PHY 171

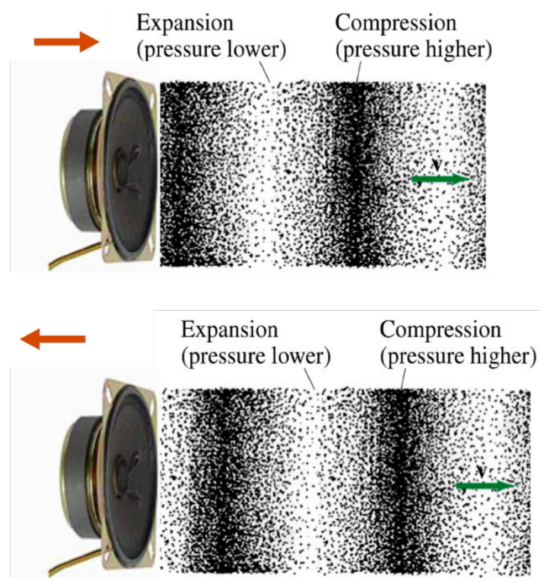
Lecture 5

(*January 16, 2012*)

Sound Waves

Sound waves are longitudinal waves.

An example is shown in the figure on the right, which depicts the sound wave generated by the motion of the diaphragm of a speaker. As the diaphragm moves to the right (top panel), it pushes the air molecules together, creating an area of higher pressure (known as a *compression*). As the diaphragm pulls back (bottom panel), the pressure in this region decreases, and causes an area of lower pressure (*rarefaction*, marked in the figure as "expansion"). This sequence of compressions and rarefactions is then communicated across the air (or the medium through which the sound wave is traveling). Since the particles are vibrating about their mean position parallel to the direction of motion of the wave, we get a longitudinal wave. When this reaches our ears, it sets membranes in the ear into vibration, thereby causing the sensation of sound.



We demonstrated the production of sound by striking a tuning fork. Unlike most of the sounds we hear in our surroundings, a tuning fork produces sound of a single frequency.

> This is because the fundamental mode (the frequency specified on the tuning fork) is due to symmetrical vibrations of the tines, and this symmetry prevents vibrations at the base of the fork where one holds the fork with their fingers. All other modes involve either a little or significant vibration of the base, and cannot exist when we hold the fork with our finger because they will be damped out. If you're interested, there is a great simulation on YouTube that was carried out by using a numerical method called Finite Element Analysis (FEA). The simulation can be viewed at: http://www.youtube.com/watch?v=m7xUtR2qevA.

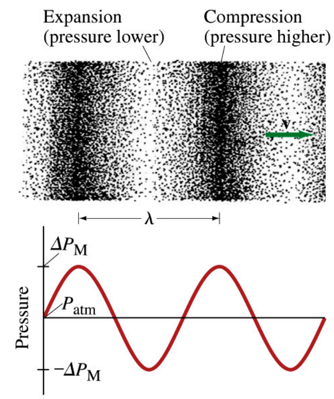
We demonstrated the to-and-fro vibrations of the tines of the tuning fork by dipping it into a beaker of water.

Note that since small objects are vibrating, the sound is not very loud. To make it louder, put it on a larger solid surface (e.g. the table). Now, you're transferring the vibrations to a larger surface (even though you're draining away the energy faster), so you will hear a louder sound. The same effect can be discerned by holding a wind-up toy music box against the desk.

Graphical Representation of sound waves

Unlike strings, where we had graphs of displacement vs. position, things get more complicated for sound waves because we have traditionally displayed two kinds of representations, as described below.

Pressure *vs.* position – PPT slide



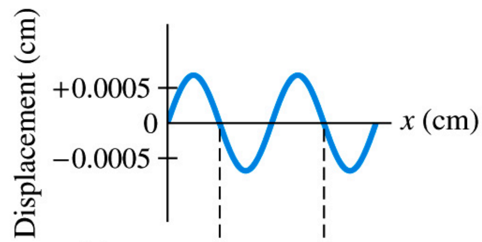
In this graphical representation shown in the figure on the right, longitudinal waves are considered from the point of view of *variations in pressure* rather than displacement.

