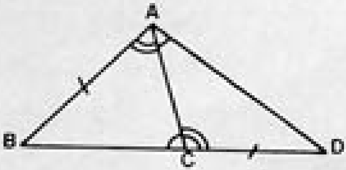

Figure B - 5.

congruence theorems (ASA, SSS, SAA), the congruence of vertical angles, the existence of right angles, the possibility of angle and segment bisection, and many other propositions of Euclid, Book I. The details appear in Hilbert [21]. As a final example we give Hilbert's proof of the exterior angle theorem.

Theorem B-17

An exterior angle of a triangle is greater than either of the interior angles not adjacent to it.

Proof. In Figure B-5, choose D on $\overline{BC}$ extended so that $AB = CD$. First, we shall show that $\sphericalangle ACD \not\cong \sphericalangle BAC$. If $\sphericalangle ACD \cong \sphericalangle BAC$ were true then $\Delta BAC \cong \Delta DCA$ by Theorem B-16. But then $\sphericalangle CAD$ would be congruent to the supplement of $\angle ACD$ and hence congruent to the supplement of $\sphericalangle BAC$. By III,4, this in turn would imply that D lies on $\overleftrightarrow{BA}$, in violation of I,2. Thus, $\sphericalangle ACD \cong \sphericalangle BAC$ is impossible.

Next suppose $\sphericalangle ACD < \sphericalangle BAC$ (Fig. B-6). Draw ray $\overrightarrow{AB'}$ on the B side of $\overleftrightarrow{AC}$ such that $\sphericalangle B'AC \cong \sphericalangle ACD$. Then $\overrightarrow{AB'}$ passes through the interior of $\sphericalangle BAC$ and so meets $\overline{BC}$ (Theorem III-17 in Section III-3) in a point, B'. But then the exterior angle $\sphericalangle ACD$ of $\Delta B'AC$ is congruent to the interior angle $\sphericalangle B'AC$. This was shown to be impossible in the first part of the proof.

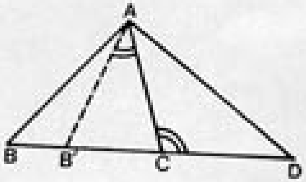

Figure B - 6.