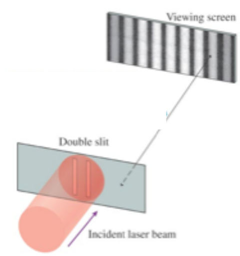
PHY 171

Discussion Session Worksheet 9

(*February 2, 2012*)

Usually, I don't post solutions to discussion worksheets (unlike homework solutions which are always posted), because I want students attending classes to discuss solutions to these problems in class (and I work most of them out after your discussion). However, I'm posting solutions for some problems on this worksheet for which I didn't have the time to go over the solutions in class.

1. Consider the double-slit interference pattern obtained on a screen shown below. Will the fringe spacing (i.e., the distance between two consecutive bright fringes) increase, decrease or stay the same if:



(a) the distance to the screen (from the slits) is increased? Explain.

(b) the spacing between the slits is increased? Explain.

(c) light of a longer wavelength is used in the experiment? Explain.

Note: No solution posted, since you'll be looking for answers to these questions in lab.

2. Two narrow slits 50 μm apart are illuminated with light of wavelength 500 nm.

(a) What is the angle of the $m = 2$ bright fringe in degrees?

Note: When a question like this is asked, they are asking you to find the angle θ made by the specified fringe with the center line from between the two slits to the screen at the position of the central ($m = 0$) bright fringe.

Solution:

Given $d = 50\ \mu m = 50 \times 10^{-6}\ m$

$\lambda = 500\ nm = 500 \times 10^{-9}\ m$

Get bright fringes for $d \sin\theta = m\lambda$

For $m = 2$ bright fringe, therefore, $d \sin\theta = 2\lambda$

So $\sin\theta = \frac{2\lambda}{d} = \frac{2(500 \times 10^{-9})}{50 \times 10^{-6}} = 0.02$

$\Rightarrow \boxed{\theta = 1.15^\circ}$

(b) If the screen is at a distance of 1.25 m from the slits, how far away will the $m = 2$ bright fringe be from the central bright fringe on the screen?

Solution: Given $L = 1.25$m, from geometry (e.g., see Figure 22.4 on page 674 of your text)

$$y = L \tan\theta = (1.25\ m) \tan 1.15^\circ = 0.0251\ m = 2.51\ cm$$

Since θ is small, you can also use the small angle approximation, $\tan\theta \approx \sin\theta \approx \theta$, provided you write θ in radians.

3. A double slit is illuminated simultaneously with orange light of wavelength 600 nm and light of an unknown wavelength. The $m = 4$ bright fringe of the unknown wavelength overlaps the $m = 3$ bright orange fringe. What is the unknown wavelength?

Solution:

The condition for bright fringes is

$$d \sin \theta = m\lambda \quad , \quad m = 0, 1, 2, 3, \ldots$$

For the orange light ($\lambda = 600$ nm), we get the $m=3$ fringe at

$$d \sin \theta_o = 3\,(600 \text{ nm})$$

For the unknown wavelength, we get the $m=4$ fringe at

$$d \sin \theta_u = 4\,(\lambda)$$

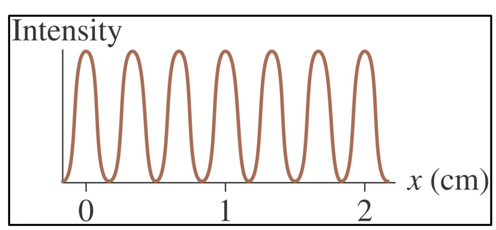
Since the positions of these fringes are the same,

$$\theta_o = \theta_u$$

$$\Rightarrow \quad 3(600 \text{ nm}) = 4\lambda$$

$$\Rightarrow \quad \lambda = \frac{3\,(600 \text{ nm})}{4} \Rightarrow \boxed{450 \text{ nm}}$$

4. Light of 600 nm wavelength illuminates a double slit. The intensity pattern shown on the right is seen on a screen 2.0 m behind the slits. What is the spacing (in mm) between the slits?



Note: No solution posted, since this was handed in, and detailed comments have been provided in the in-class worksheets handed back to students.

5. In a double slit experiment, the slit separation is 200 times the wavelength of the light. What is the angular separation (in degrees) between two adjacent bright fringes?

Solution: Since $d \sin \theta = m\lambda$ for a bright fringe, we can write for two adjacent fringes that

$$d \sin \theta_1 = m\lambda \quad \text{and} \quad d \sin \theta_2 = (m + 1)\,\lambda,$$

Using the small angle approximation ($\sin \theta = \theta$, in radians), this becomes

$$d\,\theta_1 = m\lambda \quad \text{and} \quad d\,\theta_2 = (m + 1)\,\lambda,$$

We need the angular separation between these two adjacent bright fringes, so

$$d\,\theta_2 - d\,\theta_1 = (m + 1)\,\lambda - m\,\lambda = \lambda,$$

from which we find that

$$\Delta\theta = \theta_2 - \theta_1 = \lambda/d = \lambda/200\lambda = 1/200 \text{ radians}$$

because $d = 200\,\lambda$.

Therefore, the angular separation between two adjacent fringes in degrees is

$$\Delta\theta = 1/200 \text{ radians} = (1/200)*(180°/\pi) = \mathbf{0.286°}$$

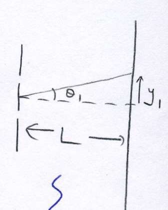
6. A double slit interference pattern is created by two narrow slits spaced 0.20 mm apart. The distance between the first and fifth minimum on a screen 60 cm behind the slits is 6.0 mm. What is the wavelength (in nm) of the light used in this experiment?

Solution:

Given $d = 0.20 \text{ mm} = 0.20 \times 10^{-3} \text{ m}$

$L = 60 \text{ cm} = 60 \times 10^{-2} \text{ m}$

$y_5 - y_1 = 6.0 \text{ mm} = 6 \times 10^{-3} \text{ m}$



Minima occur at $d \sin \theta = (m + \tfrac{1}{2})\lambda$, $m = 0, 1, 2, 3, \ldots$

From figure above $\sin\theta_1 \approx \tan\theta_1 = y_1/L$

So, 1st minimum $(m=0)$ at $d \sin \theta_1 = \lambda/2$

5th minimum $(m=4)$ at $d \sin \theta_5 = (4 + \tfrac{1}{2})\lambda = 9\lambda/2$

Subtracting,

$$d \sin \theta_5 - d \sin \theta_1 = 9\lambda/2 - \lambda/2 = 8\lambda/2 = 4\lambda$$

$$\Rightarrow d(y_5/L) - d(y_1/L) = 4\lambda$$

$$\Rightarrow \lambda = \frac{1}{4}\left[\frac{d}{L}(y_5 - y_1)\right]$$

$$= \frac{1}{4}\left[\frac{0.20 \times 10^{-3}}{60 \times 10^{-2}}(6 \times 10^{-3})\right] = 5 \times 10^{-7} \text{ m} = \boxed{500 \text{ nm}}$$