Work and Energy

Energy: a property that mechanical (and other) systems have because of *motion* or *position*.

(Not a precise definition!)

Energy may be transferred into or out of a system that has boundaries. This is WORK.

Work is a *process*. It is an “event” that changes the energy of a system.

Two questions:

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2. **What is effect of applying a force over a time interval Δt?**

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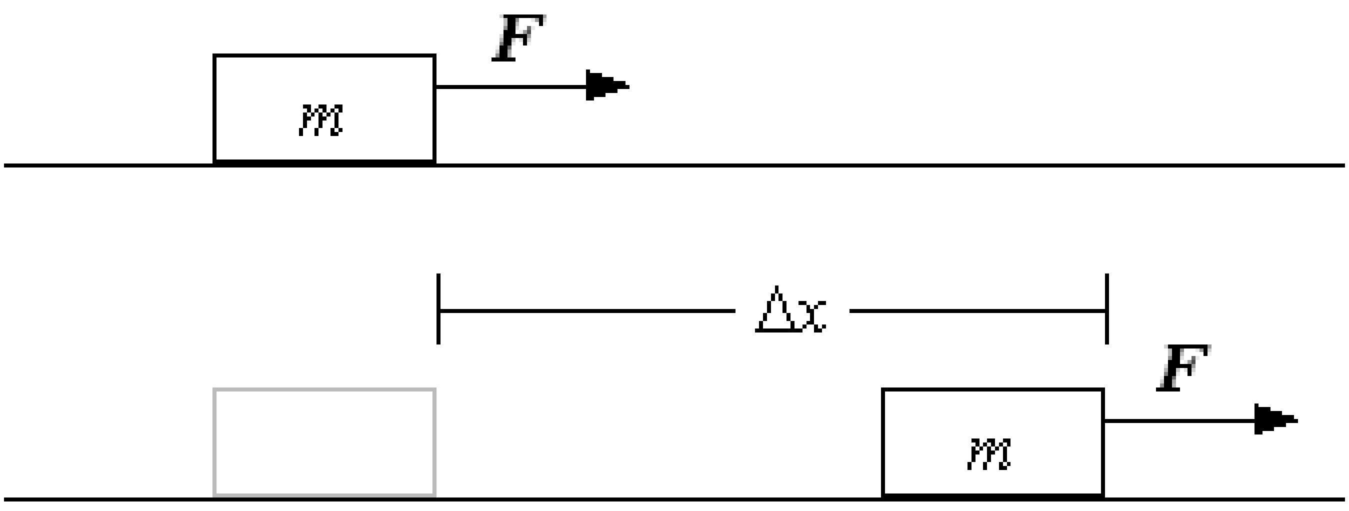
We will look at the first question.

An example:

A block of mass *m* on a frictionless surface is pulled by a constant horizontal force *F* through a displacement Δx.



Apply Newton's second law:

