GROUP III: THE CONGRUENCE AXIOMS

III, 1. If A, B are two points on a line a, and A′ is a point on the same or on another line a′ then it is always possible to find a point B′ on a given side of the line a′ through A′ such that the segment $\overline{AB}$ is congruent or equal to the segment $\overline{A'B'}$. In symbols,

$$\overline{AB} \cong \overline{A'B'}$$

III, 2. If a segment $\overline{A'B'}$ and a segment $\overline{A''B''}$ are congruent to the same segment $\overline{AB}$, then the segment $\overline{A'B'}$ is also congruent to the segment $\overline{A''B''}$, or briefly, if two segments are congruent to a third one they are congruent to each other.

Lemma B-10

Every segment is congruent to itself.

Proof. Using III, 1 construct any segment $\overline{A'B'}$ congruent to $\overline{AB}$ and then apply III, 2 to the congruences $\overline{AB} \cong \overline{A'B'}$ and $\overline{AB} \cong \overline{A'B'}$.

Theorem B-11

Congruence of segments is an equivalence relation.

Proof. Left to the reader. See Example vi of Section III-2 for the definition of equivalence relation.

III, 3. On the line a let $\overline{AB}$ and $\overline{BC}$ be two segments which except for B have no point in common. Furthermore, on the same or on another line a′ let $\overline{A'B'}$ and $\overline{B'C'}$ be two segments which except for B′ also have no point in common. In that case, if

$$\overline{AB} \cong \overline{A'B'} \text{ and } \overline{BC} \cong \overline{B'C'}$$

then

$$\overline{AC} \cong \overline{A'C'}.$$

Definition B-12

Let [α be a plane and] h, k any two distinct rays emanating from O [in α] and lying in *distinct lines*. The pair of rays h, k is called an *angle* and is denoted by ∡(h,k) or by ∡(k,h). Rays h and k are the *sides* of the angle and point O is its *vertex*. Two angles with a vertex and one side in common and whose non-common sides form a