with 10 elements to sets with the following numbers of elements?

    a) 2 b) 3 c) 4 d) 5

14. A bowl contains 10 red balls and 10 blue balls. A woman selects balls at random without looking at them.

    a) How many balls must she select to be sure of having at least three balls of the same color?
    b) How many balls must she select to be sure of having at least three blue balls?

15. Seven women and nine men are in the faculty in the department.

    a) How many ways are there to select a committee of five members if at least one woman must be on the committee?
    b) How many ways are there to select a committee of five members if at least one woman and one man must be on the committee?
    c) How many ways are there to select a committee of five members if it must have more women than men?