Vectors and Two-Dimensional Motion

Vectors and Their Properties

A *scalar* quantity can be completely specified by its magnitude with appropriate units. E.g., mass, time interval, temperature, volume.

Scalar quantities can be manipulated with the rules of arithmetic.

A *vector* quantity requires magnitude and direction.

Vector Notation

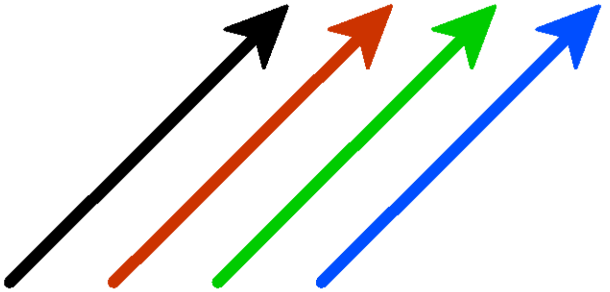
$\vec{A}$ **represents a vector.**

$\left|\vec{A}\right|$ or A (no arrow)

represents the *magnitude* of a vector.

Equality of Two Vectors

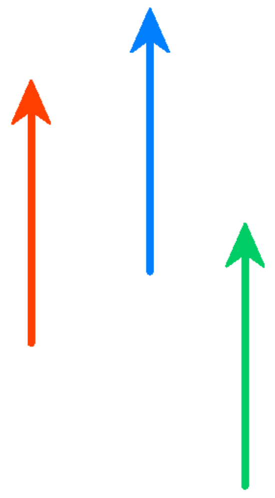
Two vectors are equal if they have the same magnitude and the same direction.



$$\vec{A} = \vec{B} = \vec{C} = \vec{D}$$

Movement of vectors in a diagram

Any vector can be moved parallel to itself without being affected; it can be moved up or down or right or left.



Negative Vectors

The *negative* of a vector is the original vector rotated through 180º.

$-\vec{A}$ $\quad \vec{A}$

$$\vec{A} + -\vec{A} = 0$$

Resultant Vector

The resultant vector is the *geometric sum* of a given set of vectors.

$$\vec{R} = \vec{A} + \vec{B}$$

Adding Vectors

When adding vectors, their directions must be taken into account.

Units must be the same.

Geometric Methods: Use scale drawings.

Analytical Methods: More convenient.

