

PHY 475

Homework 3

(Due by beginning of class on Wednesday, April 25, 2012)

Submit neat work, with answers or solutions clearly marked by the question number. Unstapled, untidy work will be charged a handling fee of 20% penalty. Writing only an answer without showing the steps used to get to that answer will fetch very few points, even if the answer is correct. Late homework will not be accepted.

1. The cosmological constant has come under renewed scrutiny in recent years (with a different value from Einstein's, of course), because it may be a contributor to the dark energy that is responsible for the acceleration of the expansion of the Universe.
 - (a) Calculate the energy density of the cosmological constant in the current epoch, assuming $\Omega_\Lambda = 0.7$ and $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$.
 - (b) What is the total energy of the cosmological constant within a sphere 1 AU in radius?
 - (c) What is the rest energy of the Sun ($E_\odot = M_\odot c^2$)?
 - (d) Comparing your answers above, do you expect the cosmological constant to have a significant effect on the motion of planets within the Solar System?
2. While Einstein introduced his cosmological constant to get a static universe, he was never satisfied with it. This wasn't merely due to the aesthetics of imposing such a solution. A significant problem with his cosmological constant was that while it made the universe static, it also made it unstable. To illustrate this, consider Einstein's static universe to be comprised only of non-relativistic matter with mass density ρ , and a cosmological constant, $\Lambda = 4\pi G\rho$. Suppose, now, that *some* of the non-relativistic matter is converted into radiation (e.g., by stars). Will the universe start to expand or contract? *You must show calculations justifying your answer; a mere statement like "expands" or "contracts" will be awarded zero points.*
3. In a *flat universe* with $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$, you observe a galaxy at a redshift $z = 7$. Carry out calculations to find the current proper distance to the galaxy, $d_p(t_0)$, in each of the following 3 cases. Also, carry out calculations to find the proper distance at the time the light was emitted, $d_p(t_e)$, again in each of the following 3 cases.
 - (a) Show your calculations for $d_p(t_0)$ and $d_p(t_e)$ if the universe contains *only radiation*?
 - (b) Show your calculations if the universe contains *only matter*?
 - (c) Show your calculations if the universe contains *only a cosmological constant*?
 - (d) Put all your answers for $d_p(t_0)$ and $d_p(t_e)$ for the three cases above in a table.