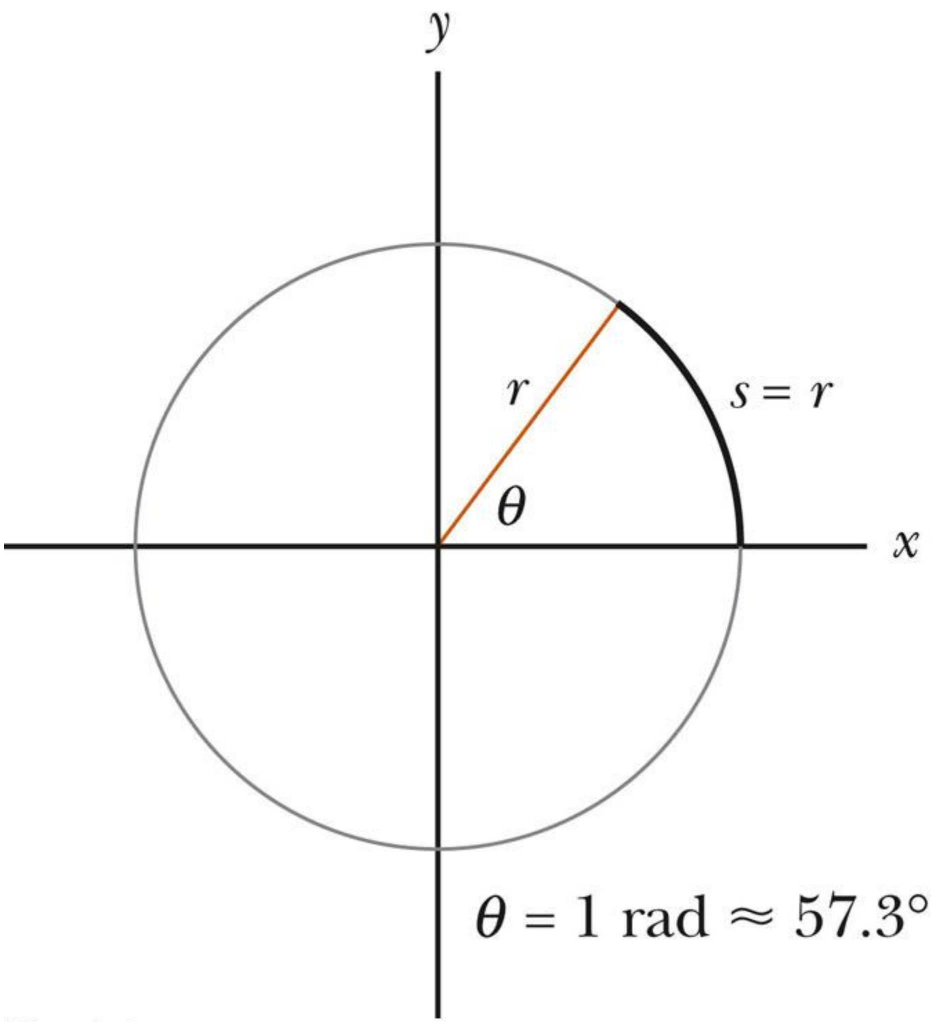
Rotational Mechanics - 1

The Radian

The radian is a unit of angular measure. The radian can be defined as the arc length *s* along a circle divided by the radius *r*.

$$\theta = \frac{s}{r}$$



Comparing degrees and radians

$$1\ \text{rad} = \frac{360^\circ}{2\pi} = 57.3^\circ$$

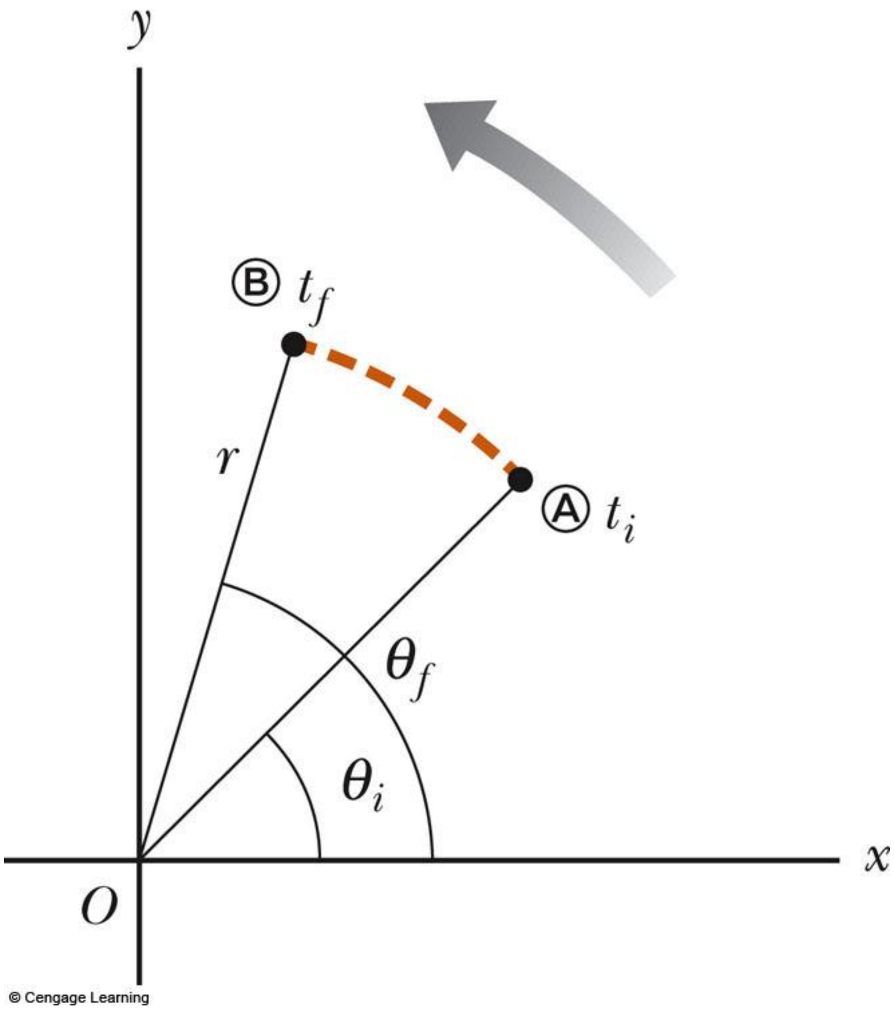
Relating degrees and radians

$$\frac{\theta_{\text{rad}}}{\pi\ \text{rad}} = \frac{\theta^\circ}{180^\circ}$$

Average Angular Speed

The average angular speed, ω, of a rotating rigid object is the ratio of the angular displacement to the time interval