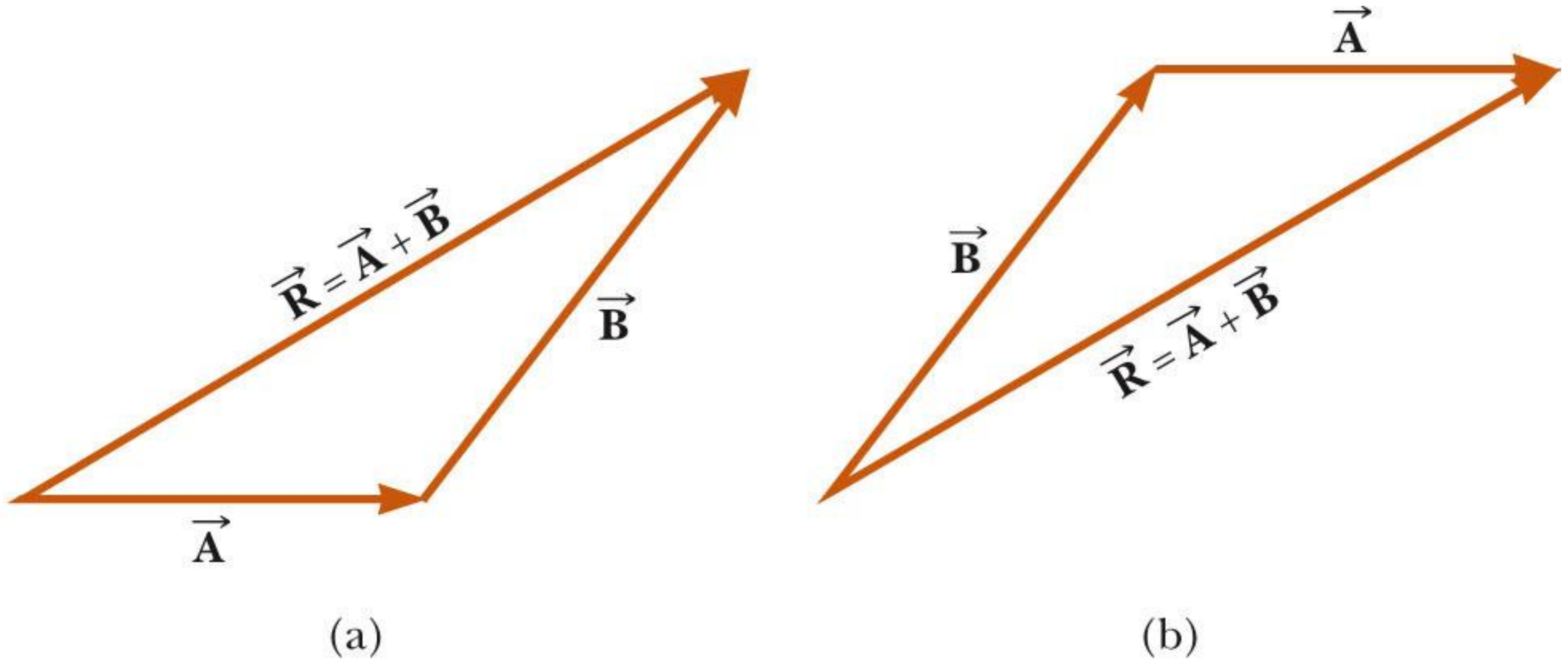
Adding Vectors Geometrically (Triangle or Polygon Method)

Choose a scale. Draw the first vector with the appropriate length and in the direction specified, with respect to a coordinate system

Draw the next vector with the appropriate length and in the direction specified, with respect to a coordinate system whose origin is the end of vector $\vec{A}$ and parallel to the coordinate system used for $\vec{A}$.

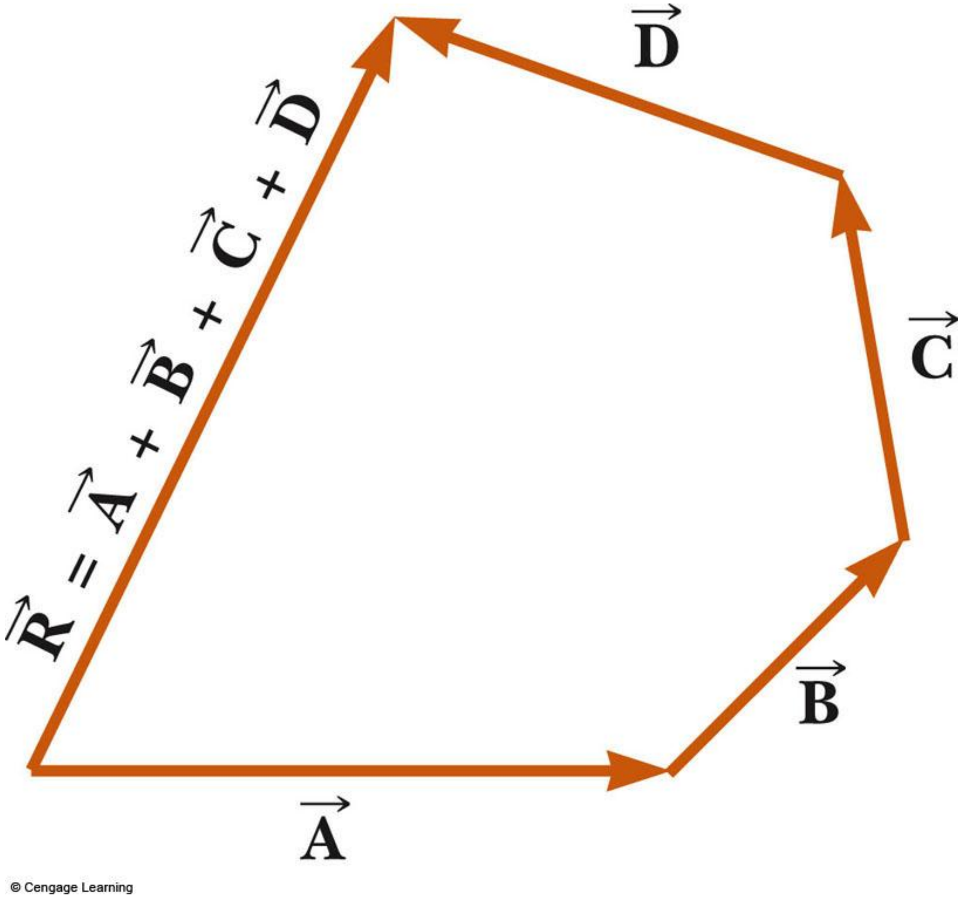
Continue drawing the vectors “tail-to-head”. The ***resultant*** is drawn from the tail of the first vector to the end of the last vector.