In a wave compression (when molecules are closer together), the pressure is higher than normal.

In a wave expansion or rarefaction (when molecules are farther apart) the pressure is lower than normal.

As seen from the figure, the usual procedure is to set an average pressure as the zero (usually atmospheric), then compression is on the (+) side, expansion (rarefaction) is on the (–) side. Note that "pressure" here really means "over-pressure," that is, the pressure above or below atmospheric pressure.

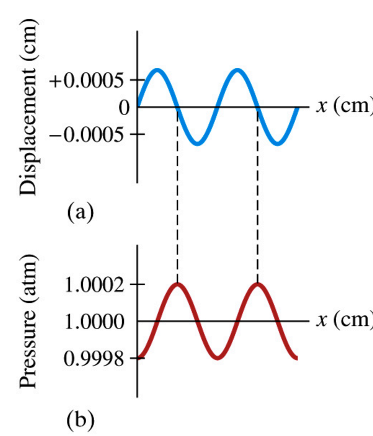
Displacement *vs.* position (*x*) – PPT slide



A graph of displacement *vs.* position for a sound wave is shown in the figure on the right. The displacement shown is that of a tiny volume element of the medium from its equilibrium position.

Remember sound is a longitudinal wave, so the particles of the medium are vibrating parallel to the direction of propagation, even though the graph looks just like that of a transverse wave.

Comparing the pressure *vs.* position and displacement *vs.* position representations



To understand how the two representations compare, consider the following:

When the pressure is a maximum or minimum, the molecules are momentarily at rest in equilibrium so the displacement from equilibrium is zero.

When the pressure variation is zero, the displacement is a maximum or minimum.

Therefore, the *displacement wave is a quarter wavelength* ($\lambda/4 \equiv \pi/2$) *out of phase with the pressure wave*, as shown in the figure on the right.

Sources of Sound

Vibrating Strings

We have already studied the vibration of strings in detail – see the previous posted lecture for transverse waves on a string. Be careful about the following point, though. While the string is vibrating in a transverse way to produce the wave, the sound wave itself is longitudinal. So you can find the frequency of the sound wave from the frequency of transverse vibration of the string (as you will do on a problem) because it is the source of the sound wave, but in order to find the wavelength of the sound wave, you will need to use this frequency and the speed of sound in air.

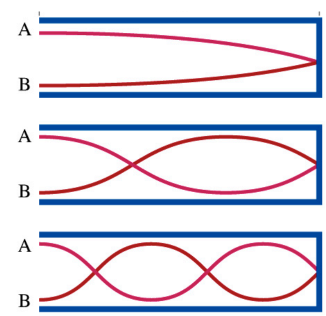
Air Columns

Standing waves can occur in the air of any cavity, but the frequencies generated can be complicated in general, except in the case of simple shapes like long, narrow tubes. Due to a disturbance at one end (e.g., vibrating lip of the player, stream of air directed against an opening), the air within the tube vibrates with a variety of frequencies, but only frequencies that correspond to standing waves will persist in the tube. To **demonstrate** this, we took tubes of different lengths and held vibrating tuning forks at one end. When the frequency of the tuning fork matched the fundamental frequency (or other harmonic in the tube, see below), we got a loud sound (i.e., resonance). If not, we could barely hear the tuning fork.

Below, we will study air columns in more detail. Intuitively, you expect tubes open at both ends to behave similar to a string (except now we have longitudinal waves).

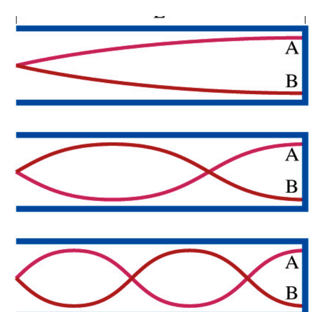
Sound in Air Columns

Can understand open tubes and tubes closed at one end in terms of *displacement picture*, or *pressure picture*.