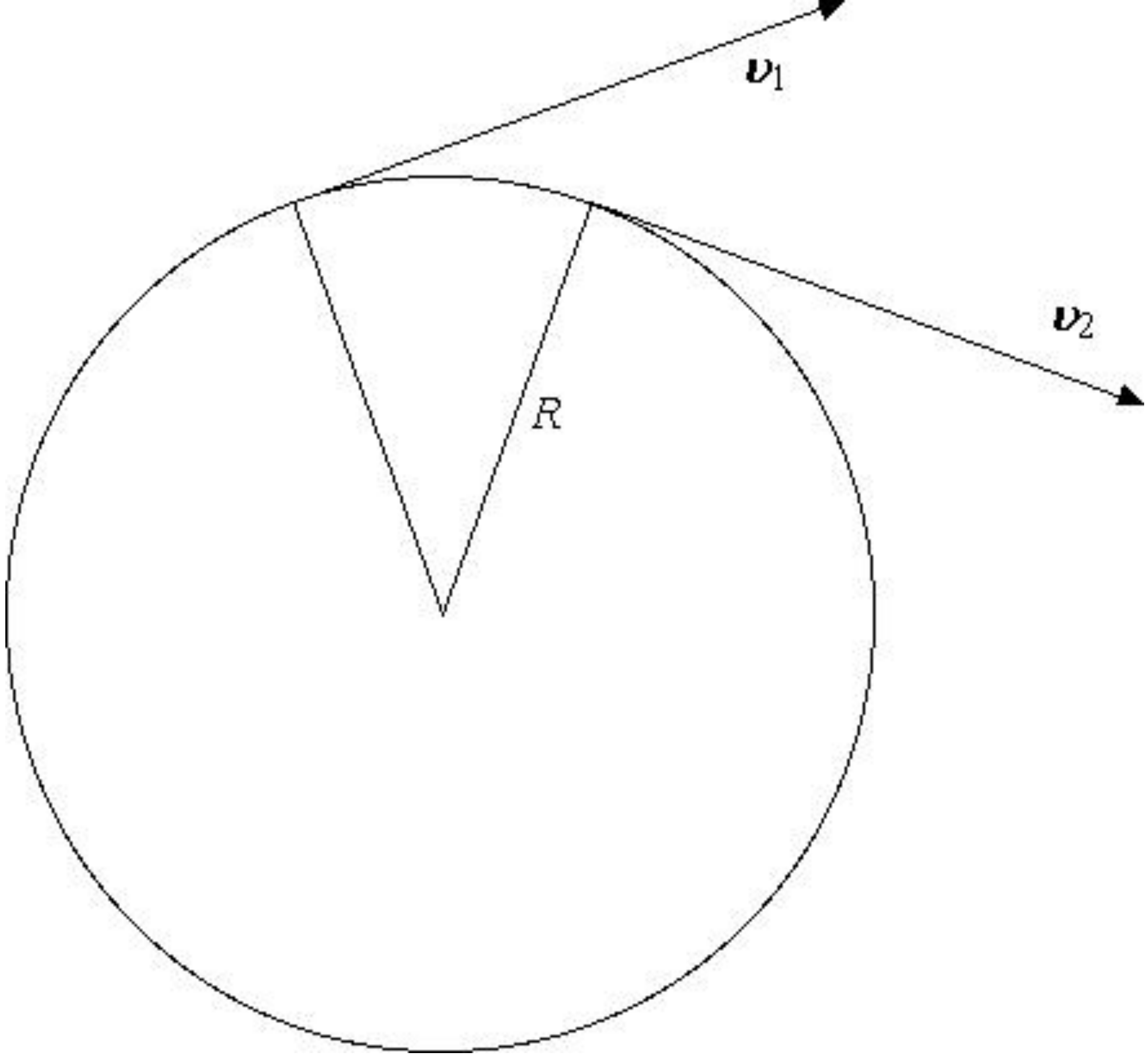
$$\omega_{\mathrm{av}} = \frac{\theta_{\mathrm{f}} - \theta_{\mathrm{i}}}{t_{\mathrm{f}} - t_{\mathrm{i}}} = \frac{\Delta\theta}{\Delta t}$$



Instantaneous **angular speed is defined as the limit of the average speed as the time interval approaches zero**

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta \theta}{\Delta t}$$

Units of angular speed are radians/sec (rad/s).
Speed is positive if θ is increasing (counterclockwise rotation).
Speed is negative if θ is decreasing (clockwise rotation).



A particle moves along a circular path of radius *R* with velocity of constant magnitude υ.
The time for one rotation is the period, *T*.

Basic Definitions & Units

Period (T): time for one rotation

Frequency (f): # of rotations/time interval

$f = \dfrac{1}{T}$ If T is in seconds (s), f is in s^{-1}

Angular velocity (magnitude): $\omega = 2\pi f = \dfrac{2\pi}{T}$

Unit for angular velocity: radians/sec

The magnitude of the particle's velocity is the ratio of distance moved in one rotation to the time (period) required for one rotation.

$$\upsilon = \frac{2\pi R}{T}$$

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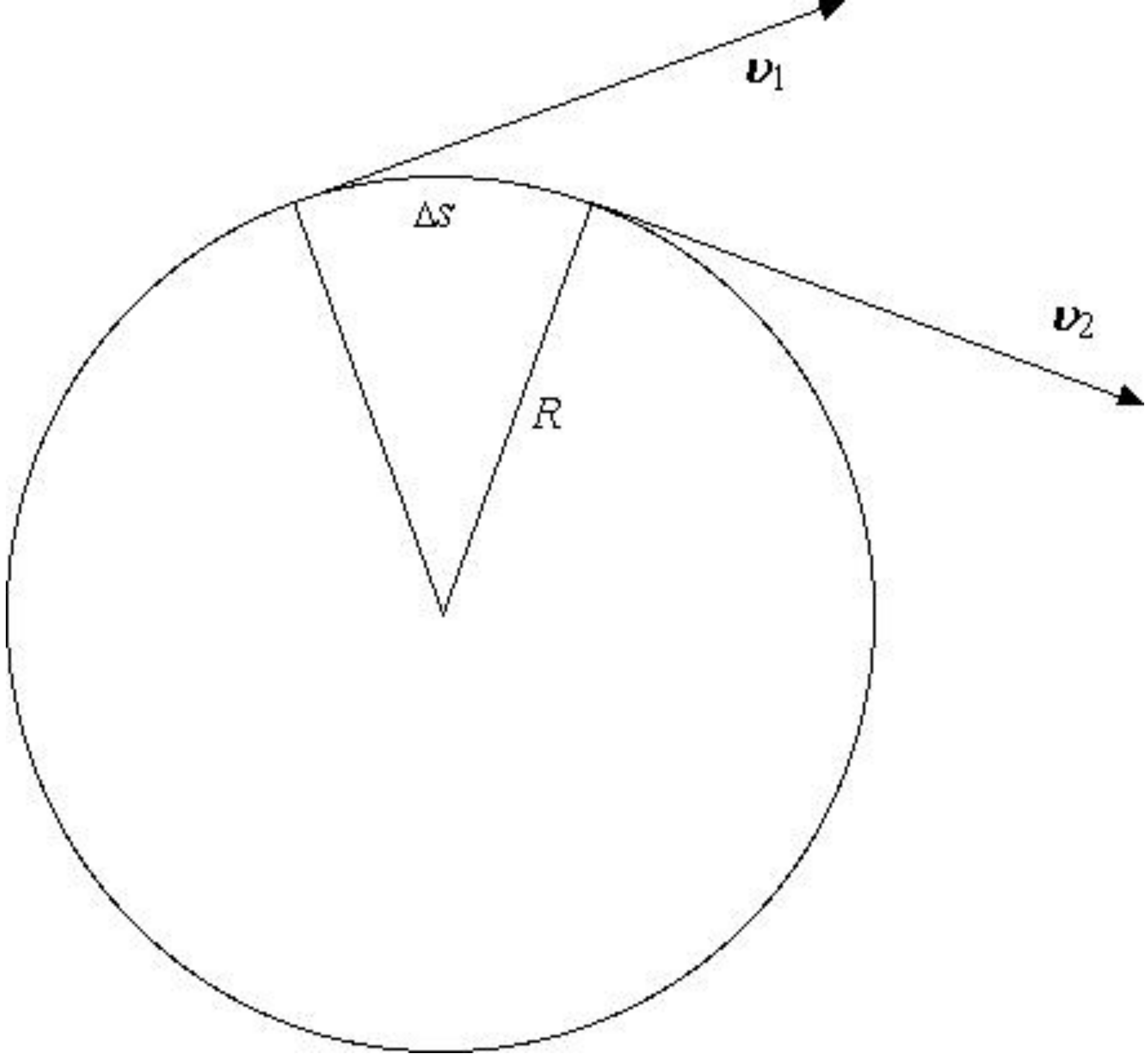
This can also be expressed in terms of the frequency f : $\upsilon = 2\pi R f$

and in terms of the angular velocity ω :

$$\upsilon = \omega R$$



In time interval Δt the particle moves along an arc of length Δs.
The magnitude of the particle's velocity is $\Delta s/\Delta t$.

The CHANGE in the velocity vector during this time interval is $\Delta\upsilon$.