- Measure the length of the resultant and its angle. Use the scale factor to convert length to actual magnitude.



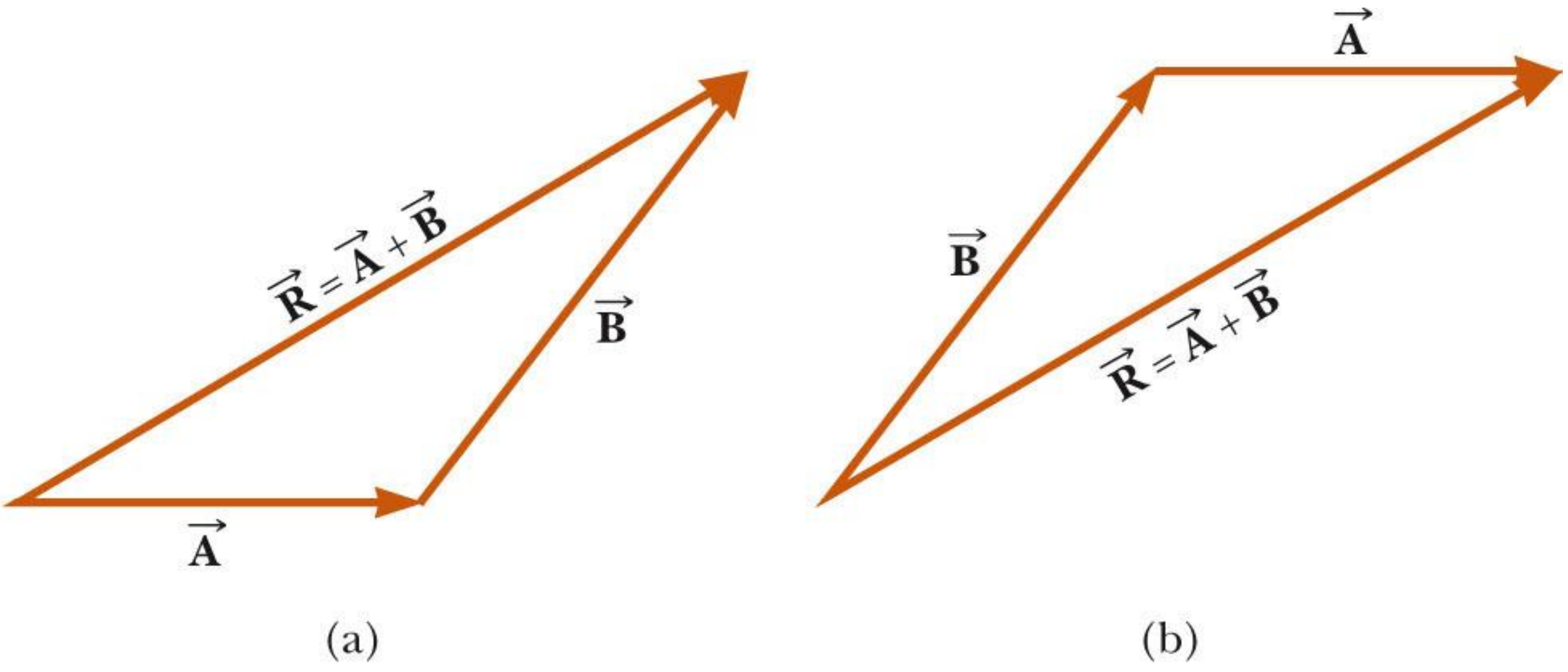
When you have many vectors, just keep repeating the process until all are included.