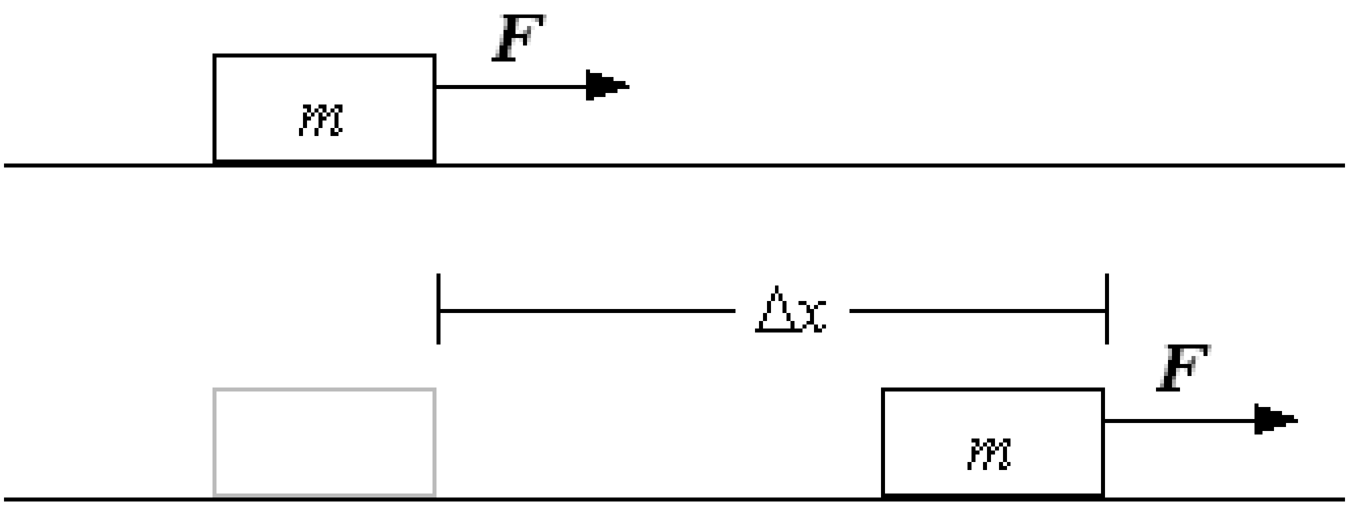
$$F\Delta x = ma\Delta x$$

If F is constant, then a is constant and

$$a = \frac{\upsilon^2}{2\Delta x}$$

Then

$$F\Delta x = m\frac{\upsilon^2}{2\Delta x}\Delta x = \frac{m\upsilon^2}{2}$$



The quantity $\frac{1}{2}m\upsilon^2$ **is the *KINETIC ENERGY* of the object.**

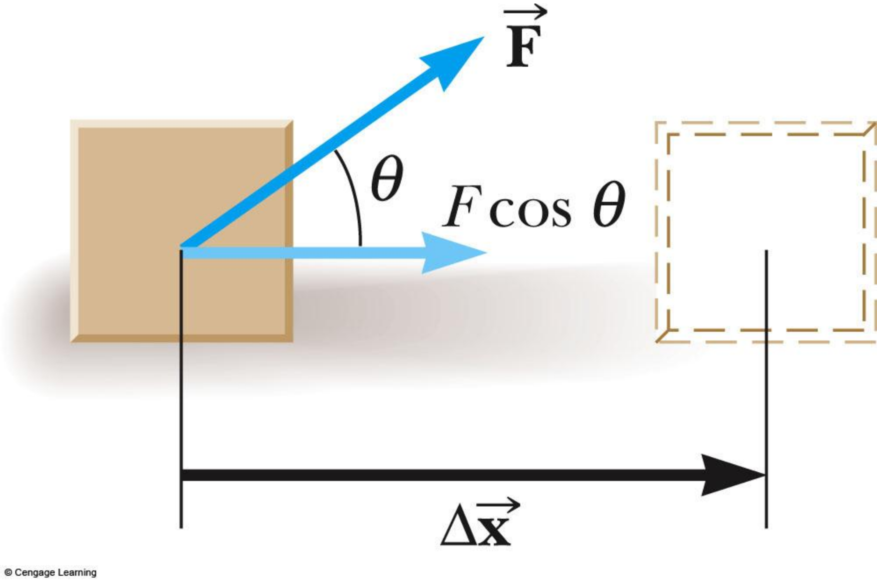
The product $F\Delta x$ **is *MECHANICAL WORK*.**

Kinetic energy (*KE*) and work (*W*) are scalar quantities.

A more general expression for work:

$W = (F\cos\theta)\Delta x$

F is the magnitude of the force.



Δx is the magnitude of the object's displacement.

θ is the angle between the force vector and the displacement vector.

Units of Work

SI: Newton·meter = Joule

N·m = J

$J = kg \cdot m^2/s^2$

US Customary: foot·pound (ft-lb)

The work done by a force is zero when the force is perpendicular to the displacement.

($\cos 90° = 0$)

If there are multiple forces acting on an object, the total work done is the *algebraic sum* of the amount of work done by each force.

