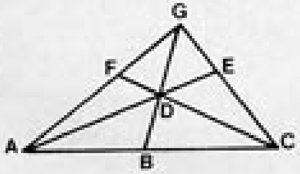

Figure B - 2.

already meets $\overleftrightarrow{AE}$ in D and can meet $\overleftrightarrow{AE}$ in no other point, then D is this point and we must have (ADE). Similarly, by II,4 applied to ΔAEC, since $\overleftrightarrow{GB}$ meets $\overline{AE}$, then $\overleftrightarrow{GB}$ must meet $\overline{AC}$ in some point, which must be B. Therefore (ABC).

Theorem B-5

Let A, B, C, and D be distinct points of a line. If (ABC) and (BCD), then (ABD) and (ACD) (Fig. B-3).

Proof. Let E and F be such that E is not on the line containing the four points, and (CEF). The remainder of the proof is a repeated application of Pasch's Axiom (II,4).

From ΔBCF, $\overleftrightarrow{AE}$ meets $\overline{BF}$ in G: (BGF).

From ΔAEC, $\overleftrightarrow{BF}$ meets $\overline{AE}$ in G: (AGE).

From ΔGBD, $\overleftrightarrow{CF}$ meets $\overline{GD}$ in H: (GHD).

From ΔAGD, $\overleftrightarrow{EH}$ meets $\overline{AD}$ [since (AEG) is false] in a point which must be C: (ACD).

We have thus proved that

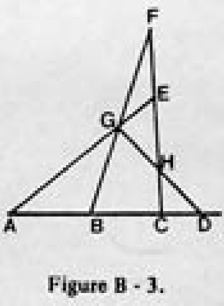

Figure B - 3.