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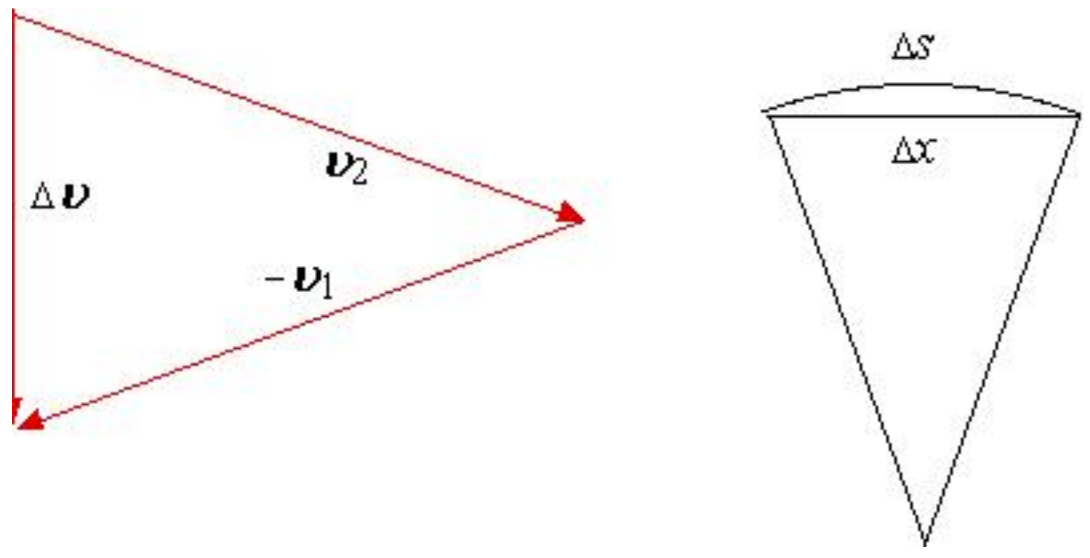

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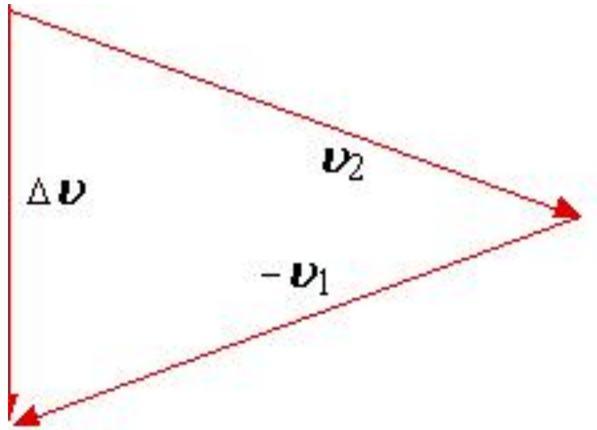
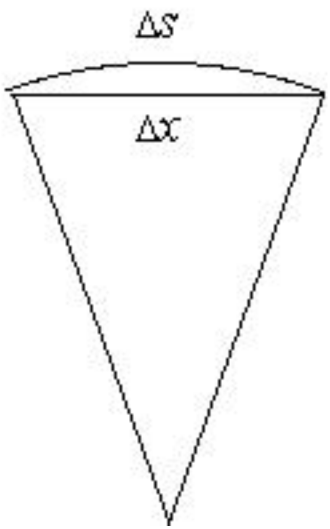
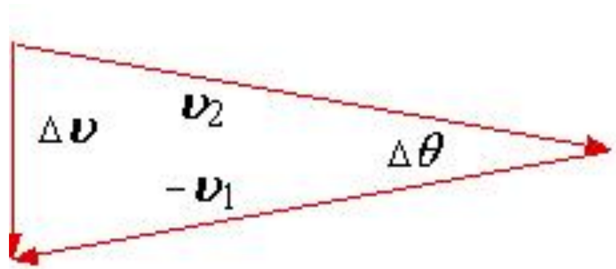


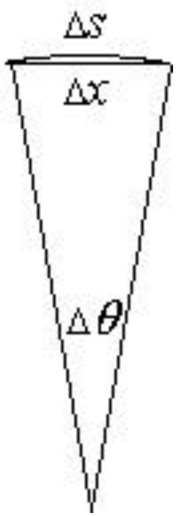


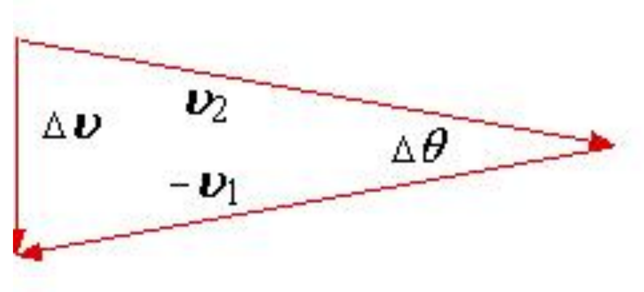
The particle moves a horizontal distance Δx during this time interval.

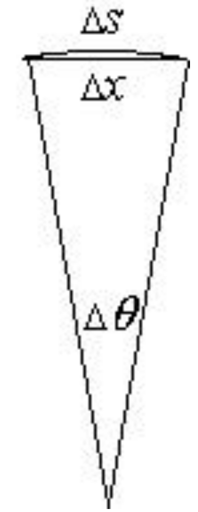
Over a shorter time interval, the length of the arc Δs approaches the length of the chord Δx. The velocity CHANGE is

$$\Delta \boldsymbol{v} = \boldsymbol{v}_2 + (-\boldsymbol{v}_1)$$









These triangles are *similar*, i.e.,

their sides have the same ratio:

$$\frac{\Delta \upsilon}{\upsilon} = \frac{\Delta x}{R} \quad \text{or} \quad \Delta \upsilon = \frac{\upsilon}{R} \Delta x$$

and as the angle gets smaller

Δs approaches Δx in length.

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$$\frac{\Delta \upsilon}{\upsilon} = \frac{\Delta x}{R} \quad \text{or} \quad \Delta \upsilon = \frac{\upsilon}{R} \Delta x$$

and as the angle gets smaller

Δs approaches Δx in length, so

$$\Delta \upsilon \approx \frac{\upsilon}{R} \Delta s.$$

The magnitude of the average acceleration is then

$$a_{\text{av}} = \frac{\Delta \upsilon}{\Delta t} = \frac{\upsilon}{R}\frac{\Delta s}{\Delta t} = \frac{\upsilon}{R} \cdot \upsilon = \frac{\upsilon^2}{R}.$$

This is the

centripetal acceleration

of the particle.

Centripetal acceleration can be expressed in these *equivalent* forms:

$$a_c = \frac{\upsilon^2}{R} = \omega^2 R = \frac{4\pi^2 R}{T^2} = 4\pi^2 R f^2$$

Average and Instantaneous Angular Acceleration

Average angular acceleration α_{av} of an object is defined as the ratio of the change in the angular speed to the time it takes for the object to undergo the change:

$$\alpha_{\mathrm{av}} = \frac{\omega_{\mathrm{f}} - \omega_{\mathrm{i}}}{t_{\mathrm{f}} - t_{\mathrm{i}}} = \frac{\Delta\omega}{\Delta t}$$

Instantaneous angular acceleration:

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t}$$

Relating Linear and Angular Motion

If υ is constant: If ω is constant:

$\Delta x = \upsilon t$ $\Delta\theta = \omega t$

Relating Linear and Angular Motion

If a is constant: If α is constant:

$$\Delta x = \upsilon_i t + \frac{1}{2}at^2 \qquad \Delta\theta = \omega_i t + \frac{1}{2}\alpha t^2$$

$$\upsilon = \upsilon_i + at \qquad \omega = \omega_i + \alpha t$$

$$\upsilon^2 = \upsilon_i^2 + 2a\Delta x \qquad \omega^2 = \omega_i^2 + 2\alpha\Delta\theta$$

$$s = r\theta$$

Tangential velocity: $\upsilon_t = r\omega$

Tangential acceleration: $a_t = r\alpha$

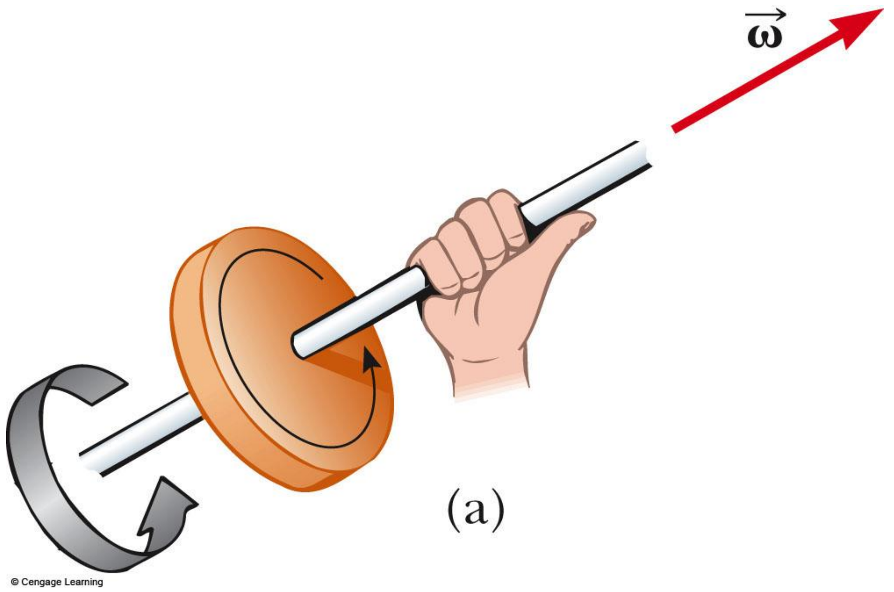
Total Acceleration

The <u>tangential</u> component of the acceleration is due to changing speed.
The <u>centripetal</u> component of the acceleration is due to changing direction.
<u>Total acceleration</u> can be found from these components

$$a = \sqrt{a_t^2 + a_c^2}$$

Vector Nature of Angular Quantities

Angular displacement, velocity and acceleration are all vector quantities. Direction can be more completely defined by using the right hand rule

