Definition B-2

Given three distinct points A, B, and C not all on the same line, ΔABC is the union of the three segments $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$.

II,4. Let A, B, C be three points that do not lie on a line and let a be a line [in the plane ABC] which does not meet any of the points A, B, C. If the line a passes through a point of the segment $\overline{AB}$, it also passes through a point of segment $\overline{AC}$, or through a point of segment $\overline{BC}$.

Let us use the notation (ABC) to indicate that B is between A and C. The following are consequences of the axioms in Groups I and II. Other consequences are deduced in the exercises of Section III-3.

Theorem B-3

Given distinct points A and C, there exists a point D such that (ADC) (Fig. B-1).

Proof: By I,3, let E be a point not on $\overleftrightarrow{AC}$. By II,2, there is a point F such that (AEF), and $F \neq C$ by I,2. Again by II,2, there is a point G such that (FCG), and by II,4, line $\overleftrightarrow{EG}$, which cannot pass through A, C, or F (why not?) but meets $\overline{AF}$, must meet $\overline{AC}$ or $\overline{CF}$ in a point D. But $\overleftrightarrow{EG}$ cannot meet $\overline{CF}$, for this would violate I,2. (G is not on $\overline{FC}$ by II,3.) Hence D is on $\overline{AC}$, or (ADC).

Theorem B-4

Of any three distinct points A, B, C on a line, at least one of them is between the other two (Fig. B-2).

Proof. Suppose (BAC) and (ACB) both do not hold. We shall show (ABC). By II-2, there exist points D and G such that (BDG). By II,4 (applied to ΔBCG), $\overleftrightarrow{AD}$ meets $\overline{CG}$ in E: (CEG). Similarly, $\overleftrightarrow{CD}$ meets $\overline{AG}$ in F: (AFG). Applying II,4 to ΔAEG (since $\overleftrightarrow{CF}$ meets $\overline{AG}$), $\overleftrightarrow{CF}$ must meet $\overline{AE}$ in a point between A and E. Because CF

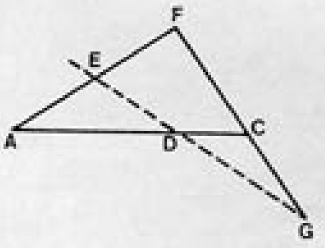

Figure B - 1.