Work, Energy and Algebraic Signs

KE is always positive.

ΔKE may be positive or negative.

W is positive if $F\cos\theta$ and Δx are in the same direction.

W is negative if $F\cos\theta$ and Δx are in the opposite direction.

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Separate (lift) a ball from the surface of the earth.

Work is done on the ball-earth system.

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What if the ball falls a distance Δy?

There is a *negative* change in PE_{grav} and a *positive* change in KE of the same magnitude, or:

$$\Delta E_{system} = \Delta PE + \Delta KE = -mg\Delta y + m\upsilon^2/2 = 0$$

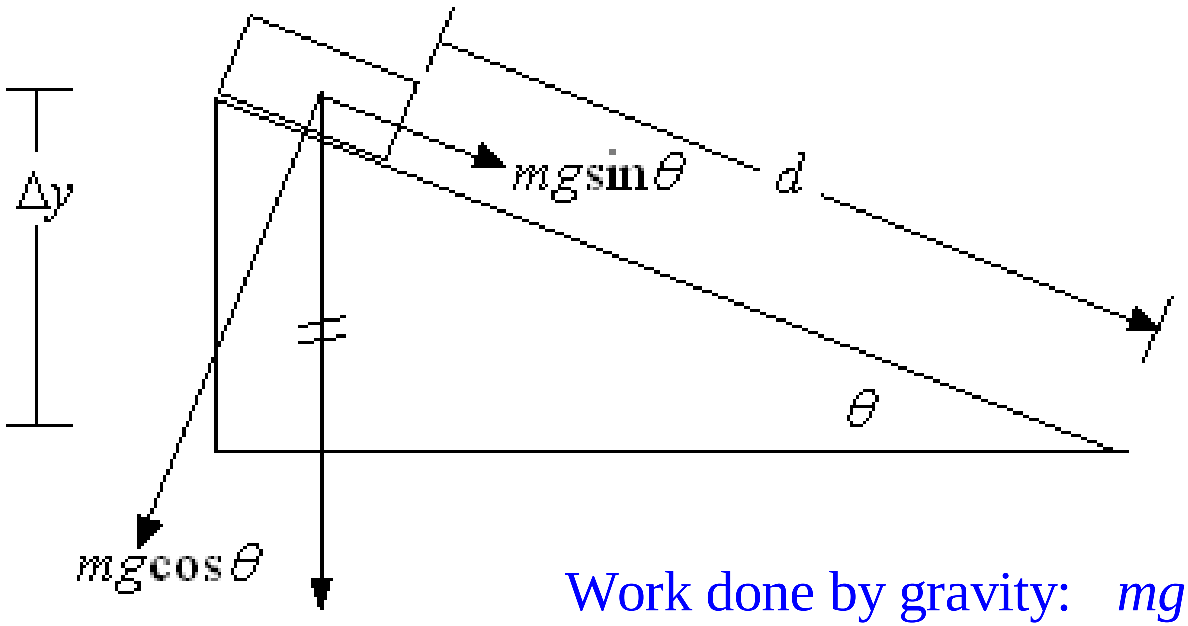
Conservative Forces

A force is conservative if the work it does on an object moving between two points is independent of the path the objects take between the points.

The work depends only upon the initial and final positions of the object.

Any conservative force can have a potential energy function associated with it.

Example: Block down an incline, no friction.



Work done by gravity: $mg\sin\theta \; d = \Delta KE$

$\Delta PE = 0 - mg\Delta y$

For a conservative force: $\Delta KE + \Delta PE = 0$

$\sin\theta = \frac{\Delta y}{d}$, so $mg\sin\theta \; d = \left(mg \frac{\Delta y}{\cancel{d}} \right) \cancel{d} = mg\Delta y$

$\Delta KE + \Delta PE = mg\Delta y - mg\Delta y = 0$

Total mechanical energy is conserved and the work done by gravity is independent of the path.