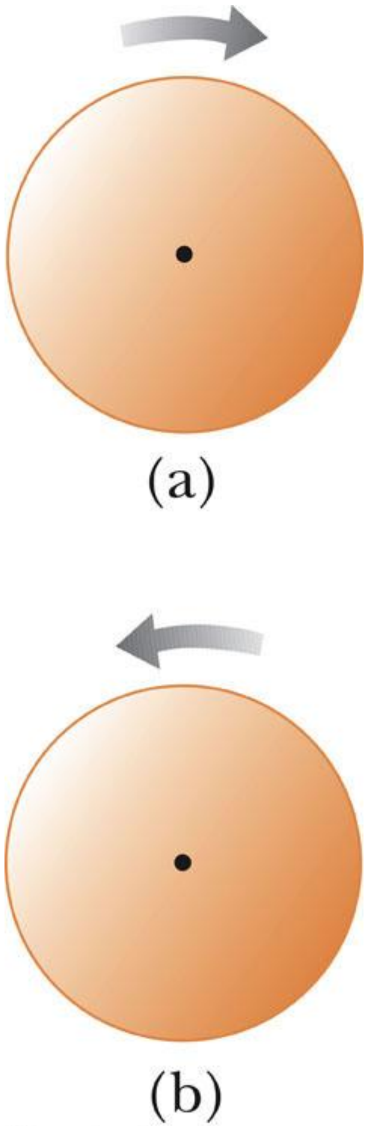


Grasp the axis of rotation with your right hand.
Wrap your fingers in the direction of rotation.
Your thumb points in the direction of ω.

Angular Velocity Directions, Example

In (a), the disk rotates clockwise. The angular velocity is into the page/screen.

In (b), the disk rotates counterclockwise. The angular velocity is out of the page/screen.



Centripetal Acceleration and Centripetal Force

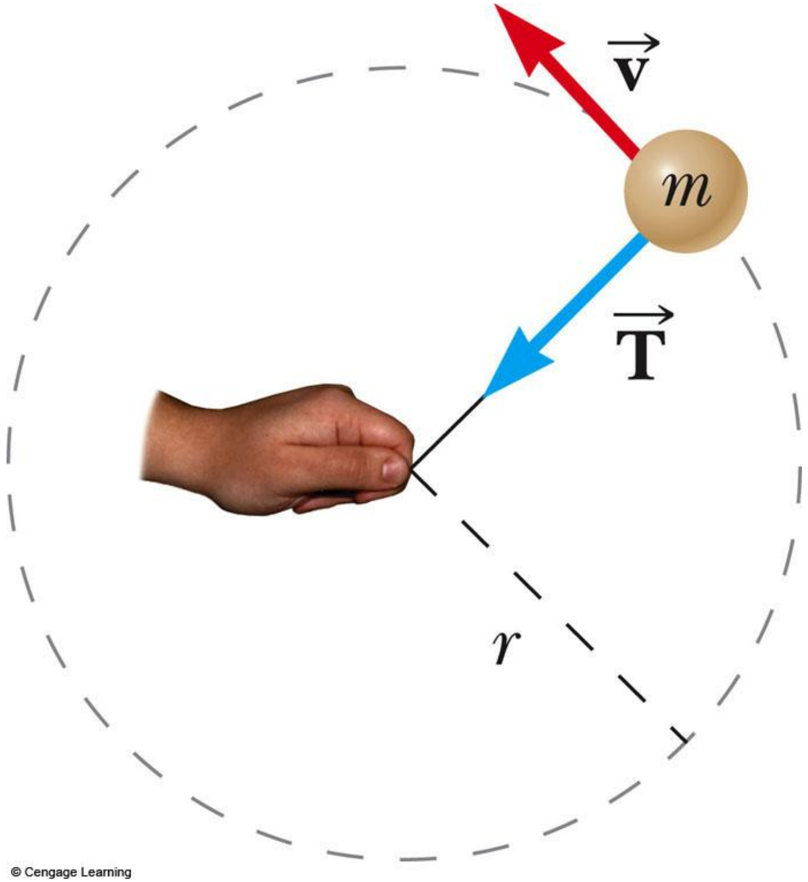
From Newton's second law: $F_c = ma_c$

Centripetal force, F_c, is not an added force. F_c can be supplied by a force that maintains an object on a circular path. Examples:

- **String tension**
- **Gravity**
- **Friction**

Centripetal Force Example 1

A ball of mass *m* is attached to a string. Its weight is supported by a frictionless table. The tension in the string causes the ball to move in a circle. The centripetal force is supplied by the string tension:



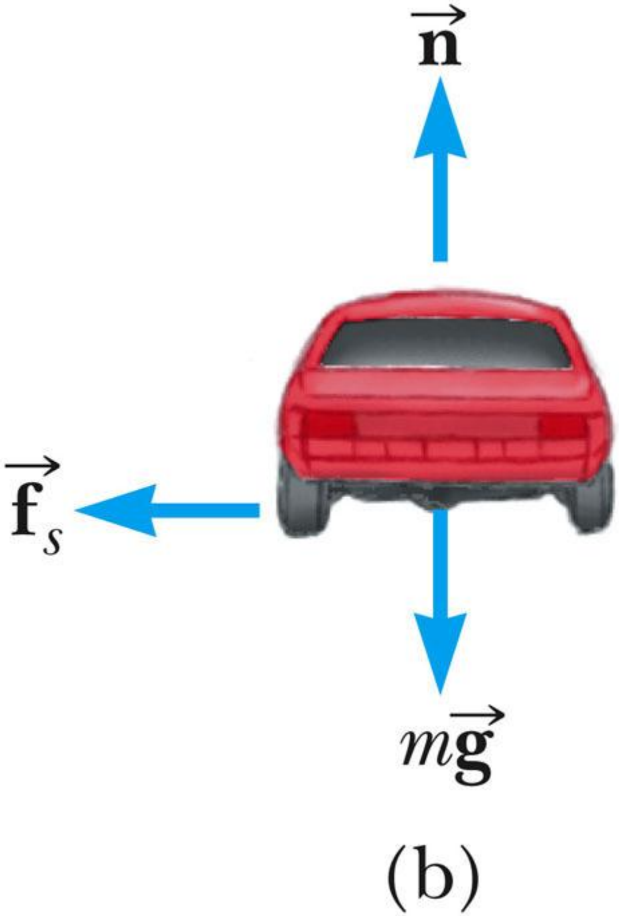
$$F_c = ma_c = T = \frac{m\upsilon^2}{r}$$

Centripetal Force Example 2
Car on a level curve

The centripetal force is supplied by the static friction force.

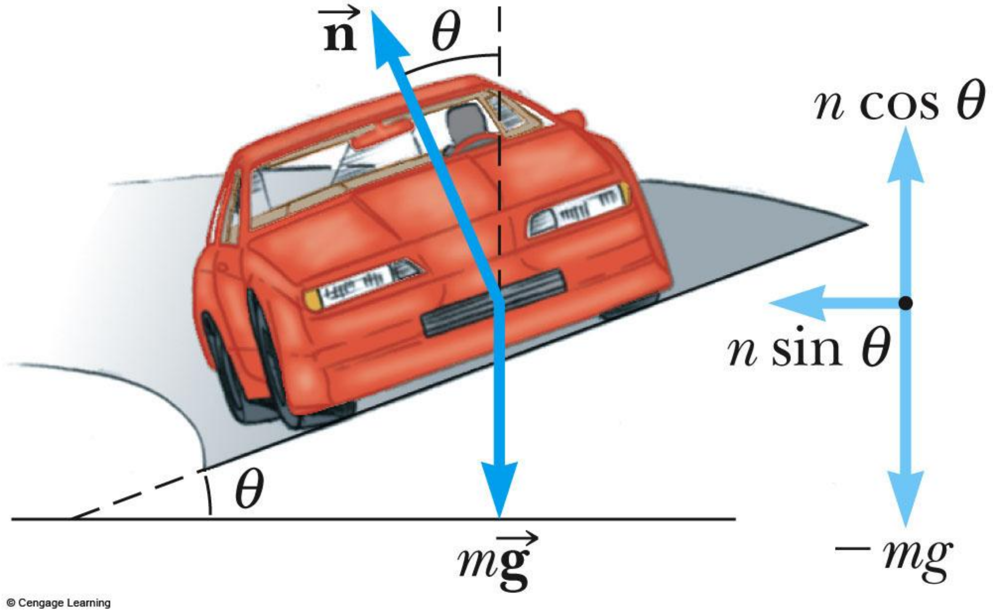
$$F_C = \frac{m\upsilon^2}{r} = f_s = \mu_s mg$$

$$\upsilon = \sqrt{\mu_s gr}$$



Centripetal Force Example 3
Car on a banked curve

The centripetal force is supplied by the horizontal component of the normal force.