The resultant is still drawn from the tail (origin) of the first vector to the head (end) of the last vector.



Vectors obey the Commutative Law of Addition.

The order in which the vectors are added doesn't affect the result.

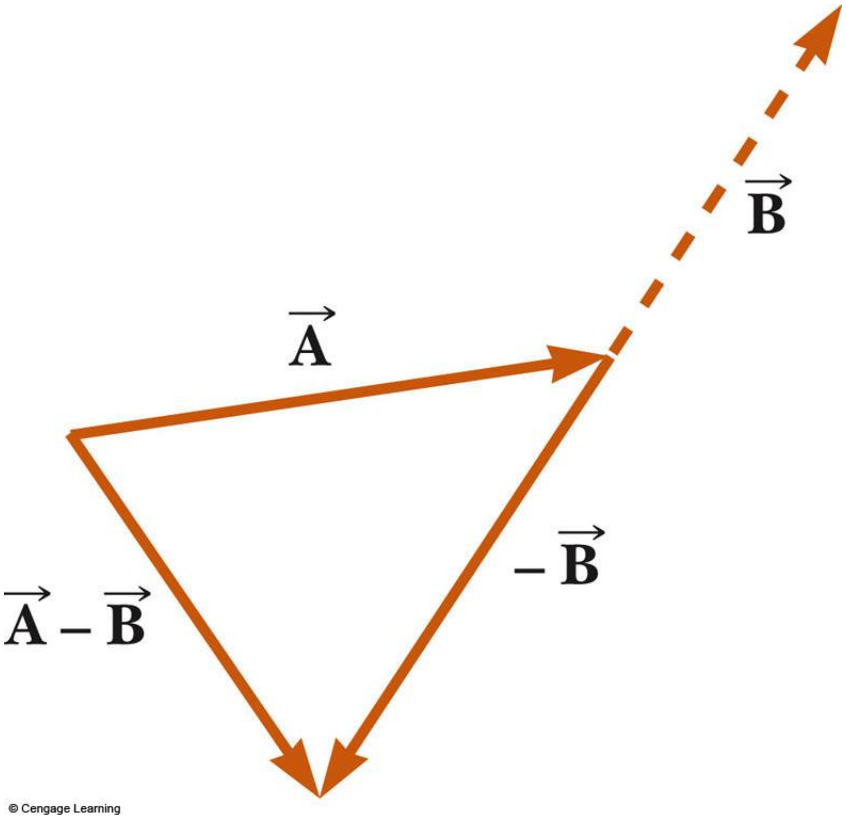


Vector Subtraction

Special case of vector addition.

Add the negative of the subtracted vector.

Continue with standard vector addition procedure.



