in $\underline{\ (4)\ }$ (by definition of $A - B$). In particular, such an element is in A.

b. **Proof:** Suppose A and B are any sets and $x \in A - B$. *[We must show that* $\underline{\ (1)\ }$*.]* By definition of set difference, $x \in \underline{\ (2)\ }$ and $x \notin \underline{\ (3)\ }$. In particular, $x \in \underline{\ (4)\ }$ *[which is what was to be shown].*

3. The following is a proof that for all sets A, B, and C, if $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$. Fill in the blanks.

   Proof: Suppose A, B, and C are sets and $A \subseteq B$ and $B \subseteq C$. To show that $A \subseteq C$, we must show that every element in $\underline{\ (a)\ }$ is in $\underline{\ (b)\ }$. But given any element in A, that element is in $\underline{\ (c)\ }$ (because $A \subseteq B$), and so that element is also in $\underline{\ (d)\ }$ (because $\underline{\ (e)\ }$). Hence $A \subseteq C$.

4. The following is a proof that for all sets A and B, if $A \subseteq B$, then $A \cup B \subseteq B$. Fill in the blanks.

   Proof: Suppose A and B are any sets and $A \subseteq B$. *[We must show that* $\underline{\ (a)\ }$*.]* Let $x \in \underline{\ (b)\ }$. *[We must show that* $\underline{\ (c)\ }$*.]* By definition of union, $x \in \underline{\ (d)\ }$ $\underline{\ (e)\ }$ $x \in \underline{\ (f)\ }$. In case $x \in \underline{\ (g)\ }$, then since $A \subseteq B$, $x \in \underline{\ (h)\ }$. In case $x \in B$, then clearly $x \in B$. So in either case, $x \in \underline{\ (i)\ }$ *[as was to be shown].*

5. Prove that for all sets A and B, $(B - A) = B \cap A^c$.

H 6. The following is a proof that for any sets A, B, and C, $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$. Fill in the blanks.

   Proof: Suppose A, B, and C are any sets.

   (1) ***Proof that*** $\boldsymbol{A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)}$**:**
   Let $x \in A \cap (B \cup C)$. *[We must show that* $x \in \underline{\ (a)\ }$*.]* By definition of intersection, $x \in \underline{\ (b)\ }$ and $x \in \underline{\ (c)\ }$. Thus $x \in A$ and, by definition of union, $x \in B$ or $\underline{\ (d)\ }$.

   Case 1 **(*x* ∈ *A and x* ∈ *B*):** In this case, by definition of intersection, $x \in \underline{\ (e)\ }$, and so, by definition of union, $x \in (A \cap B) \cup (A \cap C)$.

   Case 2 **(*x* ∈ *A and x* ∈ *C*):** In this case, $\underline{\ (f)\ }$.
   Hence in either case, $x \in (A \cap B) \cup (A \cap C)$ *[as was to be shown].*
   [So $A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$ *by definition of subset.]*

   (2) $\boldsymbol{(A \cap B) \cup (A \cap C) \subseteq A \cap (B \cup C)}$**:**
   Let $x \in (A \cap B) \cup (A \cap C)$. *[We must show that* $\underline{\ (a)\ }$*.]* By definition of union, $x \in A \cap B$ $\underline{\ (b)\ }$ $x \in A \cap C$.

   Case 1 **(*x* ∈ *A* ∩ *B*):** In this case, by definition of intersection, $x \in A$ $\underline{\ (c)\ }$ $x \in B$. Since $x \in B$, then by definition of union, $x \in B \cup C$. Hence $x \in A$ and $x \in B \cup C$, and so, by definition of intersection, $x \in \underline{\ (d)\ }$.

   Case 2 **(*x* ∈ *A* ∩ *C*):** In this case, $\underline{\ (e)\ }$.
   In either case, $x \in A \cap (B \cup C)$ *[as was to be shown]. [Thus* $(A \cap B) \cup (A \cap C) \subseteq A \cap (B \cup C)$ *by definition of subset.]*

   (3) ***Conclusion*****:** *[Since both subset relations have been proved, it follows, by definition of set equality, that* $\underline{\ (a)\ }$*.]*

Use an element argument to prove each statement in 7–19. Assume that all sets are subsets of a universal set U.

H 7. For all sets A and B, $(A \cap B)^c = A^c \cup B^c$.

8. For all sets A and B, $(A \cap B) \cup (A \cap B^c) = A$.

H 9. For all sets A, B, and C,
$$(A - B) \cup (C - B) = (A \cup C) - B.$$

10. For all sets A, B, and C,
$$(A - B) \cap (C - B) = (A \cap C) - B.$$

H 11. For all sets A and B, $A \cup (A \cap B) = A$.

12. For all sets A, $A \cup \emptyset = A$.

13. For all sets A, B, and C, if $A \subseteq B$ then $A \cap C \subseteq B \cap C$.

14. For all sets A, B, and C, if $A \subseteq B$ then $A \cup C \subseteq B \cup C$.

15. For all sets A and B, if $A \subseteq B$ then $B^c \subseteq A^c$.

H 16. For all sets A, B, and C, if $A \subseteq B$ and $A \subseteq C$ then $A \subseteq B \cap C$.

17. For all sets A, B, and C, if $A \subseteq C$ and $B \subseteq C$ then $A \cup B \subseteq C$.

18. For all sets A, B, and C,
$$A \times (B \cup C) = (A \times B) \cup (A \times C).$$

19. For all sets A, B, and C,
$$A \times (B \cap C) = (A \times B) \cap (A \times C).$$

20. Find the mistake in the following "proof" that for all sets A, B, and C, if $A \subseteq B$ and $B \subseteq C$ then $A \subseteq C$.

   "**Proof:** Suppose A, B, and C are sets such that $A \subseteq B$ and $B \subseteq C$. Since $A \subseteq B$, there is an element x such that $x \in A$ and $x \in B$. Since $B \subseteq C$, there is an element x such that $x \in B$ and $x \in C$. Hence there is an element x such that $x \in A$ and $x \in C$ and so $A \subseteq C$."

H 21. Find the mistake in the following "proof."

   "**Theorem:**" For all sets A and B, $A^c \cup B^c \subseteq (A \cup B)^c$.

   "**Proof:** Suppose A and B are sets, and $x \in A^c \cup B^c$. Then $x \in A^c$ or $x \in B^c$ by definition of union. It follows that $x \notin A$ or $x \notin B$ by definition of complement, and so $x \notin A \cup B$ by definition of union. Thus $x \in (A \cup B)^c$ by definition of complement, and hence $A^c \cup B^c \subseteq (A \cup B)^c$."

22. Find the mistake in the following "proof" that for all sets A and B, $(A - B) \cup (A \cap B) \subseteq A$.

   "**Proof:** Suppose A and B are sets, and suppose $x \in (A - B) \cup (A \cap B)$. If $x \in A$ then $x \in A - B$. Then, by definition of difference, $x \in A$ and $x \notin B$. Hence $x \in A$, and so $(A - B) \cup (A \cap B) \subseteq A$ by definition of subset."