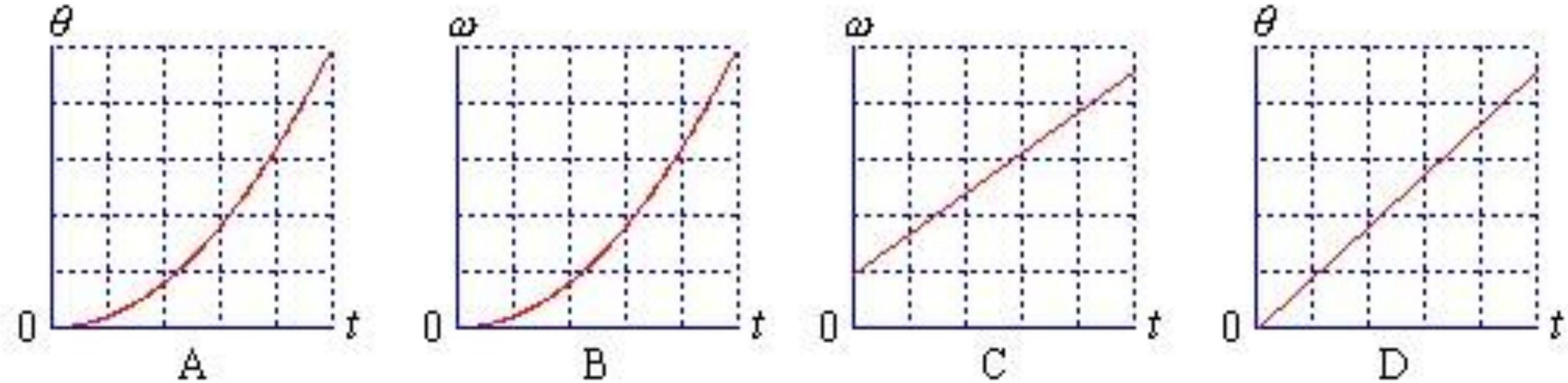
$$n\sin\theta = \frac{m\upsilon^2}{r} \qquad n\cos\theta = mg$$

$$\frac{n\sin\theta}{n\cos\theta} = \tan\theta = \frac{\upsilon^2}{gr}$$

$$a_c = g\tan\theta$$

Rotational Mechanics Questions

A particle rotating in a circle undergoes uniform angular acceleration. Which (one or more) of these graphs could be associated with this kind of motion?



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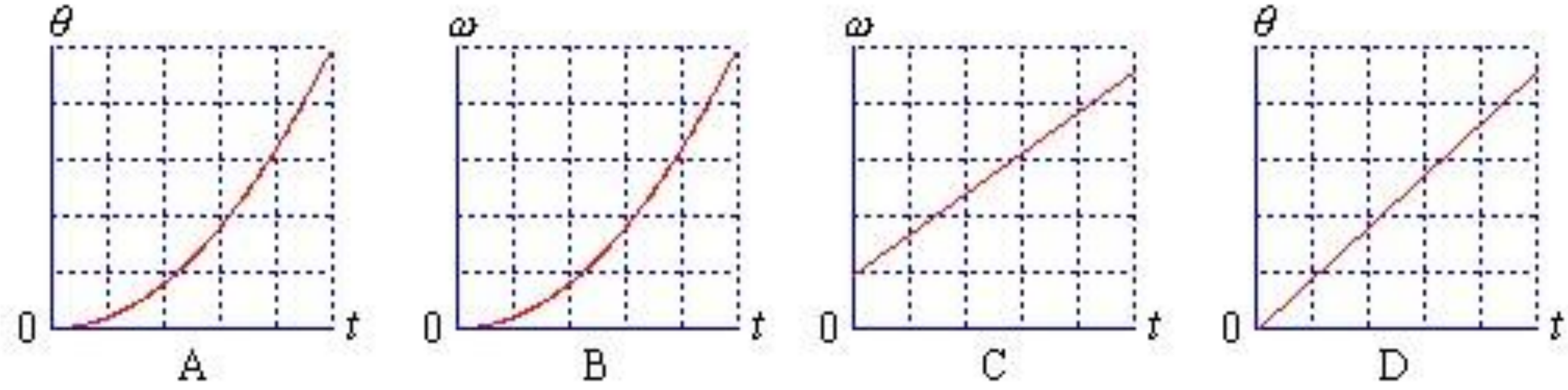
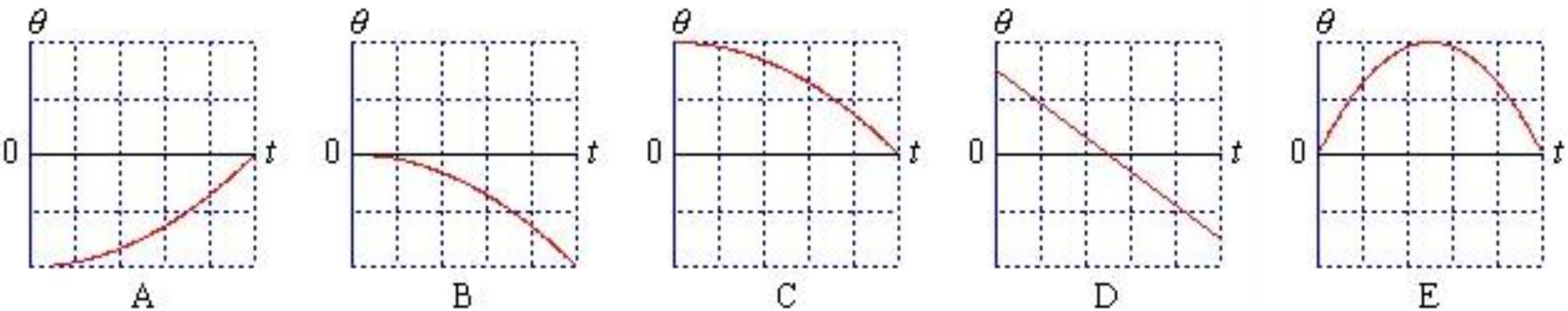


A, C

A particle rotating in a circle undergoes uniform angular acceleration. Which (one or more) of these graphs could be associated with this kind of motion?



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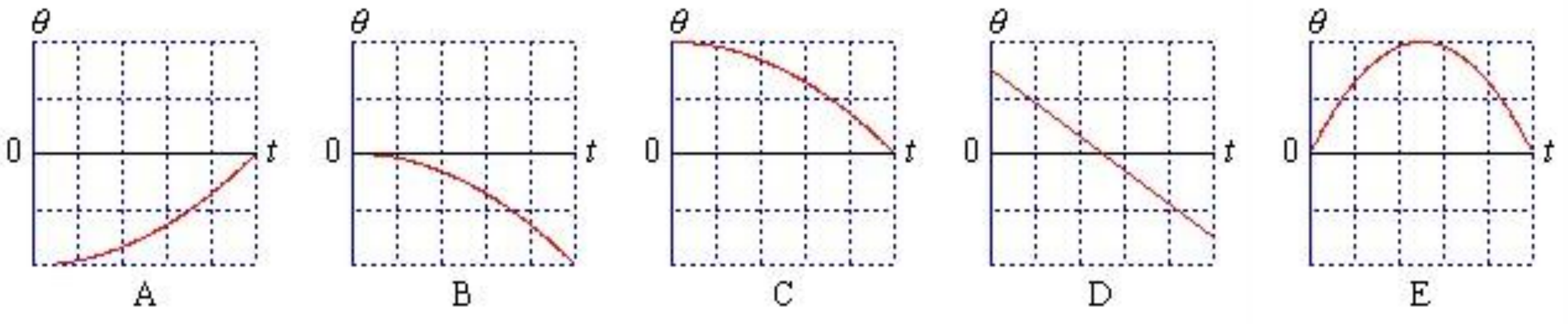

