line are called *supplementary*. An angle which is congruent to one of its supplementary angles is called a *right angle*.

The interior of an angle (or of a triangle) is defined in part d of Section III-3.

III,4. Let $\measuredangle(h,k)$ be an angle [in a plane α] and a′ a line [in a plane α'] and let a definite side of a′ [in α'] be given. Let h′ be a ray on the line a′ that emanates from the point O′. Then there exists [in the plane α'] one and only one ray k′ such that the angle $\measuredangle(h,k)$ is congruent or equal to the angle $\measuredangle(h',k')$ and at the same time all interior points of the angle $\measuredangle(h',k')$ lie on the given side of a′. Symbolically, $\measuredangle(h,k) \cong \measuredangle(h',k')$. Every angle is congruent to itself, i.e., $\measuredangle(h,k) \cong \measuredangle(h,k)$ is always true.

Following standard practice, if A and B are points (other than O) on rays h and k, respectively, we shall usually write $\measuredangle AOB$ or $\measuredangle BOA$ rather than $\measuredangle(h,k)$.

III,5. If for two triangles $\triangle ABC$ and $\triangle A'B'C'$ the congruences

$$\overline{AB} \cong \overline{A'B'}, \quad \overline{AC} \cong \overline{A'C'}, \quad \measuredangle BAC \cong \measuredangle B'A'C'$$

hold, then the congruence $\measuredangle ABC \cong \measuredangle A'B'C'$ is also satisfied.

Note: from an interchange of the letters B and C in III,5, it follows that under the same hypotheses $\measuredangle ACB \cong \measuredangle A'C'B'$ holds also.

From now on, we shall write $AB = CD$ to indicate $\overline{AB} \cong \overline{CD}$. It is a consequence of Hilbert's postulates that real numbers (lengths) can be assigned to segments in such a way that segments are congruent if and only if they have the same length, and so that if the distance between two points A and B is defined to be the length of $\overline{AB}$, then the usual properties of a distance function hold. (We shall not go into the details here.) In this text, we shall use AB to denote the distance from A to B. Thus $AB = CD$ indicates both congruence of segments and (equivalently) equality of distance.

Theorem B-13

The point B′ whose existence is asserted in Axiom III,1 is unique (for a given side of A′).

Proof. Suppose there were two (distinct) such points, B′ and B″ Choose any point C not on $\overleftrightarrow{A'B'}$, and apply III,5 to $\triangle A'B'C$ and $\triangle A'B''C$ to show that $\measuredangle A'CB' \cong \measuredangle A'C'B''$. Then deduce a contradiction to the uniqueness asserted in III,4.