Theorem B-14

If, in ΔABC, $AB = AC$, then $\measuredangle ABC \cong \measuredangle ACB$.

Proof. Apply III,5 (and the last part of III,4) with $A' = A$, $B' = C$, $C' = B$.

Definition B-15

Two triangles will be called *congruent* if there exists a one-to-one correspondence between their vertices relative to which corresponding sides are congruent and corresponding angles are congruent. In other words, the triangles are congruent if we can label the vertices A, B, C in one triangle and A′, B′, C′ in the other in such a way that the congruences

$$AB = A'B', \quad AC = A'C', \quad BC = B'C'$$

$$\measuredangle BAC \cong \measuredangle B'A'C', \quad \measuredangle ABC \cong \measuredangle A'B'C', \quad \measuredangle ACB \cong \measuredangle A'C'B'$$

all hold.

Theorem B-16 (Side-Angle-Side, or SAS)

If, in ΔABC and $\Delta A'B'C'$, $AB = A'B'$, $AC = A'C'$, and $\measuredangle BAC \cong \measuredangle B'A'C'$, then $\Delta ABC = \Delta A'B'C'$.

Proof. We already know, from III,5 and the note following it, that $\measuredangle ABC \cong \measuredangle A'B'C'$ and $\measuredangle ACB \cong \measuredangle A'C'B'$ (Fig. B-4). It remains to prove $BC = B'C'$. Suppose on the contrary, that this were false. By III,1, choose D′ on $\overrightarrow{B'C'}$ such that $B'D' = BC$. Applying III,5 to ΔABC and $\Delta A'B'D'$ (where $\measuredangle ABC \cong \measuredangle A'B'D'$), we deduce that $\measuredangle BAC \cong \measuredangle B'A'D'$. Since $\measuredangle BAC \cong \measuredangle B'A'C'$ is given, we then have a contradiction to the uniqueness asserted in III,4, since $\overrightarrow{A'D'}$ and $\overrightarrow{A'C'}$ are distinct rays (why?).

From the preceding postulates and theorems, it is possible to deduce the transitivity of angle congruence, the remaining triangle

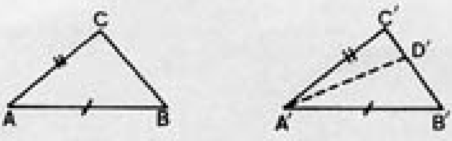

Figure B - 4.