A, B, C, E

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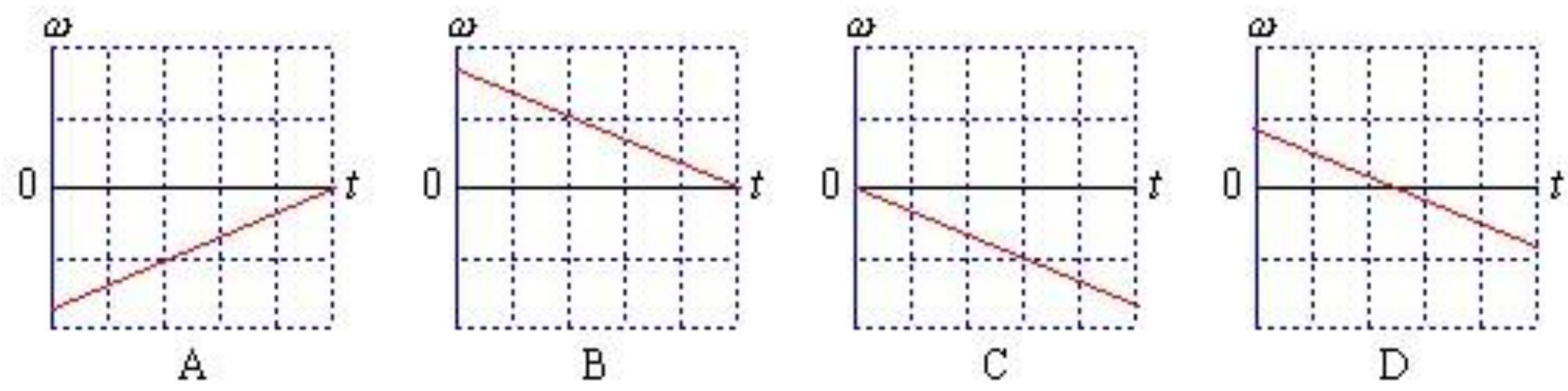

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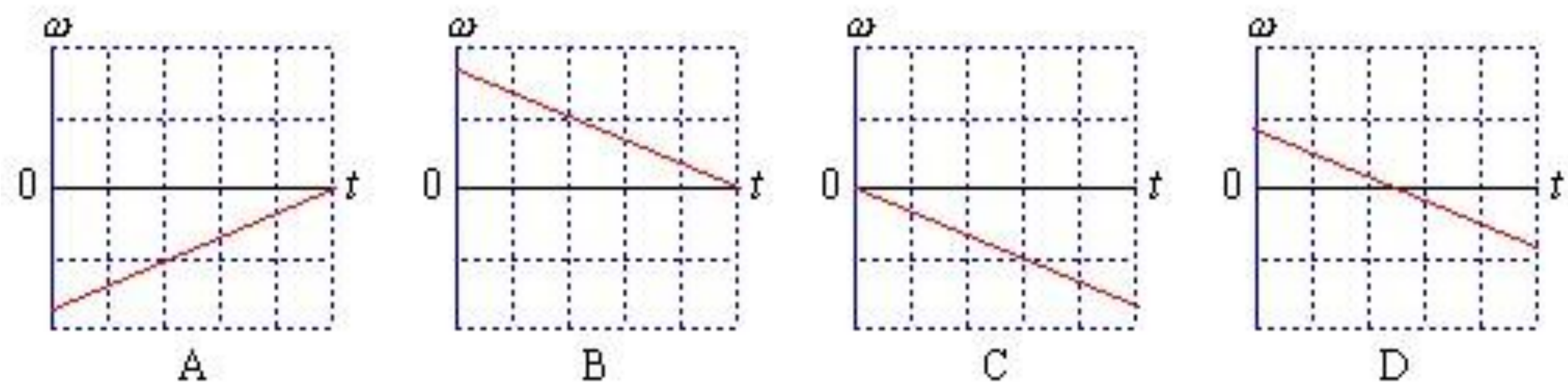

A, B, C, D

A disk is rotating at a constant rate about a vertical axis through its center. Point Q is twice as far from the center of the disk as point P is. The angular velocity of Q at a given time is

A. twice as big as P's.

B. the same as P's.

C. half as big as P's.

D. none of the above.

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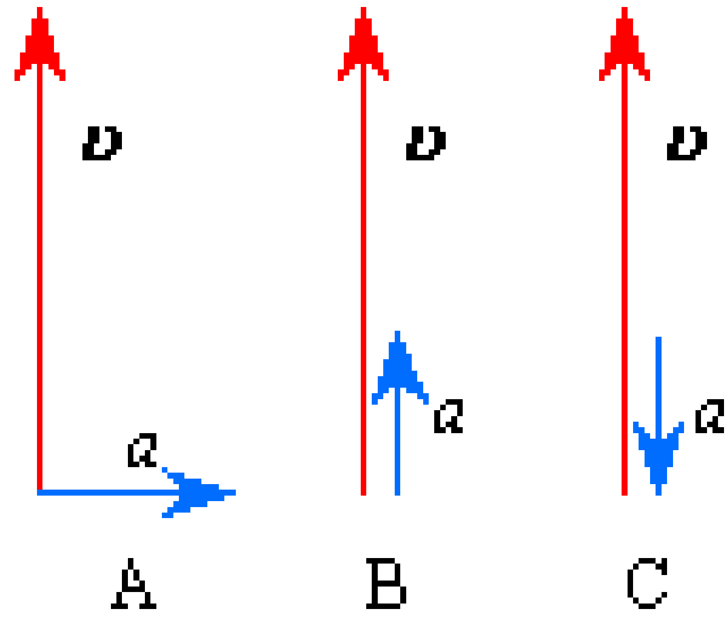
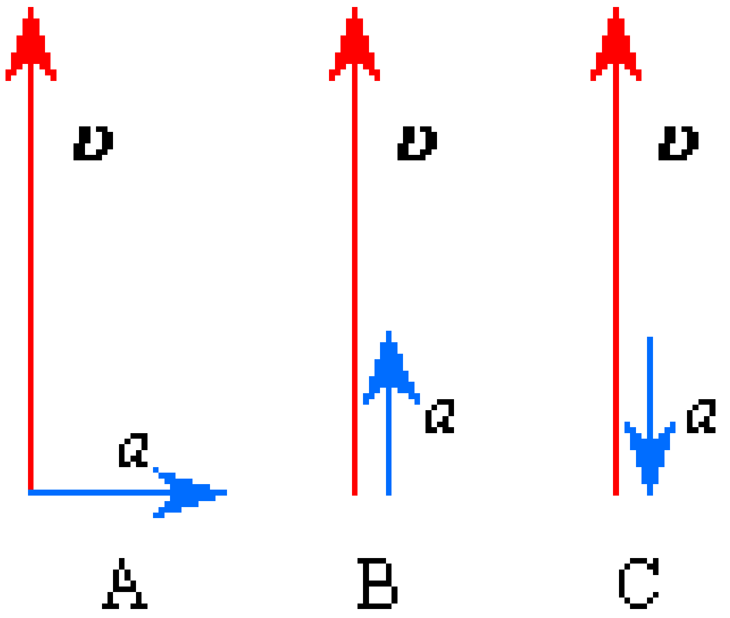
The figure shows the velocity and acceleration of a particle at a particular instant in three situations



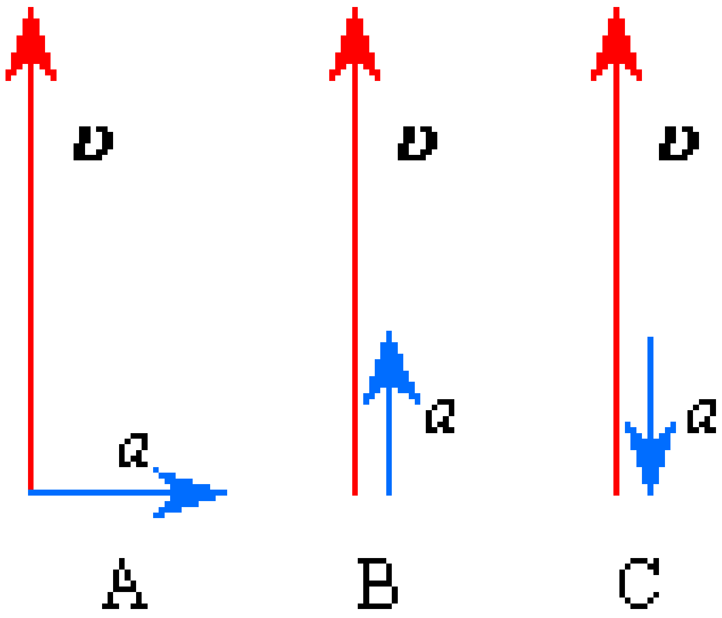
In which situation, and at that instant, is the speed increasing?

