PHY 475
Homework 4
(Due by beginning of class on Wednesday, May 16, 2012)

It took me a while searching for good problems on dark matter, and in the end, I decided to make up my own. So, this homework is being posted two days later than I would have liked to have it online. Consequently, it ***will be accepted without penalty until noon on Friday (5/18)****. Please leave the HW in my mailbox if you're submitting it on Thursday or Friday. I will clear my mailbox at noon on Friday, and no homework will be accepted after that time.*

1. The surface brightness Σ of an astronomical object is defined as its observed flux divided by its observed angular area; that is, $\Sigma \propto f/(\delta\theta)^2$.

(a) For a class of objects which are both standard candles and standard yardsticks, what is Σ as a function of redshift?

(b) Would observing the surface brightness of this class of objects be a useful way of determining the deceleration parameter q_0? Explain clearly why or why not.

2. When a photon passes an object of mass M at an impact parameter b (see Figure 8.5 in your text), it will be deflected by an angle $\alpha = 4GM/c^2b$ (equation 8.48).

Suppose now that an object of mass M is acting as a gravitational lens. To keep things simple, assume that this lens is exactly along the line of sight between the observer and the lensed object. The distance from observer to the lensed object is d, and the distance from observer to the lens is xd, where $0 < x < 1$.

(a) Show that the angular radius of the Einstein ring for this configuration is given by

$$\theta_E = \left(\frac{4GM}{c^2 d} \frac{1-x}{x} \right)^{1/2}$$

Remember that due to the large distances involved, all angles are small, and consequently, $\sin\theta \approx \tan\theta \approx \theta$, provided θ is expressed in radians.

(b) For what value of the fraction x would you get the best Einstein ring (i.e., the Einstein ring with the largest angular radius). Show your calculations/derivations clearly.

(c) Find the angular radius θ_E of the Einstein ring (in arcsec) for a microlensing event in which the lens is a *MACHO* of mass $0.3M_\odot$ in the Galactic halo at a distance of 10 kpc from the Earth, and the lensed object is a star in the LMC at a distance of 50 kpc from the Earth.

Note that $1M_\odot = 1.99 \times 10^{30}$ kg, and π radians $\equiv 180 \times 3600$ arcsec.

(d) If the timescale for a microlensing event is the time it takes for a *MACHO* to travel through an angular distance equal to the Einstein radius θ_E as seen from Earth, calculate the time (in days) for the microlensing event described in part (c), if the *MACHO* is moving at 200 km s^{-1}.

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3. Study the paper by Riess et al. (2000) on Type Ia supernovae that has been posted at the undergrad (PHY 375) website at the link under "Writing Assignments."

(a) Figures 1 and 2 (which have been reproduced as Figures 7.5 and 7.6 in your text) present complementary interpretations of the Type Ia supernovae data, but each presents one or more aspects of the data that is missing in the other. Can you identify some of these?

(b) In Chapter 7, Ryden mentions that while H_0 can be measured by observing Type Ia supernovae at $z \sim 0.1$, the acceleration (or deceleration, in principle) must be measured by observing Type Ia supernovae at higher redshift. From your study of the Riess et al. (2000) paper, why would you need to observe at higher redshift to measure the acceleration (or deceleration)?

(c) While an accelerating universe has been adopted as the most likely interpretation of the Type Ia supernovae data, what alternative scenarios do Riess et al. (2000) discuss that could account for the same observations? For each scenario, discuss in brief how it would mimic the observed effect.

Submit neat work, with answers or solutions clearly marked by the question number. Unstapled, untidy work will be charged a handling fee of 20% penalty. Writing only an answer without showing the steps used to get to that answer will fetch very few points, even if the answer is correct. Late homework will not be accepted.

The following problem is for practice on Chapter 6 only. Do not submit this problem.

A. Consider an expanding, positively curved universe containing only a cosmological constant ($\Omega_0 = \Omega_{\Lambda,0} > 1$).

(a) Show that such a universe underwent a "Big Bounce" at a scale factor: $a_{\rm bounce} = \left(\dfrac{\Omega_0 - 1}{\Omega_0}\right)^{1/2}$

(b) Show that the scale factor as a function of time is

$$a(t) = a_{\rm bounce} \cosh\left[\sqrt{\Omega_0}\, H_0 \left(t - t_{\rm bounce}\right)\right]$$

where $t_{\rm bounce}$ is the time at which the Big Bounce occurred.

Note: You will need to integrate an expression at some stage, and just to save you time, the integral table will give you the answer in ln. So, you should substitute the limits and then use $\cosh^{-1}(x) = \ln\left(x + \sqrt{x^2 - 1}\right)$.

(c) What is the time $(t_0 - t_{\rm bounce})$ which has elapsed since the Big Bounce, expressed as a function of H_0 and Ω_0?