In the **displacement picture**,

- Always have a displacement node at the *closed* end, because air is not free to move there.
- Always have a displacement antinode at the *open* end, because air is free to move there.



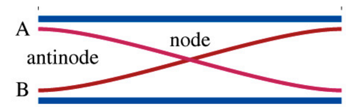
In the **pressure picture**,

- Always have a pressure node at the *open* end, because pressure remains at outside atmospheric pressure there.
- Always have a pressure antinode at the *closed* end, where pressure can readily alternate to be above or below atmospheric pressure.

Tube open at both ends (e.g., flute)

We will now look in detail at the *displacement picture* for an open tube.

Recall that an *open end always has displacement antinodes*, so an open tube will have displacement antinodes at both ends.



At a minimum, therefore, there must be at least one node within an open tube if there is to be a standing wave at all – so a single node within the tube corresponds to the fundamental frequency of the tube.

Now, the distance between two successive antinodes is $\lambda/2$. You can work this out easily by looking at the figure above. From antinode to node is a quarter of the wavelength, and we are adding two of these, so we will get $\lambda/2$ from one antinode to the next antinode. Therefore, we can write for the fundamental mode (recall from strings that this is also called the first harmonic) that

$$L = \lambda_1/2, \text{ so that } \lambda_1 = 2L$$

In other words, the longest wavelength of a standing wave that can be supported by an open tube is equal to twice the length of the tube. Notice that we have put a subscript ("1") on the wavelength to keep track of the harmonic number.

So, the *fundamental frequency* (*first harmonic*) of a tube open at both ends is

$$f_1 = \frac{v}{\lambda_1} = \frac{v}{2L}$$

where v is the velocity of sound in air.

The standing wave with two nodes is called the *second harmonic*. In this case, notice from the figure that there are two quarter waves at each end, and half a wave in between (node to node), so that the length L of the tube has in total: $\lambda/4 + \lambda/4 + \lambda/2 = \lambda$.

Therefore, since $L = \lambda_2$, the frequency of the 2nd harmonic will be

$$f_2 = \frac{v}{\lambda_2} = \frac{v}{L}$$

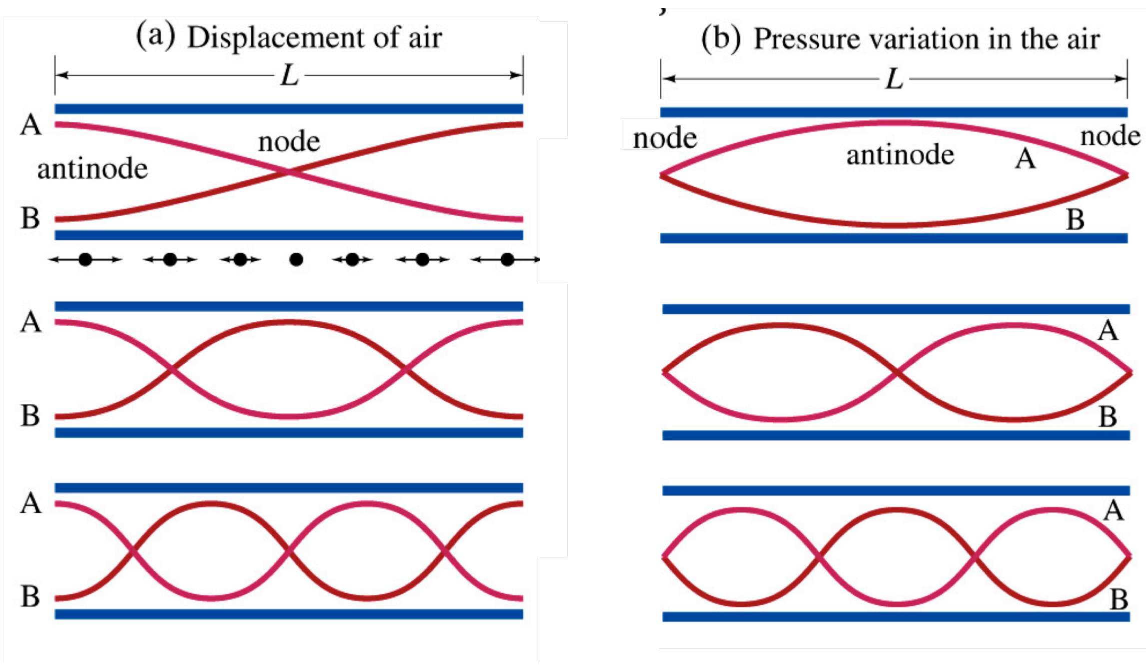
Notice something interesting? As in the transverse vibrations of a string, we have

