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// Sample Solutions to Lecture Note #2 Exercise Questions
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6. Lecture note #2, Methods of Proof

Exercise a.

Proposition:
For all real numbers x and y, if x + y >= 2, then either x >= 1 or y >= 1.

Proof: By contradiction. Suppose the negation of the proposition is true, that is, x + y >= 2 and x < 1 and y < 1.

Since x < 1 and y < 1, we have x + y < 1 + 1 = 2, which gives x + y < 2.
This contradicts with the other assumption x + y >= 2.
Thefore the original proposition was true. QED.

Exercise b.[Section 4.6, Exercise #18, p. 206]

The textbook has a solution for this problem at the end of the book, but here is mine.

(a) 5 | n (or "5 divides n" or "n is divisible by 5").
(b) 5 | n^2 (or "5 divides n^2" or "n^2 is divisible by 5")
(c) 5k
(d) (5k)^2
(e) 5 | n^2

Exercise c. c. [Section 4.1, Exercise #47, p. 162]
Determine whether the statement is true or false. Justify your answer with a proof or a counterexample.

If a sum of two integers is even, then one of the summands is even.

Proof: The statement is false.
[Give a counterexample which satisfies the negation of the statement -- A sum of two integers is even, but both summans are odd.]

Take 1 and 3. The sum is 4, which is even, but both 1 and 3 are odd. QED.