Nonconservative Forces

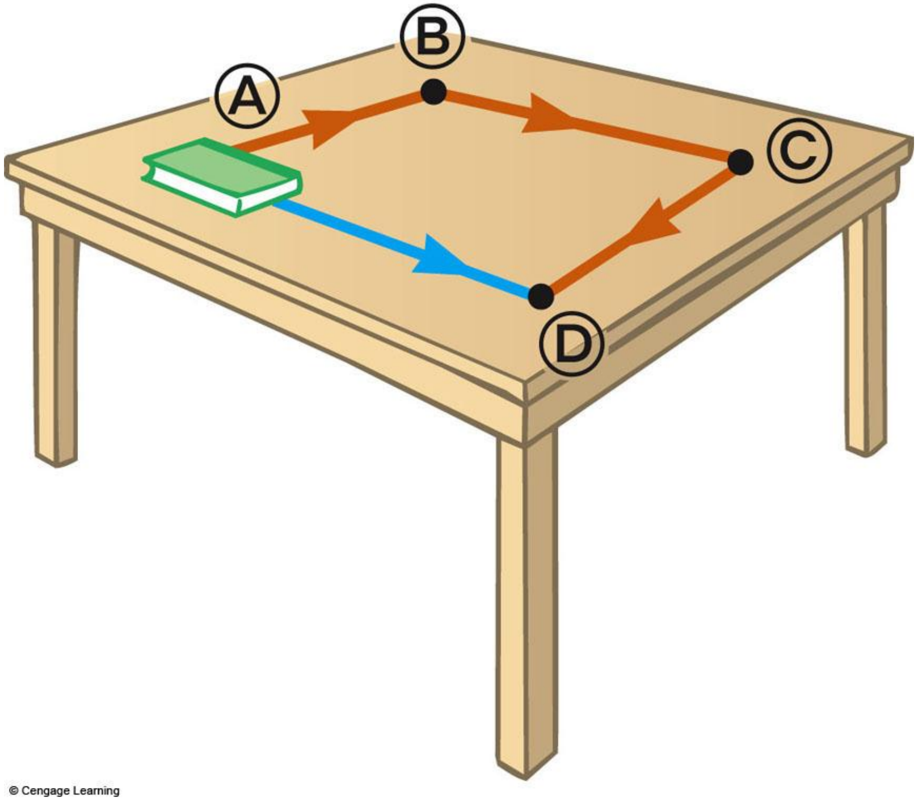
A force is nonconservative if the work it does on an object depends on the path taken by the object between its final and starting points.

Examples of nonconservative forces:

Kinetic friction, air drag, propulsive forces

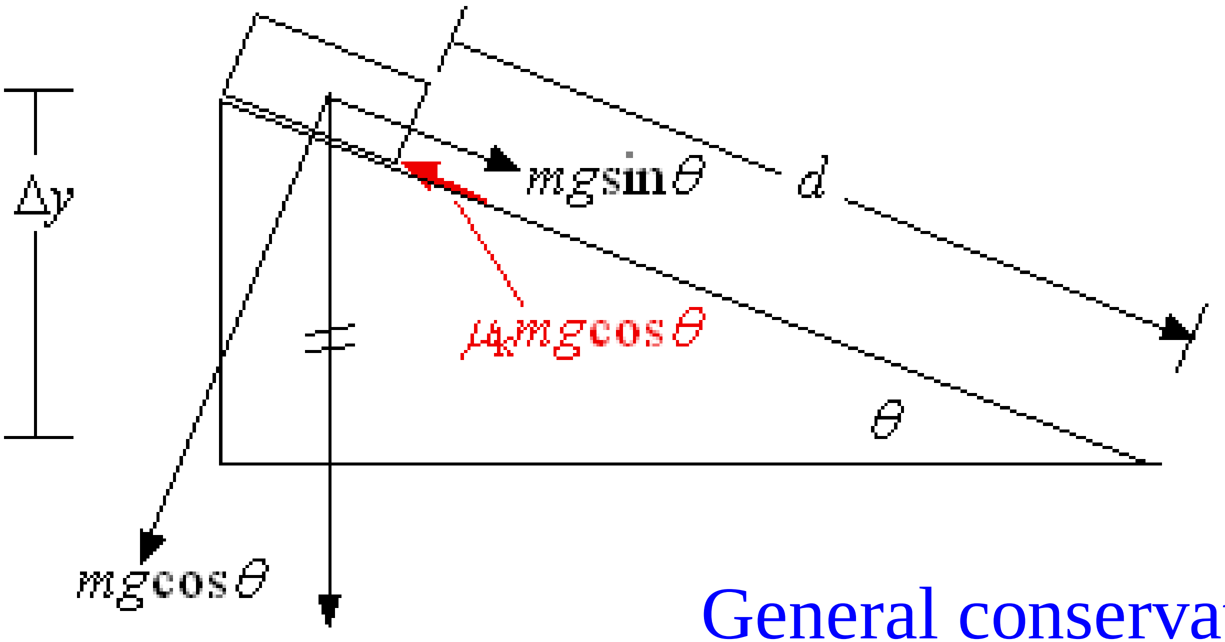
Friction Depends on the Path

The blue path is shorter than the red path.



The work required is less on the blue path than on the red path. Friction depends on the path and so is a non-conservative force.

Example: Block down an incline with friction.



General conservation of energy:

$$W_{\text{nc}} = \Delta KE + \Delta PE$$

Here, $W_{\text{nc}} = -\mu_{\text{k}} mg\cos\theta \cdot d$

W_{nc} appears as thermal energy.

(Block and ramp get warmer.)

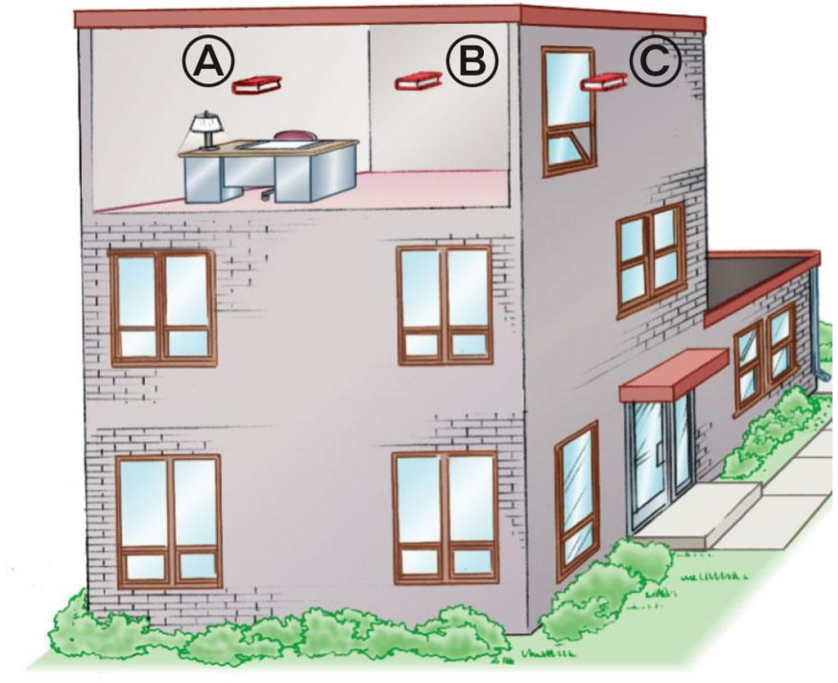
Reference Levels for Gravitational Potential Energy

A location where the gravitational potential energy is zero must be chosen for each problem.

