Lecture Note #4 (Set Theory)
Exercises (partial)

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- Example 6.1.3 [Set Equality, p. 339]

Define sets A and B as follows:

A = {m ∈ Z | m = 2a for some integer a}
B = {n ∈ Z | n = 2b − 2 for some integer b}

Is A = B?

Part 1: Show A ⊆ B. [This part is in the lecture note.]

Part 2: Show B ⊆ A.

We must show that, ∀x, if x ∈ B, then x ∈ A.

Since x ∈ B, we can write x as x = 2b – 2, for some integer b.

Here we have x = 2b – 2 = 2 * (b - 1).

Let a = b – 1. Then we have x = 2 * (b - 1) = 2*a.

Since a is an integer (because b is an integer, and adding 1 makes an integer), x is an integer too.

By definition of A, we can conclude that x ∈ A.

By Part 1 and 2, we can conclude that A = B. Q.E.D.

- Exercise [Section 6.1, #31, p. 351]

Suppose A = {1, 2} and B = {2, 3}. Find each of the following:

- ℘ (A ∩ B)
- ℘ (A)
- ℘ (A ∪ B) = P({1,2,3})
  = { {}, {1}, {2}, {3}, {1,2}, {2,3}, {1,3}, {1,2,3} }
- ℘ (A × B) = P({ (1,2), (1,3), (2,2), (2,3) })
  = **{** {}, {(1,2)}, {(1,3)}, {(2,2)}, {(2,3)},
  {(1,2), (1,3)}, {(1,2), (2,2)}, {(1,2), (2,3)},
  {(1,3), (2,2)}, {(1,3), (2,3)}, {(2,2), (2,3)},
  {(1,2), (1,3), (2,2)}, {(1,2), (1,3), (2,3)}, { (1,3), (2,2), (2,3)},
  {(1,2), (1,3), (2,2), (2,3)} **}**

- Exercise #1 [Section 6.2, #3, p. 365]

The following is a proof that for all sets A, B, and C, if A ⊆ B and B ⊆ C, then A ⊆ C. Fill in the blanks.

… Will work on this in the class, but you can find an answer at the back of the textbook.

Proof: Suppose A, B, and C are sets and A ⊆ B and B ⊆ C. To show that A ⊆ C, we must show that every element in __(a) A__ is in __(b) C__. But given any element in A, that element is in __(c) B__ (because A ⊆ B), and so that element is also in __(d) C__ (because __(e) B ⊆ C__). Hence A ⊆ C.

- Exercise #2 [Section 6.2, #5, p. 365]

Prove that for all sets A and B, $(B - A) = B \cap A^c$.

Proof:

(1) Show $B - A \subseteq B \cap A^c$.

To show, we must show that, for all x, if x is in (B-A), then x is in $B \cap A^c$.

By definition of set difference, x is in B but x is not in A. So we know that x is in B and x is in A^c.
By definition of set intersection, we get x is in $(B \cap A^c)$.

(2) Show $B \cap A^c \subseteq B - A$.

To show, we must show that, for all x, if x is in $B \cap A^c$, then x is in (B-A).

By definition of set intersection, x is in B and x is in A^c. And by definition of set complement, x is in B but x is not in A. Then by definition of set difference, we get x is in (B-A).

By (1) and (2), we can conclude that $(B - A) = B \cap A^c$. Q.E.D.