PHY 475/375
Lecture 3
(April 2, 2012)

Observational Anchors

At this stage, it would be a good idea to remind ourselves of what we are trying to accomplish.

We wish to find a cosmological model that tells us how the Universe got to where it is today, and where it might possibly be going.

Any such model must be constrained by the following observations and assumptions based on observations.

- Galaxies are moving away from us, and the farther they are, the faster they appear to be moving.
- The night sky is dark (Olbers' paradox).
- On large scales, the Universe is *isotropic* and *homogenous*.
- The Universe contains different types of particles.
- The Universe is filled with a background radiation (that we observe over the entire sky, and observe to be similar in all directions), called the *Cosmic Microwave Background* (CMB).

We have already discussed how galaxies are moving away from us according to Hubble's law: $v = H_0\, r$, so that the farther away the galaxies are (larger r), the faster they appear to be moving (higher v).

Let us now look at some details of the others one by one.

The night sky is dark (*Olbers' paradox*)

We learned in the last lecture that as we look farther and farther at more distant galaxies, we are looking further and further back in time. So, how far back do we go? Did the Universe always exist, or did it have a beginning?

A deceptively simple observation provides an answer. If you look up at the night sky, you'll see a couple of thousand stars scattered about the sky, but mostly you'll notice that the sky is dark. The fact that the night sky is dark at visible wavelengths is known as Olbers' Paradox.

So, *why should it be a paradox that the night sky is dark at visible wavelengths*?

The darkness of the night sky certainly posed no problem for the ancients, since to them the stars were just points of light stuck on a celestial dome. But after Copernicus, stars didn't need to be stuck on a celestial dome, and could be at any distance from the Sun (as we know they are today).

Let us first figure out how bright we would expect the night sky to be in an infinite universe.

Let L be the average luminosity of a star. *Luminosity* is the rate at which energy is radiated away from an object in the form of light. (Note that we are really talking about the luminosity at optical frequencies here, i.e., frequencies visible to the human eye).

Let n be the average number density of stars in the Universe, i.e., the average number of stars per unit volume.

The flux received at the Earth from a star of luminosity L at a distance r is given by an inverse square law:

$$f(r) = \frac{L}{4\pi r^2} \tag{2.1}$$

Let us now consider a thin spherical shell of stars centered on the Earth, as shown in Figure 2.1 of your text (page 7). The shell has radius r and thickness dr.

The intensity of radiation received at the Earth from this shell of stars (i.e., the power per unit area per steradian of the sky) is then

$$dJ(r) = \frac{L}{4\pi r^2}\Big(n\Big)\, r^2\, dr = \frac{nL}{4\pi}\, dr \tag{2.2}$$

because the volume of the shell will be $r^2\, dr$, so there will be $n\, r^2\, dr$ stars in it. One point of interest is that the total intensity of starlight from the shell depends only on the thickness of the shell, and not its distance from us.

In order to find the total intensity of starlight from all the stars in the universe, we will need to integrate from $r = 0$ to $r = \infty$:

$$J = \int_0^\infty dJ = \frac{nL}{4\pi}\int_0^\infty dr = \infty \tag{2.3}$$

We have demonstrated that the night sky should be infinitely bright!

Since this is clearly not the case, some of the assumptions that have gone into the above calculation must be wrong. Let us take a closer look at some of our assumptions.

- One assumption in the above calculation was that we have an unobstructed line of sight to every star in the Universe. Of course, this can't be, and nearby stars should hide the ones behind them. Nevertheless, every line of sight should end on a star, so even if we don't get an infinitely bright sky, we should still get a sky that is very different from the dark one we see.

  Note, the above can't be resolved by allowing for interstellar matter that absorbs starlight, since the interstellar matter would be heated until it had the same temperature as starlight, and would glow as brightly as the stars.

- In taking n and L outside the integral in equation (2.3), we have made a second assumption that they are constant throughout the Universe, or to put it more appropriately, that the product nL is constant and does not depend on r. This might not be true if distant stars are less luminous, or if there are fewer stars per unit volume at larger distances than nearby.

- A third assumption made in the above calculation is that the Universe is infinitely large (by taking the upper limit $r = \infty$). If, instead, the Universe extends to a maximum distance r_{max} from us, then the total intensity of starlight will only be $J \sim nLr_{\text{max}}/4\pi$. Note, however, that this would also be the case if the Universe were infinite in space, but devoid of stars beyond a distance r_{max}.

- A fourth assumption made in the above treatment is that the Universe is infinitely old. If instead the Universe was of a finite age t_0, then the upper limit in equation (2.3) above would be $r_{\text{max}} \sim ct_0$, so that we would obtain $J \sim nLct_0/4\pi$. Of course, this would also be true if the Universe was infinitely old but has had stars in it for a finite time t_0.

