

and  $a$  is a line, then in place of " $A$  lies on  $a$ ," we will often substitute " $a$  contains  $A$ ," " $a$  passes through  $A$ ," " $A$  is on  $a$ ," etc. Two lines containing  $A$  are said to "intersect in  $A$ ," or "have  $A$  in common." Similar terminological substitutions will be used for incidence relations involving planes.

#### GROUP I: THE INCIDENCE AXIOMS

I,1. For every two points  $A, B$  there exists a line  $a$  that contains each of the points  $A, B$ .

I,2. For every two points  $A, B$  there exists no more than one line that contains each of the points  $A, B$ .

I,3. There exist at least two points on a line. There exist at least three points that do not lie on a line.

[I,4. For any three points  $A, B, C$  that do not lie on the same line there exists a plane  $\alpha$  that contains each of the three points  $A, B, C$ . For every plane there exists a point which it contains.]

[I,5. For any three points  $A, B, C$  that do not lie on one and the same line there exists no more than one plane that contains each of the three points  $A, B, C$ .]

[I,6. If two points  $A, B$  of a line  $a$  lie in a plane  $\alpha$  then every point of  $a$  lies in the plane  $\alpha$ .]

[I,7. If two planes  $\alpha, \beta$  have a point  $A$  in common then they have at least one more point  $B$  in common.]

[I,8. There exist at least four points which do not lie in a plane.]

If  $A$  and  $B$  are two distinct points of a line, we will often denote this line by  $\overleftrightarrow{AB}$ .

#### GROUP II: THE ORDER AXIOMS

II,1. If a point  $B$  lies between a point  $A$  and a point  $C$  then the points  $A, B, C$  are three distinct points of a line, and  $B$  also lies between  $C$  and  $A$ .

II,2. For two points  $A$  and  $C$ , there always exists at least one point  $B$  on the line  $\overleftrightarrow{AC}$  such that  $C$  lies between  $A$  and  $B$ .

II,3. Of any three points on a line there exists no more than one that lies between the other two.

#### Definition B-1

Given two distinct points  $A$  and  $B$ , the segment  $\overline{AB}$  consists of the points  $A$  and  $B$  together with all points which are between  $A$  and  $B$ .