*Exercise Set 9.1**

1. Toss two coins 30 times and make a table showing the relative frequencies of 0, 1, and 2 heads. How do your values compare with those shown in Table 9.1.1?

2. In the example of tossing two quarters, what is the probability that at least one head is obtained? that coin *A* is a head? that coins *A* and *B* are either both heads or both tails?

In 3–6 use the sample space given in Example 9.1.1. Write each event as a set, and compute its probability.

3. The event that the chosen card is red and is not a face card.

4. The event that the chosen card is black and has an even number on it.

5. The event that the denomination of the chosen card is at least 10 (counting aces high).

6. The event that the denomination of the chosen card is at most 4 (counting aces high).

In 7–10, use the sample space given in Example 9.1.2. Write each of the following events as a set and compute its probability.

7. The event that the sum of the numbers showing face up is 8.

8. The event that the numbers showing face up are the same.

9. The event that the sum of the numbers showing face up is at most 6.

10. The event that the sum of the numbers showing face up is at least 9.

11. Suppose that a coin is tossed three times and the side showing face up on each toss is noted. Suppose also that on each toss heads and tails are equally likely. Let *HHT* indicate the outcome heads on the first two tosses and tails on the third, *THT* the outcome tails on the first and third tosses and heads on the second, and so forth.
   a. List the eight elements in the sample space whose outcomes are all the possible head–tail sequences obtained in the three tosses.
   b. Write each of the following events as a set and find its probability:
      (i) The event that exactly one toss results in a head.
      (ii) The event that at least two tosses result in a head.
      (iii) The event that no head is obtained.

12. Suppose that each child born is equally likely to be a boy or a girl. Consider a family with exactly three children. Let *BBG* indicate that the first two children born are boys and the third child is a girl, let *GBG* indicate that the first and third children born are girls and the second is a boy, and so forth.
   a. List the eight elements in the sample space whose outcomes are all possible genders of the three children.
   b. Write each of the events in the next column as a set and find its probability.
      (i) The event that exactly one child is a girl.
      (ii) The event that at least two children are girls.
      (iii) The event that no child is a girl.

13. Suppose that on a true/false exam you have no idea at all about the answers to three questions. You choose answers randomly and therefore have a 50–50 chance of being correct on any one question. Let *CCW* indicate that you were correct on the first two questions and wrong on the third, let *WCW* indicate that you were wrong on the first and third questions and correct on the second, and so forth.
   a. List the elements in the sample space whose outcomes are all possible sequences of correct and incorrect responses on your part.
   b. Write each of the following events as a set and find its probability:
      (i) The event that exactly one answer is correct.
      (ii) The event that at least two answers are correct.
      (iii) The event that no answer is correct.

14. Three people have been exposed to a certain illness. Once exposed, a person has a 50–50 chance of actually becoming ill.
   a. What is the probability that exactly one of the people becomes ill?
   b. What is the probability that at least two of the people become ill?
   c. What is the probability that none of the three people becomes ill?

15. When discussing counting and probability, we often consider situations that may appear frivolous or of little practical value, such as tossing coins, choosing cards, or rolling dice. The reason is that these relatively simple examples serve as models for a wide variety of more complex situations in the real world. In light of this remark, comment on the relationship between your answer to exercise 11 and your answers to exercises 12–14.

16. Two faces of a six-sided die are painted red, two are painted blue, and two are painted yellow. The die is rolled three times, and the colors that appear face up on the first, second, and third rolls are recorded.
   a. Let *BBR* denote the outcome where the color appearing face up on the first and second rolls is blue and the color appearing face up on the third roll is red. Because there are as many faces of one color as of any other, the outcomes of this experiment are equally likely. List all 27 possible outcomes.
   b. Consider the event that all three rolls produce different colors. One outcome in this event is *RBY* and another *RYB*. List all outcomes in the event. What is the probability of the event?

*For exercises with blue numbers or letters, solutions are given in Appendix B. The symbol ***H*** indicates that only a hint or a partial solution is given. The symbol ✶ signals that an exercise is more challenging than usual.

c. Consider the event that two of the colors that appear face up are the same. One outcome in this event is *RRB* and another is *RBR*. List all outcomes in the event. What is the probability of the event?

17. Consider the situation described in exercise 16.
a. Find the probability of the event that exactly one of the colors that appears face up is red.
b. Find the probability of the event that at least one of the colors that appears face up is red.