- A fifth assumption is that the inverse square law for the flux of light $f \propto 1/r^2$ is valid throughout the universe, no matter how far away the light may be coming from. There may be something in the geometry of the universe that changes this at large distances, or if the universe is expanding or contracting, the light from distant sources could be redshifted to lower photon energies or blueshifted to higher photon energies respectively.

In fact, our fourth assumption turns out to be the primary resolution to Olbers' paradox, that the Universe in fact has a finite age. The stars beyond some finite distance, called the horizon distance, are invisible to us because their light hasn't had the time to reach us yet. As we set up our cosmology, we will see how it will be consistent with the resolution of Olbers' paradox.

For now, we will move on to stating and understanding two of the fundamental assumptions of cosmology, and see how they hold up to observations.

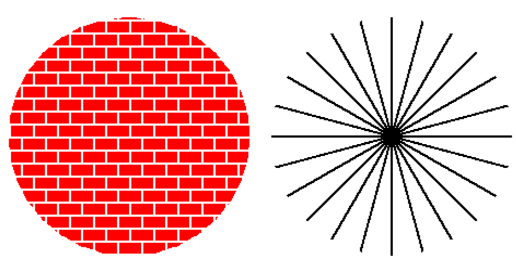
Isotropy and Homogeneity

Recall that we said that cosmologists consider the Universe to be isotropic and homogenous on the largest scales. So, what is isotropy and homogeneity?

- Isotropy means that there is no preferred direction in the Universe; it looks the same no matter which direction you point your telescope.

- Homogeneity means there are no preferred locations in the Universe; it looks the same no matter where you set up your telescope.

Note that one doesn't imply the other, as shown in the figure on the right (taken from Ned Wright's page at UCLA). The red brick pattern in the left is an example of a homogenous pattern that is not isotropic, whereas the spoke-like pattern on the right is an example of an isotropic pattern that is not homogenous.

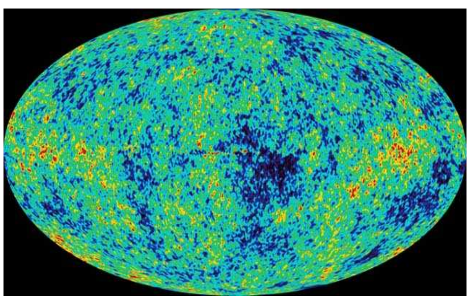


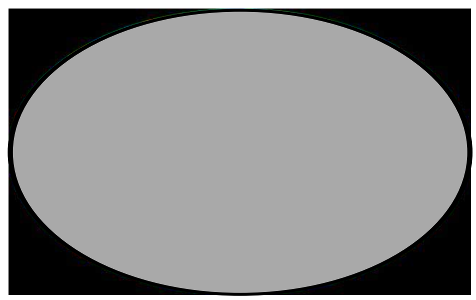
Now, recall we examined structure in our Universe up to scales of 50 Mpc, and there certainly wasn't isotropy or homogeneity at that level.

So, when we say the Universe is isotropic and homogenous on the largest scales, we mean scales of the order of 100 Mpc, or greater. Statistically speaking, most volumes at this level do contain the same proportion of superclusters and voids, as revealed by large redshift surveys (e.g., the 2dF redshift survey we looked at in an earlier lecture).

The stipulation of isotropy and homogeneity was imposed mainly because it was felt necessary to avoid having a special location in the Universe. It is called the *Cosmological Principle.*

Eventually, isotropy and homogeneity are borne out by the cosmic microwave background (CMB), about which we will learn more shortly. For now, let us just say that it is microwave radiation that pervades the entire Universe, and comes to us uniformly from all directions. Usually, the map most people are used to seeing of the CMB is the one generated by the WMAP satellite, and it looks like the one on the left.





However, the fluctuations you see in the WMAP image on the left are at the miniscule level of 10^{-5}, and are, if anything, a credit to the capability of the detectors on WMAP and the people who designed them. If you didn't have this precision, though, the CMB would simply look like the grey image on the right — the ultimate proof of isotropy and homogeneity of our Universe.

As an aside for now, note that this doesn't mean we will ignore these tiny fluctuations however, since they are eventually the seeds of all the structure that exists. It is just for now that we choose to ignore them, to illustrate the concept of isotropy and homogeneity.

We are now at an interesting point. Theory, supported by observations on the largest scales, stipulates the cosmological principle, namely that the Universe is homogenous and isotropic. Meanwhile, observations tell us that galaxies are moving away from us, and the more distant the galaxy is, the faster it is moving away. Doesn't this put us at a special location in the Universe though, thereby violating the cosmological principle?