$$f_2 = \frac{v}{L} = 2\left(\frac{v}{2L}\right) = 2f_1$$

That is, the frequency of the 2nd harmonic f_2 is equal to twice that of the fundamental (f_1).

Continuing in this way, we will get for the frequency f_n of the n^{th} harmonic that $f_n = n\, f_1$. Just as for the transverse vibrations in a string, therefore, the frequency of the longitudinal standing waves in a tube open at both ends is an integer multiple of the fundamental frequency.

In the above, we derived the frequencies of the harmonics of standing waves in a tube open at both ends from the displacement picture. Recall, however, that the displacement and pressure pictures are out of phase by a quarter wavelength. Where we have a displacement antinode we will get a pressure node, and where we have a displacement node, we will get a pressure antinode. To see what the standing waves in an open tube look like in these two representations, let us plot the displacement and pressure pictures side by side:



Again, note in the figure on the left above that even though the curves look identical to the transverse vibrations of a string, the vibrations of the particles are parallel to the direction of motion. To reinforce this concept, the displacement directions (and magnitudes) are displayed below the fundamental mode of the tube on the left.

Next, notice how the quarter wavelength lag between displacement and pressure pictures results in the patterns shown. Each mode has been displayed in the two representations, side by side. On the left is the displacement picture, and on the right is the pressure picture.

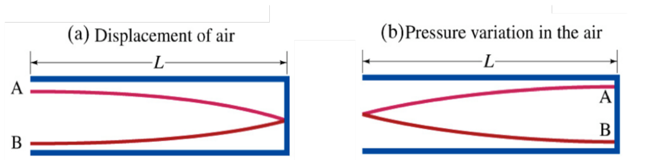
Now, no matter which picture you work with, you should get the same fundamental frequency for a tube of length L, and hence the same harmonics. The mathematics in the pressure description must match up with the displacement description – the physical phenomenon cannot change due to a different model being used to describe it.

> As an example, consider the fundamental mode in the pressure description (top right image above). From node to node is half a wavelength, so the length L of the tube is accommodating half a wavelength, that is, $L = \lambda_1/2$. This gives for the fundamental frequency of the tube:
> $$f_1 = \frac{v}{\lambda_1} = \frac{v}{2L}$$
> which is the same as the fundamental frequency obtained from the displacement picture.

Tube closed at one end (e.g., clarinet)

Recall that we always have a displacement node at the closed end (where air cannot move) and a displacement antinode at the open end (where air can move freely) – see figures on page 3 above. Conversely, we have a pressure node at the open end (which must be at atmospheric pressure) and a pressure antinode at the closed end (where the pressure can vary and be greater or less than the atmospheric pressure).

So, for the fundamental mode, the length L of the tube accommodates a quarter wavelength, $\lambda/4$. As we discussed for the open tube, this must be true whether we consider the displacement picture or the pressure picture. To illustrate this, we have shown both pictures side by side on the right.