- **The choice is arbitrary since the *change* in the potential energy is the important quantity.**
- **Choose a convenient location for the zero reference height, often the Earth's surface.**
  - **It may be some other point suggested by the problem.**
- **Once the position is chosen, it must remain fixed for the entire problem.**

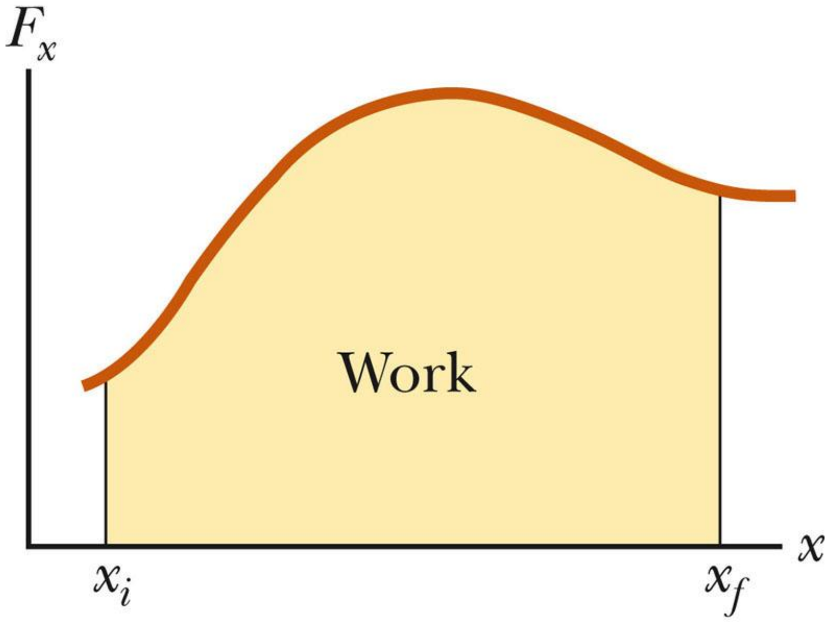
Reference Levels

At location A, the desk may be the convenient reference level.
At location B, the floor could be used.
At location C, the ground would be the most logical reference level.
Still, the choice is arbitrary.



Work Done by Varying Forces

The work done by a variable force acting on an object that undergoes a displacement is equal to the area under the graph of F_x versus x.



(b)

Example: Energy stored in a spring.

Potential Energy Stored in a Spring

Hooke's Law **for an ideal spring:**

$$F = -kx$$

F **is the restoring force.**

F **is in the opposite direction of the stretch or compression distance** ***x*****.**

k **is the elastic constant or spring constant. It depends on how the spring was formed, the material it is made from, thickness of the wire, etc.**

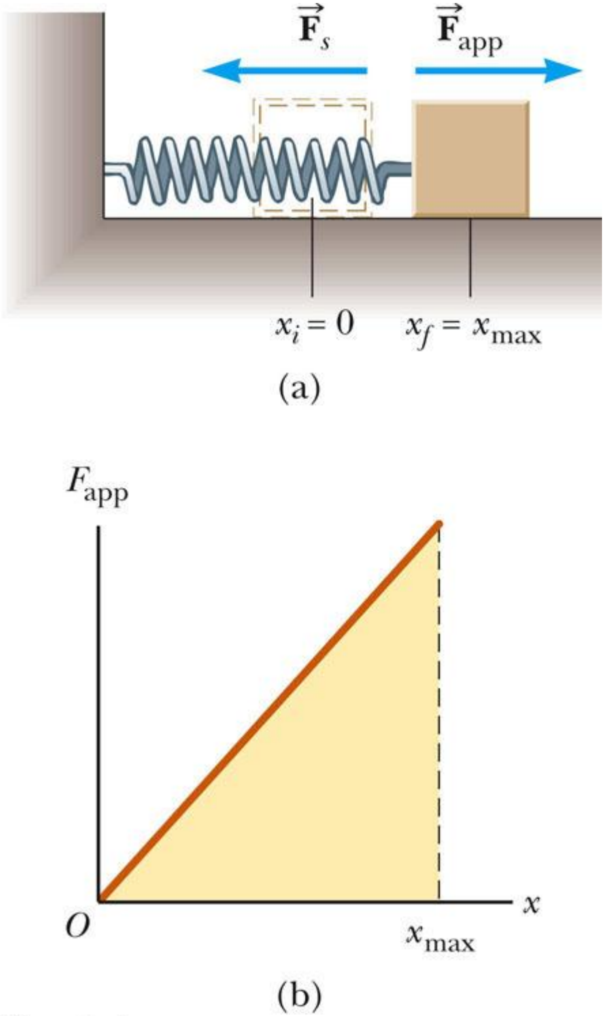
The spring is slowly stretched from 0 to x_{max}.

