

## Exercise Set 1.2

1. Which of the following sets are equal?

$$\begin{array}{ll} A = \{a, b, c, d\} & B = \{d, e, a, c\} \\ C = \{d, b, a, c\} & D = \{a, a, d, e, c, e\} \end{array}$$

2. Write in words how to read each of the following out loud.

- $\{x \in \mathbf{R}^+ \mid 0 < x < 1\}$
- $\{x \in \mathbf{R} \mid x \leq 0 \text{ or } x \geq 1\}$
- $\{n \in \mathbf{Z} \mid n \text{ is a factor of } 6\}$
- $\{n \in \mathbf{Z}^+ \mid n \text{ is a factor of } 6\}$

- Is  $4 = \{4\}$ ?
  - How many elements are in the set  $\{3, 4, 3, 5\}$ ?
  - How many elements are in the set  $\{1, \{1\}, \{1, \{1\}\}\}$ ?
- Is  $2 \in \{2\}$ ?
  - How many elements are in the set  $\{2, 2, 2, 2\}$ ?
  - How many elements are in the set  $\{0, \{0\}\}$ ?
  - Is  $\{0\} \in \{\{0\}, \{1\}\}$ ?
  - Is  $0 \in \{\{0\}, \{1\}\}$ ?

- H 5. Which of the following sets are equal?

$$\begin{array}{l} A = \{0, 1, 2\} \\ B = \{x \in \mathbf{R} \mid -1 \leq x < 3\} \\ C = \{x \in \mathbf{R} \mid -1 < x < 3\} \\ D = \{x \in \mathbf{Z} \mid -1 < x < 3\} \\ E = \{x \in \mathbf{Z}^+ \mid -1 < x < 3\} \end{array}$$

- H 6. For each integer  $n$ , let  $T_n = \{n, n^2\}$ . How many elements are in each of  $T_2$ ,  $T_{-3}$ ,  $T_1$  and  $T_0$ ? Justify your answers.

7. Use the set-roster notation to indicate the elements in each of the following sets.

- $S = \{n \in \mathbf{Z} \mid n = (-1)^k, \text{ for some integer } k\}$ .
- $T = \{m \in \mathbf{Z} \mid m = 1 + (-1)^i, \text{ for some integer } i\}$ .

- $U = \{r \in \mathbf{Z} \mid 2 \leq r \leq -2\}$
- $V = \{s \in \mathbf{Z} \mid s > 2 \text{ or } s < 3\}$
- $W = \{t \in \mathbf{Z} \mid 1 < t < -3\}$
- $X = \{u \in \mathbf{Z} \mid u \leq 4 \text{ or } u \geq 1\}$

8. Let  $A = \{c, d, f, g\}$ ,  $B = \{f, j\}$ , and  $C = \{d, g\}$ . Answer each of the following questions. Give reasons for your answers.

- Is  $B \subseteq A$ ?
- Is  $C \subseteq A$ ?
- Is  $C \subseteq C$ ?
- Is  $C$  a proper subset of  $A$ ?

- Is  $3 \in \{1, 2, 3\}$ ?
  - Is  $\{2\} \in \{1, 2\}$ ?
  - Is  $1 \in \{1\}$ ?
  - Is  $\{1\} \subseteq \{1, 2\}$ ?
- Is  $1 \subseteq \{1\}$ ?
  - Is  $\{3\} \in \{1, \{2\}, \{3\}\}$ ?
  - Is  $\{2\} \subseteq \{1, \{2\}, \{3\}\}$ ?
  - Is  $1 \in \{\{1\}, 2\}$ ?
  - Is  $\{1\} \subseteq \{1, \{2\}\}$ ?
  - Is  $\{1\} \subseteq \{1\}$ ?

- Is  $((-2)^2, -2^2) = (-2^2, (-2)^2)$ ?
  - Is  $(5, -5) = (-5, 5)$ ?
  - Is  $(8 - 9, \sqrt[3]{-1}) = (-1, -1)$ ?
  - Is  $(\frac{-2}{-4}, (-2)^3) = (\frac{3}{6}, -8)$ ?

11. Let  $A = \{w, x, y, z\}$  and  $B = \{a, b\}$ . Use the set-roster notation to write each of the following sets, and indicate the number of elements that are in each set:

- $A \times B$
- $B \times A$
- $A \times A$
- $B \times B$

12. Let  $S = \{2, 4, 6\}$  and  $T = \{1, 3, 5\}$ . Use the set-roster notation to write each of the following sets, and indicate the number of elements that are in each set:

- $S \times T$
- $T \times S$
- $S \times S$
- $T \times T$

## Answers for Test Yourself

- does not matter
- the set of all real numbers
- the set of all integers
- the set of all rational numbers
- the set of all  $x$  such that  $P(x)$
- every element in  $A$  is an element in  $B$
- the set of all ordered pairs  $(a, b)$  where  $a$  is in  $A$  and  $b$  is in  $B$

## 1.3 The Language of Relations and Functions

*Mathematics is a language.* — Josiah Willard Gibbs (1839–1903)

There are many kinds of relationships in the world. For instance, we say that two people are related by blood if they share a common ancestor and that they are related by marriage if one shares a common ancestor with the spouse of the other. We also speak of the relationship between student and teacher, between people who work for the same employer, and between people who share a common ethnic background.

Similarly, the objects of mathematics may be related in various ways. A set  $A$  may be said to be related to a set  $B$  if  $A$  is a subset of  $B$ , or if  $A$  is not a subset of  $B$ , or if  $A$  and  $B$  have at least one element in common. A number  $x$  may be said to be related to a number  $y$  if  $x < y$ , or if  $x$  is a factor of  $y$ , or if  $x^2 + y^2 = 1$ . Two identifiers in a computer