Applied and Computational Math Concentration

Formula Sheet for Part II of the comprehensive exam

MAT 451 - Probability

I. Probability

1. The Additive Law of Probability: The probability of the union of two events A and B is

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

2. Bayes Theorem: If events $E_1, \ldots, E_k$ form a partition of sample space S and $E \subseteq S$,

$$P(E_i \mid E) = \frac{P(E \mid E_i)P(E_i)}{P(E)} = \frac{P(E \mid E_i)P(E_i)}{\sum_{j=1}^{k} P(E \mid E_j)P(E_j)}$$

II. Discrete Distributions

Discrete Distributions X	P(x)	Values of x	Mgf
Binomial	$P(x) = \binom{n}{x} p^x (1-p)^{n-x}$	$x = 0, 1, 2, \cdots, n$	$\left[pe^t + (1-p)\right]^n$
Geometric	$P(x) = p\ (1-p)^{x-1}$	$x = 1,\ 2,\ \cdots$	$\left[\frac{pe^t}{1-(1-p)e^t}\right]$
Negative Binomial	$P(x) = \binom{x-1}{r-1} p^r (1-p)^{x-r}$	$x = \{r, r+1, \ldots\}$	$\left[\frac{pe^t}{1-(1-p)e^t}\right]^r$
Poisson	$P(x) = \frac{\lambda^x e^{-\lambda}}{x!}$	$x = 0, 1, 2, \cdots$	$\exp\left[\lambda(e^t - 1)\right]$
Hypergeometric	$P(x) = \frac{\binom{r}{x}\binom{N-r}{n-x}}{\binom{N}{n}}$	x is an integer 0, 1,…,n subject to the restrictions $x \le r$ and n - x ≤ N - r	Undefined

III. Continuous Distributions

Continuous Distributions X	f(x)	Values of x	Mgf
Normal	$f(x)=\frac{1}{\sigma\sqrt{2\pi}}\exp\left[-\left(\frac{1}{2\sigma^2}\right)(x-\mu)^2\right]$	$-\infty \le x \le \infty$	$\exp\left[\mu t+\frac{t^2\sigma^2}{2}\right]$
Gamma	$f(x)=\frac{1}{\Gamma(\alpha)\beta^\alpha}x^{\alpha-1}e^{-x/\beta}$	$x \ge 0$	$(1-\beta t)^{-\alpha}$
Exponential	$f(x)=\frac{1}{\beta}e^{-x/\beta}$	$x \ge 0$	$(1-\beta t)^{-1}$
Chi-square	$f(x)=\frac{1}{\Gamma(v/2)2^{v/2}}x^{(v/2)-1}e^{-x/2}$	$x \ge 0$	$(1-2t)^{-v/2}$
Beta	$f(x)=\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}x^{\alpha-1}(1-x)^{\beta-1}$	$0 \le x \le 1$	Does not exist in closed form

The Gamma function $\Gamma(\alpha)=\int_0^\infty u^{\alpha-1}e^{-u}du$

The moment-generating function $m(t)=E(e^{tX})$

IV. Useful Theorems

1. Tchebysheff's Theorem: *Let X be a random variable with mean* μ and finite variance σ^2. *Then, for any constant k,*

$$P(|X-\mu|<k\sigma)\ge 1-\frac{1}{k^2}$$

2. Central Limit Theorem

Suppose $X_1,\cdots,X_n$ are i.i.d random variables with mean μ and variance σ^2. If $\bar{X}_n$ is defined by

$$\bar{X}_n=\frac{\sum_{i=1}^{n}X_i}{n} \quad \text{and} \quad Z_n=\frac{\bar{X}_n-\mu}{\sigma/\sqrt{n}}$$

Then, as $n\to\infty$, $Z_n \to N(0,1)$

MAT 484 – Mathematical Modeling

Absorbing Markov Chains: Suppose that the system has *a* absorbing states and *b* non-absorbing (transient) states. Arrange the transition matrix T so that the first *b* columns contain the transition probabilities associated to the non-absorbing states:

$$T = \begin{pmatrix} A_{b\times b} & O_{b\times a} \\ B_{a\times b} & I_{a\times a} \end{pmatrix}$$

Then the limiting steady-state matrix $L = \lim_{n\to\infty} T^n$ exists and it has the following structure:

$$L = \begin{pmatrix} O_{b\times b} & O_{b\times a} \\ B_{a\times b}(I_{b\times b} - A_{b\times b})^{-1} & I_{a\times a} \end{pmatrix}$$

Zero-sum games: For a matrix game with pay-off matrix

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

the optimal mixed strategies for the row player [p1,p2] and column player [q1,q2] are given by

$$p_1 = \frac{a_{22} - a_{21}}{a_{11} + a_{22} - a_{12} - a_{21}} \qquad p_2 = \frac{a_{11} - a_{12}}{a_{11} + a_{22} - a_{12} - a_{21}}$$

$$q_1 = \frac{a_{22} - a_{12}}{a_{11} + a_{22} - a_{12} - a_{21}} \qquad q_2 = \frac{a_{11} - a_{21}}{a_{11} + a_{22} - a_{12} - a_{21}}$$

The value of the game is $v = \dfrac{a_{11}a_{22} - a_{12}a_{21}}{a_{11} + a_{22} - a_{12} - a_{21}}$