Table 2.3.1 Valid Argument Forms

Modus Ponens	$p \to q$ p $\therefore q$	**Elimination**	**a.** $p \lor q$ $\sim q$ $\therefore p$ **b.** $p \lor q$ $\sim p$ $\therefore q$
Modus Tollens	$p \to q$ $\sim q$ $\therefore \sim p$	**Transitivity**	$p \to q$ $q \to r$ $\therefore p \to r$
Generalization	**a.** p $\therefore p \lor q$ **b.** q $\therefore p \lor q$	**Proof by Division into Cases**	$p \lor q$ $p \to r$ $q \to r$ $\therefore r$
Specialization	**a.** $p \land q$ $\therefore p$ **b.** $p \land q$ $\therefore q$		
Conjunction	p q $\therefore p \land q$	**Contradiction Rule**	$\sim p \to c$ $\therefore p$

Test Yourself

1. For an argument to be valid means that every argument of the same form whose premises ____ has a ____ conclusion.

2. For an argument to be invalid means that there is an argument of the same form whose premises ____ and whose conclusion ____.

3. For an argument to be sound means that it is ____ and its premises ____. In this case we can be sure that its conclusion ____.

Exercise Set 2.3

Use modus ponens or modus tollens to fill in the blanks in the arguments of 1–5 so as to produce valid inferences.

1. If $\sqrt{2}$ is rational, then $\sqrt{2} = a/b$ for some integers a and b.
It is not true that $\sqrt{2} = a/b$ for some integers a and b.
$\therefore$ ______________________.

2. If $1 - 0.99999\ldots$ is less than every positive real number, then it equals zero.
______________________.
$\therefore$ The number $1 - 0.99999\ldots$ equals zero.

3. If logic is easy, then I am a monkey's uncle.
I am not a monkey's uncle.
$\therefore$ ______________________.

4. If this figure is a quadrilateral, then the sum of its interior angles is 360°.
The sum of the interior angles of this figure is not 360°.
$\therefore$ ______________________.

5. If they were unsure of the address, then they would have telephoned.
______________________.
$\therefore$ They were sure of the address.

Use truth tables to determine whether the argument forms in 6–11 are valid. Indicate which columns represent the premises and which represent the conclusion, and include a sentence explaining how the truth table supports your answer. Your explanation should show that you understand what it means for a form of argument to be valid or invalid.

6. $p \to q$
$q \to p$
$\therefore p \lor q$

7. p
$p \to q$
$\sim q \lor r$
$\therefore r$

8. $p \lor q$
$p \to \sim q$
$p \to r$
$\therefore r$

9. $p \land q \to \sim r$
$p \lor \sim q$
$\sim q \to p$
$\therefore \sim r$

10. $p \to r$
$q \to r$
$\therefore p \vee q \to r$

11. $p \to q \vee r$
$\sim q \vee \sim r$
$\therefore \sim p \vee \sim r$

12. Use truth tables to show that the following forms of argument are invalid.
   a. $p \to q$
   q
   $\therefore p$
   (converse error)
   b. $p \to q$
   $\sim p$
   $\therefore \sim q$
   (inverse error)

Use truth tables to show that the argument forms referred to in 13–21 are valid. Indicate which columns represent the premises and which represent the conclusion, and include a sentence explaining how the truth table supports your answer. Your explanation should show that you understand what it means for a form of argument to be valid.

13. Modus tollens:
$$p \to q$$
$$\sim q$$
$$\therefore \sim p$$

14. Example 2.3.3(a)
15. Example 2.3.3(b)
16. Example 2.3.4(a)
17. Example 2.3.4(b)
18. Example 2.3.5(a)
19. Example 2.3.5(b)
20. Example 2.3.6
21. Example 2.3.7

Use symbols to write the logical form of each argument in 22 and 23, and then use a truth table to test the argument for validity. Indicate which columns represent the premises and which represent the conclusion, and include a few words of explanation showing that you understand the meaning of validity.

22. If Tom is not on team A, then Hua is on team B.
If Hua is not on team B, then Tom is on team A.
∴ Tom is not on team A or Hua is not on team B.

23. Oleg is a math major or Oleg is an economics major.
If Oleg is a math major, then Oleg is required to take Math 362.
∴ Oleg is an economics major or Oleg is not required to take Math 362.

Some of the arguments in 24–32 are valid, whereas others exhibit the converse or the inverse error. Use symbols to write the logical form of each argument. If the argument is valid, identify the rule of inference that guarantees its validity. Otherwise, state whether the converse or the inverse error is made.

24. If Jules solved this problem correctly, then Jules obtained the answer 2.
Jules obtained the answer 2.
∴ Jules solved this problem correctly.

25. This real number is rational or it is irrational.
This real number is not rational.
∴ This real number is irrational.

26. If I go to the movies, I won't finish my homework. If I don't finish my homework, I won't do well on the exam tomorrow.
∴ If I go to the movies, I won't do well on the exam tomorrow.

27. If this number is larger than 2, then its square is larger than 4.
This number is not larger than 2.
∴ The square of this number is not larger than 4.