Multiplying or Dividing a Vector by a Scalar

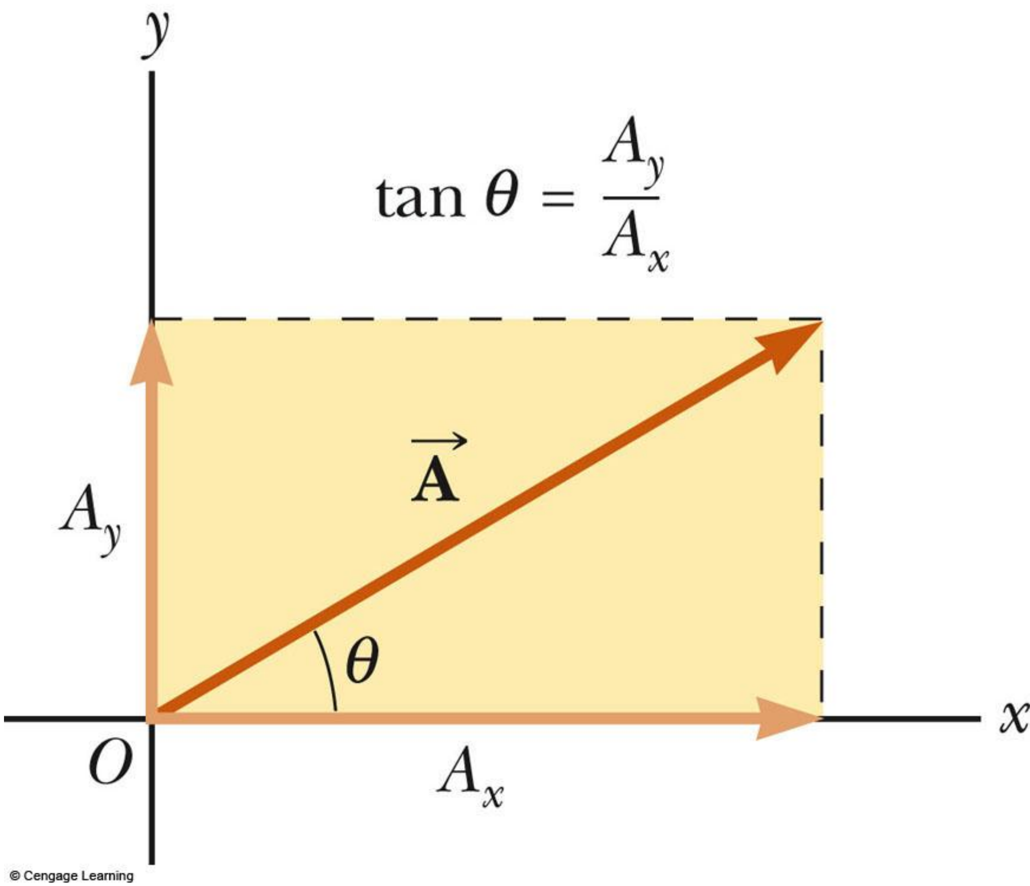
The result of the multiplication or division is a *vector*.

The magnitude of the vector is multiplied or divided by the scalar.

If the scalar is positive, the direction of the result is the same as of the original vector. If the scalar is negative, the direction of the result is opposite that of the original vector.

Components of a Vector

It is useful to use ***rectangular components***. These are the projections of the vector along the *x*- and *y*-axes.



$$A_x = |\vec{A}|\cos\theta \qquad A_y = |\vec{A}|\sin\theta$$

θ must be measured counterclockwise from the +*x*-axis.

Adding Vectors Algebraically

It is useful to sketch the vectors.

Add the *x*- and *y*-components of all the vectors.

$$R_x = \sum V_x \qquad R_y = \sum V_y$$

$$\text{Magnitude of } \vec{R}: \; R = \sqrt{R_x^2 + R_y^2}$$

$$\theta_R = \tan^{-1}\frac{R_y}{R_x}$$ If R_x and R_y are both positive

Adding Vectors Algebraically

To find θ_R if $\vec{R}$ is not in the first quadrant:

Quadrant	R_x	R_y	θ_R
2	$-$	$+$	$180^o - \tan^{-1}\left\|\frac{R_y}{R_x}\right\|$
3	$-$	$-$	$180^o + \tan^{-1}\left\|\frac{R_y}{R_x}\right\|$
4	$+$	$-$	$370^o - \tan^{-1}\left\|\frac{R_y}{R_x}\right\|$