$$\vec{F}_{applied} = -\vec{F}_s = kx$$

The work done to stretch the spring is the area of the triangle.

$$W = \frac{1}{2}kx^2 = PE_s$$

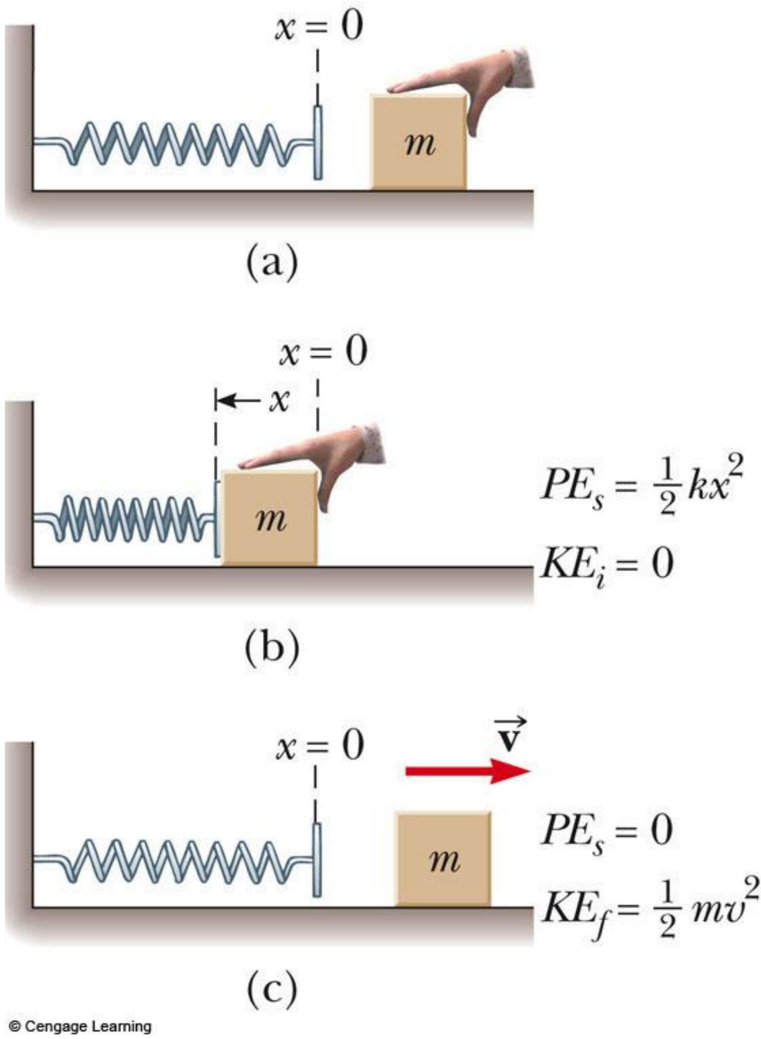
This is the elastic potential energy stored in the spring.



(a) The spring is in equilibrium, neither stretched or compressed.

(b) The spring is compressed, storing potential energy.

(c) The block is released and the potential energy is transformed to kinetic energy of the block.



Work-Energy Theorem Including a Spring

$$W_{\mathrm{nc}} = \left(KE_{\mathrm{f}} - KE_i\right) + \left(PE_{\mathrm{gf}} - PE_{gi}\right) + \left(PE_{\mathrm{sf}} - PE_{si}\right)$$

Power…

…is the rate of energy transfer or the rate of doing work.

$$\bar{\mathscr{P}} = \frac{W}{t} = F\bar{\upsilon}$$

Power Units

SI unit: 1 watt (W) = 1 J/s = 1 kg·m^2/s^2

US Customary unit is the horsepower (hp)

1 hp = 550 ft·lb/s = 746 W