The figure shows the velocity and acceleration of a particle at a particular instant in three situations	

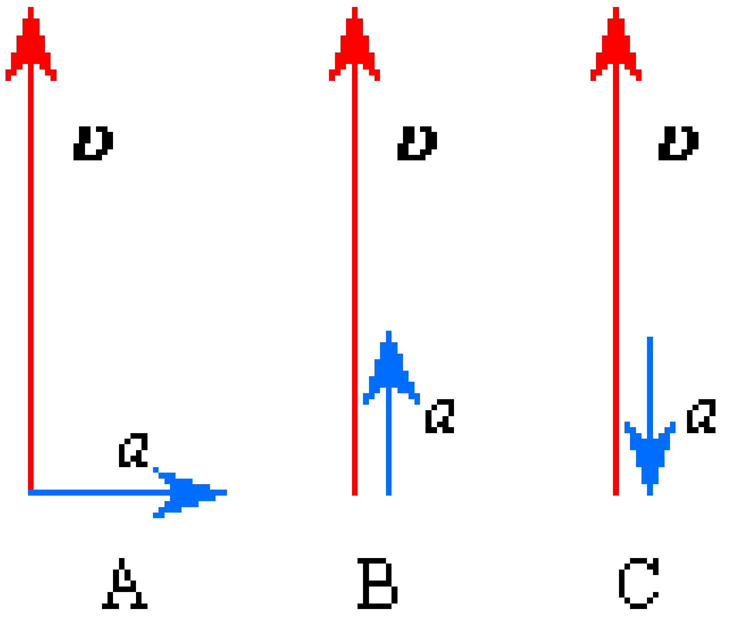
In which situation, and at that instant, is the speed increasing?

B

The figure shows the velocity and acceleration of a particle at a particular instant in three situations

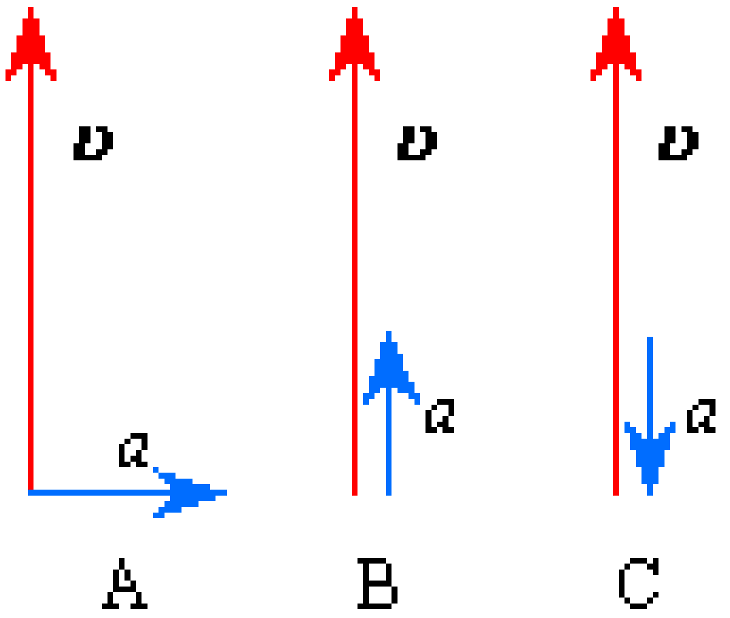


In which situation, and at that instant, is the speed decreasing?

The figure shows the velocity and acceleration of a particle at a particular instant in three situations	

In which situation, and at that instant, is the speed increasing?

C

The figure shows the velocity and acceleration of a particle at a particular instant in three situations	

In which situation, and at that instant, is the speed not changing?

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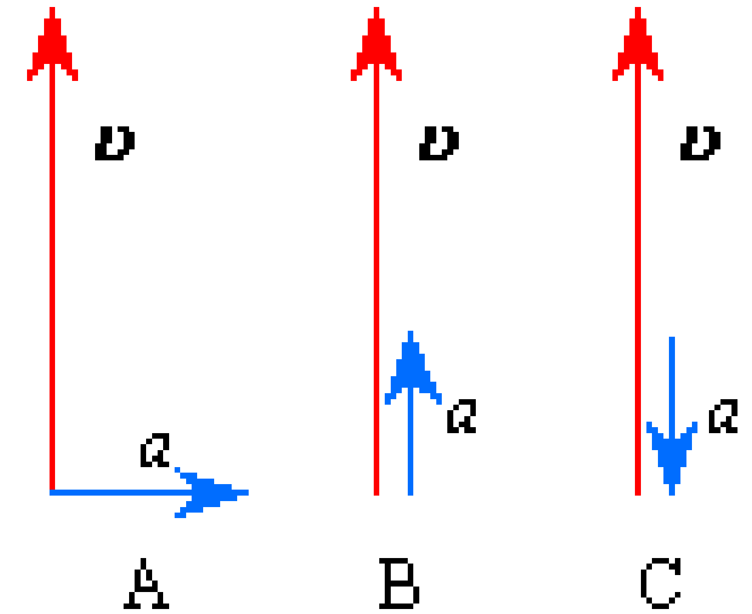
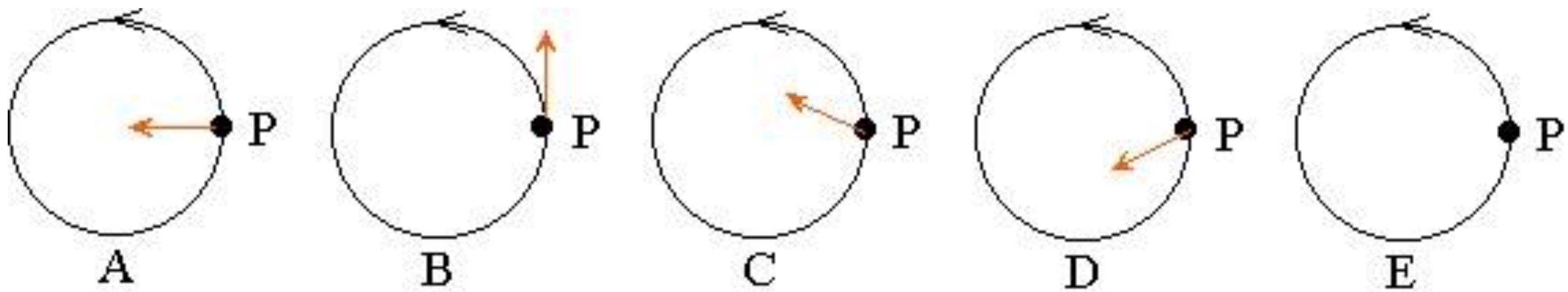


In which situation, and at that instant, is the speed not changing?

A (*a* is centripetal)

A particle P moves in a circle in a horizontal plane. The motion is counterclockwise, as viewed from above.



If the particle undergoes positive angular acceleration, which drawing correctly represents the net horizontal force (red arrow) applied to the particle?

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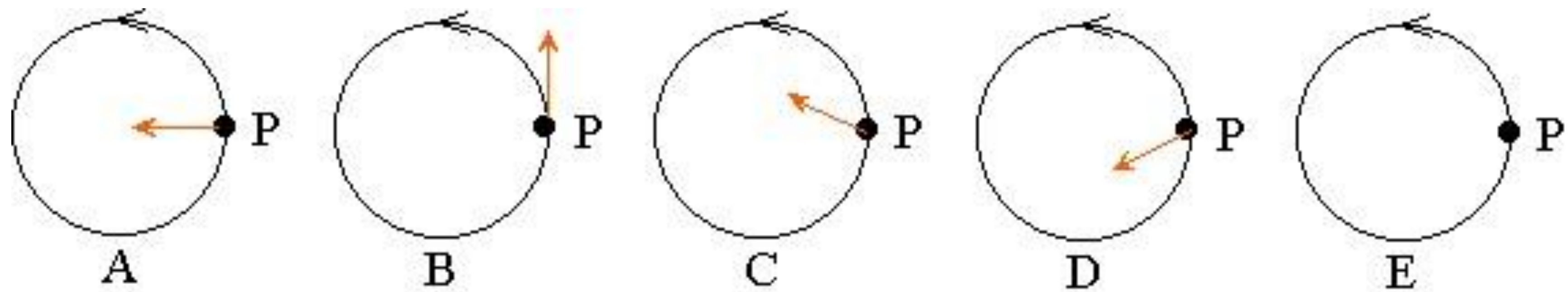
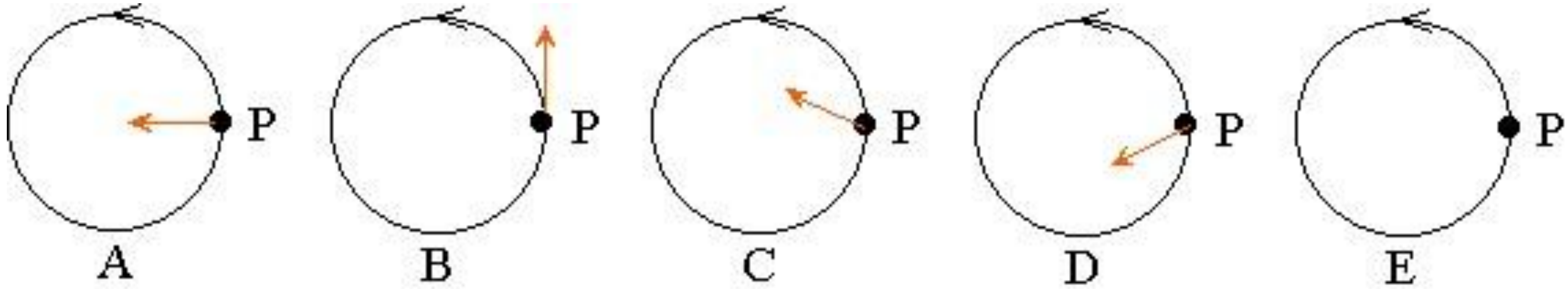