18. An urn contains two blue balls (denoted B_1 and B_2) and one white ball (denoted W). One ball is drawn, its color is recorded, and it is replaced in the urn. Then another ball is drawn, and its color is recorded.
a. Let B_1W denote the outcome that the first ball drawn is B_1 and the second ball drawn is W. Because the first ball is replaced before the second ball is drawn, the outcomes of the experiment are equally likely. List all nine possible outcomes of the experiment.
b. Consider the event that the two balls that are drawn are both blue. List all outcomes in the event. What is the probability of the event?
c. Consider the event that the two balls that are drawn are of different colors. List all outcomes in the event. What is the probability of the event?

19. An urn contains two blue balls (denoted B_1 and B_2) and three white balls (denoted W_1, W_2, and W_3). One ball is drawn, its color is recorded, and it is replaced in the urn. Then another ball is drawn and its color is recorded.
a. Let B_1W_2 denote the outcome that the first ball drawn is B_1 and the second ball drawn is W_2. Because the first ball is replaced before the second ball is drawn, the outcomes of the experiment are equally likely. List all 25 possible outcomes of the experiment.
b. Consider the event that the first ball that is drawn is blue. List all outcomes in the event. What is the probability of the event?
c. Consider the event that only white balls are drawn. List all outcomes in the event. What is the probability of the event?

20. Refer to Example 9.1.3. Suppose you are appearing on a game show with a prize behind one of five closed doors: A, B, C, D, and E. If you pick the right door, you win the prize. You pick door A. The game show host then opens one of the other doors and reveals that there is no prize behind it. Then the host gives you the option of staying with your original choice of door A or switching to one of the other doors that is still closed.
a. If you stick with your original choice, what is the probability that you will win the prize?
b. If you switch to another door, what is the probability that you will win the prize?

21. a. How many positive two-digit integers are multiples of 3?
b. What is the probability that a randomly chosen positive two-digit integer is a multiple of 3?
c. What is the probability that a randomly chosen positive two-digit integer is a multiple of 4?

22. a. How many positive three-digit integers are multiples of 6?
b. What is the probability that a randomly chosen positive three-digit integer is a multiple of 6?
c. What is the probability that a randomly chosen positive three-digit integer is a multiple of 7?

23. Suppose $A[1], A[2], A[3], \ldots, A[n]$ is a one-dimensional array and $n \geq 50$.
a. How many elements are in the array?
b. How many elements are in the subarray

$$A[4], A[5], \ldots, A[39]?$$

c. If $3 \leq m \leq n$, what is the probability that a randomly chosen array element is in the subarray

$$A[3], A[4], \ldots, A[m]?$$

d. What is the probability that a randomly chosen array element is in the subarray shown below if $n = 39$?

$$A[\lfloor n/2 \rfloor], A[\lfloor n/2 \rfloor + 1], \ldots, A[n]$$

24. Suppose $A[1], A[2], \ldots, A[n]$ is a one-dimensional array and $n \geq 2$. Consider the subarray

$$A[1], A[2], \ldots, A[\lfloor n/2 \rfloor].$$

a. How many elements are in the subarray (i) if n is even? and (ii) if n is odd?
b. What is the probability that a randomly chosen array element is in the subarray (i) if n is even? and (ii) if n is odd?

25. Suppose $A[1], A[2], \ldots, A[n]$ is a one-dimensional array and $n \geq 2$. Consider the subarray

$$A[\lfloor n/2 \rfloor], A[\lfloor n/2 \rfloor + 1], \ldots, A[n].$$

a. How many elements are in the subarray (i) if n is even? and (ii) if n is odd?
b. What is the probability that a randomly chosen array element is in the subarray (i) if n is even? and (ii) if n is odd?

26. What is the 27th element in the one-dimensional array $A[42], A[43], \ldots, A[100]$?

27. What is the 62nd element in the one-dimensional array $B[29], B[30], \ldots, B[100]$?

28. If the largest of 56 consecutive integers is 279, what is the smallest?

29. If the largest of 87 consecutive integers is 326, what is the smallest?

30. How many even integers are between 1 and 1,001?

31. How many integers that are multiples of 3 are between 1 and 1,001?

32. A certain non-leap year has 365 days, and January 1 occurs on a Monday.
a. How many Sundays are in the year?
b. How many Mondays are in the year?

✶ 33. Prove Theorem 9.1.1. (Let m be any integer and prove the theorem by mathematical induction on n.)