28. If there are as many rational numbers as there are irrational numbers, then the set of all irrational numbers is infinite.
The set of all irrational numbers is infinite.
∴ There are as many rational numbers as there are irrational numbers.

29. If at least one of these two numbers is divisible by 6, then the product of these two numbers is divisible by 6.
Neither of these two numbers is divisible by 6.
∴ The product of these two numbers is not divisible by 6.

30. If this computer program is correct, then it produces the correct output when run with the test data my teacher gave me.
This computer program produces the correct output when run with the test data my teacher gave me.
∴ This computer program is correct.

31. Sandra knows Java and Sandra knows C++.
∴ Sandra knows C++.

32. If I get a Christmas bonus, I'll buy a stereo.
If I sell my motorcycle, I'll buy a stereo.
∴ If I get a Christmas bonus or I sell my motorcycle, then I'll buy a stereo.

33. Give an example (other than Example 2.3.11) of a valid argument with a false conclusion.

34. Give an example (other than Example 2.3.12) of an invalid argument with a true conclusion.

35. Explain in your own words what distinguishes a valid form of argument from an invalid one.

36. Given the following information about a computer program, find the mistake in the program.
   a. There is an undeclared variable or there is a syntax error in the first five lines.
   b. If there is a syntax error in the first five lines, then there is a missing semicolon or a variable name is misspelled.
   c. There is not a missing semicolon.
   d. There is not a misspelled variable name.

37. In the back of an old cupboard you discover a note signed by a pirate famous for his bizarre sense of humor and love of logical puzzles. In the note he wrote that he had hidden treasure somewhere on the property. He listed five true statements (a–e below) and challenged the reader to use them to figure out the location of the treasure.

a. If this house is next to a lake, then the treasure is not in the kitchen.
b. If the tree in the front yard is an elm, then the treasure is in the kitchen.
c. This house is next to a lake.
d. The tree in the front yard is an elm or the treasure is buried under the flagpole.
e. If the tree in the back yard is an oak, then the treasure is in the garage.

Where is the treasure hidden?

38. You are visiting the island described in Example 2.3.14 and have the following encounters with natives.

a. Two natives A and B address you as follows:
A says: Both of us are knights.
B says: A is a knave.
What are A and B?

b. Another two natives C and D approach you but only C speaks.
C says: Both of us are knaves.
What are C and D?

c. You then encounter natives E and F.
E says: F is a knave.
F says: E is a knave.
How many knaves are there?

H **d.** Finally, you meet a group of six natives, U, V, W, X, Y, and Z, who speak to you as follows:
U says: None of us is a knight.
V says: At least three of us are knights.
W says: At most three of us are knights.
X says: Exactly five of us are knights.
Y says: Exactly two of us are knights.
Z says: Exactly one of us is a knight.
Which are knights and which are knaves?

39. The famous detective Percule Hoirot was called in to solve a baffling murder mystery. He determined the following facts:

a. Lord Hazelton, the murdered man, was killed by a blow on the head with a brass candlestick.
b. Either Lady Hazelton or a maid, Sara, was in the dining room at the time of the murder.
c. If the cook was in the kitchen at the time of the murder, then the butler killed Lord Hazelton with a fatal dose of strychnine.
d. If Lady Hazelton was in the dining room at the time of the murder, then the chauffeur killed Lord Hazelton.
e. If the cook was not in the kitchen at the time of the murder, then Sara was not in the dining room when the murder was committed.
f. If Sara was in the dining room at the time the murder was committed, then the wine steward killed Lord Hazelton.

Is it possible for the detective to deduce the identity of the murderer from these facts? If so, who did murder Lord Hazelton? (Assume there was only one cause of death.)

40. Sharky, a leader of the underworld, was killed by one of his own band of four henchmen. Detective Sharp interviewed the men and determined that all were lying except for one. He deduced who killed Sharky on the basis of the following statements:

a. Socko: Lefty killed Sharky.
b. Fats: Muscles didn't kill Sharky.
c. Lefty: Muscles was shooting craps with Socko when Sharky was knocked off.
d. Muscles: Lefty didn't kill Sharky.

Who did kill Sharky?

In 41–44 a set of premises and a conclusion are given. Use the valid argument forms listed in Table 2.3.1 to deduce the conclusion from the premises, giving a reason for each step as in Example 2.3.8. Assume all variables are statement variables.

41.
a. $\sim p \vee q \to r$
b. $s \vee \sim q$
c. $\sim t$
d. $p \to t$
e. $\sim p \wedge r \to \sim s$
f. $\therefore \sim q$

42.
a. $p \vee q$
b. $q \to r$
c. $p \wedge s \to t$
d. $\sim r$
e. $\sim q \to u \wedge s$
f. $\therefore t$

43.
a. $\sim p \to r \wedge \sim s$
b. $t \to s$
c. $u \to \sim p$
d. $\sim w$
e. $u \vee w$
f. $\therefore \sim t$

44.
a. $p \to q$
b. $r \vee s$
c. $\sim s \to \sim t$
d. $\sim q \vee s$
e. $\sim s$
f. $\sim p \wedge r \to u$
g. $w \vee t$
h. $\therefore u \wedge w$

Answers for Test Yourself

1. are all true; true 2. are all true; is false 3. valid; are all true; is true