If the particle undergoes positive angular acceleration, which drawing correctly represents the net horizontal force (red arrow) applied to the particle? C



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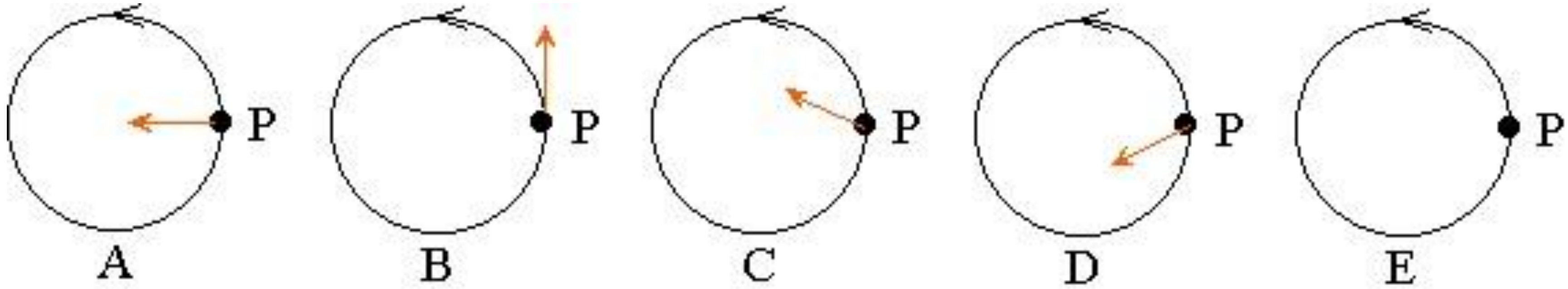
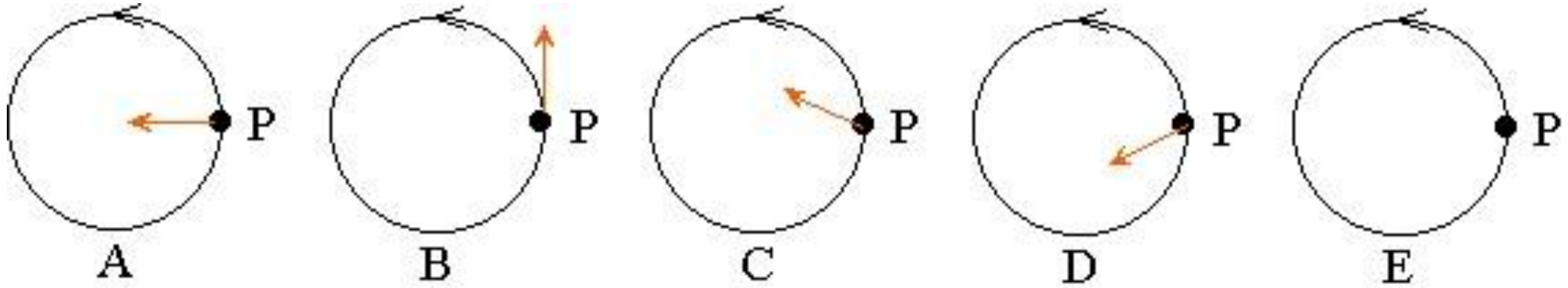
(The net force is the vector sum of the tangential and centripetal forces.)



If the particle moves with constant angular velocity, which drawing correctly represents the net horizontal force (red arrow) applied to the particle?



If the particle moves with constant angular velocity, which drawing correctly represents the net horizontal force (red arrow) applied to the particle? A

A nut is located 0.250 m from the axis of a machine part that rotates uniformly at a rate of 120. times per second. Calculate the period of rotation.

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$T = (1/120)$ s

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$$\upsilon = \frac{2\pi r}{T}$$

$$s = \upsilon t = \left(\frac{2\pi r}{T}\right)t$$

$$= \frac{(2\pi)(.25\text{ m})(2\text{ s})}{(1/120\text{ s})} = 120\pi\text{ m} = 377\text{ m}$$

Calculate the time required for the nut to move through an angle of π radians.

Calculate the time required for the nut to move through an angle of π radians.

$$\Delta\theta = \omega t = \frac{2\pi}{T}t$$

$$t = \frac{T\Delta\theta}{2\pi} = \frac{T\cancel{\pi}}{2\cancel{\pi}} = \frac{T}{2} = \frac{1/120\ \text{s}}{2} = \frac{1}{240}\ \text{s}$$

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Write down everything you know:

M **= 55 kg,** υ **= 4 m/s,** R **= .8 m**

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(a) Determine the force exerted by the horizontal rope on her arms.

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This force is

A. tangent to the circular path.

B. directed inward.

C. directed outward.

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B. directed inward (centripetal force).

C. directed outward.

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$$F_c = \frac{m\upsilon^2}{R} = \frac{(55\text{ kg})(4\text{ m/s})^2}{.8\text{ m}} = \boxed{1100\text{ N}}$$

7.21 A 55.0-kg ice skater is moving at 4.00 m/s when she grabs the loose end of a rope, the opposite end of which is tied to a pole. She then moves in a circle of radius 0.800 m around the pole.

(b) Compare this force (1100 N) with her weight.

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(b) Compare this force (1100 N) with her weight.

$$\frac{F_c}{mg} = \frac{1100\ \text{N}}{(55\ \text{kg})\left(9.8\ \text{m/s}^2\right)} = \boxed{2.04/1}$$

7.23 A truck can go around a flat curve of radius 150 m with a maximum speed of 32.0 m/s. With what maximum speed can it go around a curve of 75 m?

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The centripetal force is supplied by

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B. The car’s brakes
C. Static friction

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$$R_1 = 150 \text{ m}, \; \upsilon_1 = 32 \text{ m/s}, \; R_2 = 75 \text{ m}$$

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<u>Assumption</u>:

7.23 A truck can go around a flat curve of radius 150 m with a maximum speed of 32.0 m/s. With what maximum speed can it go around a curve of 75 m?

Assumption:

Centripetal force (static friction force) does not change.

7.23 A truck can go around a flat curve of radius 150 m with a maximum speed of 32.0 m/s. With what maximum speed can it go around a curve of 75 m?

<u>Equate expressions for centripetal acceleration</u>:

$$\frac{\upsilon_1^2}{R_1} = \frac{\upsilon_2^2}{R_2}$$

$$\upsilon_2 = \upsilon_1\sqrt{\frac{R_2}{R_1}} = (32\ \text{m/s})\sqrt{\frac{75\ \text{m}}{150\ \text{m}}} = \boxed{22.6\ \text{m/s}}$$

7.24 A sample of blood is in a centrifuge of radius 15 cm. The mass of a red blood cell is 3.0×10^{-16} kg and the magnitude of the force acting on it is 4.0×10^{-11} N. At how many revolutions per second should the centrifuge be operated?

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Given:

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$$F_c = 4\pi^2 R f^2 m$$

$$f = \sqrt{\frac{F_c}{4\pi^2 R m}} = \sqrt{\frac{4\times10^{-11}\ \text{N}}{4\pi^2\ \ .15\ \text{m}\ \ 3\times10^{-16}\ \text{kg}}}$$

$$= \boxed{150\ \text{s}^{-1}\quad \text{rev/s}}$$