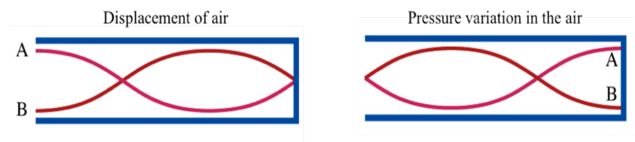
So, for the fundamental mode, we get $\lambda_1 = 4L$ (note that again we have subscripted λ with the number of the harmonic). Therefore, the fundamental frequency of a tube closed at one end is

$$f_1 = \frac{v}{\lambda_1} = \frac{v}{4L}$$

Compare this to the fundamental frequency of a tube open at both ends, which $f_{1o} = \frac{v}{2L}$ is

Therefore, the fundamental frequency of a tube closed at one end is ½ that for a tube of the same length that is open at both ends.

Now, consider the next harmonic. The displacement and pressure pictures are shown on the right. We have a quarter wavelength plus a half-wavelength ($\lambda/4 + \lambda/2$); so we get $L = 3\lambda_h/4$; for now, we have subscripted λ with "h", we'll see why below.



Therefore, the frequency of the this harmonic is

$$f_h = \frac{v}{\lambda_h} = \frac{v}{4L/3} = 3\left(\frac{v}{4L}\right)$$

Notice something odd? (I won't pretend the pun is unintended.) The next higher harmonic from the fundamental is not the 2$^{\text{nd}}$ harmonic, but the third! Its frequency is $f_3 = 3f_1$.

If you continue by sketching the next higher pattern and repeat the above, you will find its frequency is $5f_1$.

In other words, only the odd harmonics are present in a tube closed at one end – they have frequencies corresponding to 3, 5, 7, … times the fundamental frequency. Waves with 2, 4, 6, … times the fundamental frequency cannot exist in a tube closed at one end because they cannot have a node at one end and an antinode at the other.

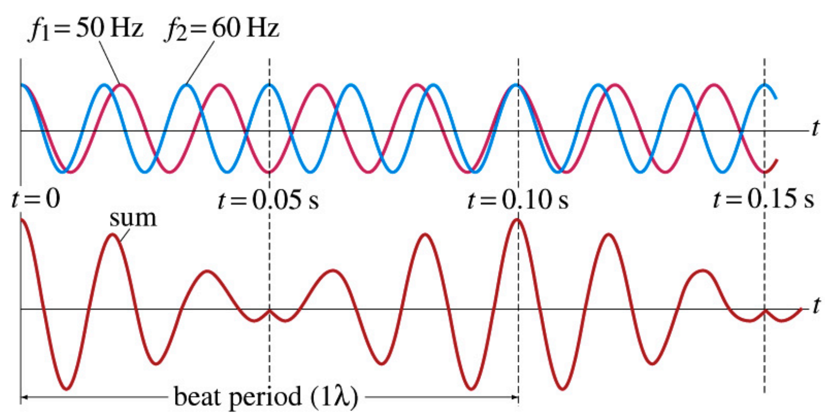
Is there a counterpart to the tube closed at one end for a vibrating string?

This is a Physics GRE favorite, so you should remember it. If one end of a string is in free vibration, then the results will match that for a tube closed at one end. Theoretically, this arrangement is obtained by attaching the string to a massless ring that slides on a frictionless pole. In practice, this is hard to do, so while there are a number of musical instruments that utilize the properties of a stretched string (guitar, violin, cello, harp, one-stringed instruments, two-stringed instruments), there are none that I know of that utilize free vibrations at one end.

Interference of Sound Waves in time

When interference of sound waves takes place in the space domain, we get the patterns we have seen above in open tubes, and tubes closed at one end.

Another interesting phenomenon also results when we have waves interfering in the time domain. Such interference in time gives rise to the phenomenon of beats.



An example is shown in the figure above. In this example, the frequency of one wave is f_1 = 50 Hz, and that of the other wave is f_2 = 60 Hz (as shown in the upper graph). The lower graph shows the sum of the two waves. Note the beat frequency (which is the difference in frequency between the two waves) – waves interfere constructively after 0.1 s, meaning beat frequency is 10 per second, or 10 Hz. In class, we demonstrated how the above pattern is perceived by the human ear – we hear an alternating rise and fall in the sound.

Appendix A

Sound waves require a material medium:

Unlike light, sound waves *require a material medium* in order to propagate. Therefore, sound waves cannot travel in outer space.

> This can be demonstrated by putting a bell inside a glass enclosure and pumping out the air from the enclosure. The sound becomes fainter, and eventually you stop hearing it. (In reality, you will still hear a faint buzzing from sound conveyed along the wires and glass enclosure).

Speed of Sound

For all problems in this class, we will write the speed of sound as 340 m/s (unless specified otherwise; e.g., *some of your homework problems may require you to use 343 m/s*).

> The speed of sound can be measured easily. Just send a square wave of sound (generated in a speaker using a Pasco function generator) down a tube closed at one end (for a 1.5 m long tube, I usually use a square wave of frequency 0.5 Hz). The speaker is placed at the open end of the tube, and the sound wave is reflected at the closed end. The sound wave itself and its reflection are then picked up by a microphone placed at the open end of the tube; one usually plots a graph of sound intensity vs. time, and in it you see the square sound wave and its reflection as peaks in an otherwise nearly flat-line signal. Twice the length of the tube divided by the time interval between consecutive peaks in the graph then gives us the speed of sound in air.

The speed of sound does depend on the temperature of the air through which it is propagating. The speed of sound in air of temperature T (in Kelvin) is given by

$$v = 331\sqrt{\frac{T \text{ (in K)}}{273}}$$

Expression for the speed of sound

We will write down an equation for the speed of a sound wave by analogy with waves on a string. Recall that the velocity of transverse waves on a string is given by

$$v_{\mathrm{T}} = \sqrt{\frac{F_{\mathrm{T}}}{\mu}}$$

where F_{T} is the tension in the string, and μ is the mass per unit length of the string (also known as linear density).

For longitudinal waves like sound, the counterpart of μ in the denominator is the volume density ρ. In order to find a counterpart to F_{T}, consider the following: as the compression wave passes an element of the medium, the amount of compression depends on the elastic property of the medium. This property is called the *bulk modulus*, B. It measures how much change in pressure occurs as a ratio of the change in volume to the original volume. The bulk modulus is given by

$$B = -\frac{\Delta P}{\Delta V/V}$$

The minus sign indicates that an increase in pressure on the medium causes a decrease in the volume of an element.

Therefore, the speed of longitudinal waves (including sound) is given by

$$v = \sqrt{\frac{B}{\rho}}$$

where B is the bulk modulus defined above, and ρ is the volume density.

Now, notice that you have to be careful about figuring out how the velocity of sound depends on the density of the medium. Since the density is in the denominator, a naïve consideration might suggest incorrectly that sound travels slower in a medium of higher density. However, the effect of density is offset by the increase in the bulk modulus, thereby ensuring that the speed of sound is greater in a denser medium.

For an example, consider the speed of sound in air at 20°C and 1 atm. pressure, for which the density of air is 1.21 kg/m^3, and the bulk modulus is 1.4×10^5 N/m^2 (Table 18-1). This gives for the velocity of sound in air:

$$v_{\mathrm{air}} = \sqrt{\frac{1.4 \times 10^5}{1.21}} = 340 \mathrm{\ m/s}$$

Meanwhile, at 20°C and 1 atm. pressure, the density of water is 990 kg/m^3, and the bulk modulus is 2.2×10^9 N/m^2 (Table 18-1). This gives for the velocity of sound in water:

$$v_{\mathrm{water}} = \sqrt{\frac{2.2 \times 10^9}{990}} = 1490 \mathrm{\ m/s}$$

Since 1490/340 ~ 4, this means that sound travels *four times faster in water* compared to air.