The answer is that the cosmological principle is valid, and in fact, not only do we see distant galaxies receding from us, every observer in every galaxy (if they exist) would also see distant galaxies receding from them. Therefore, rather than being a contradiction, the recession of distant galaxies is a consequence of the isotropy and homogeneity of the Universe.

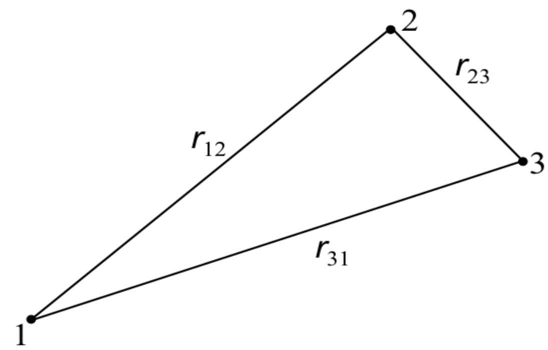
Let us now look at this in a quantitative way. Consider three galaxies at positions $\vec{r}_1$, $\vec{r}_2$, and $\vec{r}_3$.

As shown in the figure on the right, the positions of these galaxies define a triangle with sides of length

$$r_{12} \equiv \left|\vec{r}_1 - \vec{r}_2\right| \tag{2.8}$$

$$r_{23} \equiv \left|\vec{r}_2 - \vec{r}_3\right| \tag{2.9}$$

$$r_{31} \equiv \left|\vec{r}_3 - \vec{r}_1\right| \tag{2.10}$$



If the Universe is to remain homogenous and isotropic as it expands, the shape of the triangle in the figure above must be preserved. In order to maintain the relative lengths of the sides of the triangle, the expansion law must be of the form

$$r_{12}(t) = a(t)\, r_{12}(t_0) \tag{2.11}$$

$$r_{23}(t) = a(t)\, r_{23}(t_0) \tag{2.12}$$

$$r_{31}(t) = a(t)\, r_{31}(t_0) \tag{2.13}$$

where $a(t)$ is a scale factor, equal to one at the present moment ($t = t_0$), and totally independent of location or direction. The scale factor tells us how the expansion (or contraction) of the Universe depends on time.

At any time t, an observer in galaxy 1 will see the other galaxies receding with speeds:

$$v_{2\,\mathrm{from}\,1}(t) = \frac{dr_{12}}{dt} = \frac{d}{dt}\Big[a(t)\, r_{12}(t_0)\Big] = \dot{a}\, r_{12}(t_0) = \dot{a}\left[\frac{r_{12}(t)}{a}\right]$$

where, in the usual notation, $\dot{a} = da/dt$, and we have substituted equation (2.11) in the last step on the right hand side. We get finally

$$v_{2\,\mathrm{from}\,1}(t) = \left(\frac{\dot{a}}{a}\right) r_{12}(t) \tag{2.14}$$

Likewise, we get

$$v_{3\,\mathrm{from}\,1}(t) = \frac{dr_{31}}{dt} = \left(\frac{\dot{a}}{a}\right) r_{31}(t) \tag{2.15}$$

Equations (2.14) and (2.15) both have the form of the Hubble law we wrote earlier: $v = H_0\, r$, with the Hubble constant (or, more properly, Hubble parameter) given by $\dot{a}/a$, and r being the distance between the two galaxies.

Likewise, it is easy to show that an observer in galaxy 2 or 3 will see the same Hubble law, with galaxies moving away with speeds proportional to distance. For example, for galaxy 2, we get

$$v_{1\,\mathrm{from}\,2}(t) = \frac{dr_{12}}{dt} = \left(\frac{\dot{a}}{a}\right) r_{12}(t)$$

and

$$v_{3\,\mathrm{from}\,2}(t) = \frac{dr_{23}}{dt} = \left(\frac{\dot{a}}{a}\right) r_{23}(t)$$

which is the same linear relation $v = H_0\, r$ as found by the observer in galaxy 1, with $H_0 \equiv \dot{a}/a$, and r being the distance between the two galaxies.

Since the argument above can be applied to any 3 galaxies, it implies that in a universe undergoing homogenous, isotropic expansion, the velocity-distance relation must take the linear form $v = H_0\, r$, where r is the distance between any pair of galaxies, and $H_0 \equiv \dot{a}/a$. Every observer in that universe sees every other galaxy moving away with the speed given by the Hubble law.

You may have noticed in the above that the movement of galaxies away from each other is *not* a motion through space. Rather, it is *space itself that is expanding*, and carrying the galaxies away with it. One way to visualize this is to think about baking a raisin cake, where the raisins move away from each other as the cake is baked (although you need to be careful to understand that all of the Universe is the raisin cake; there is no Universe outside the cake).

Now, if galaxies are moving away from each other, they must have been closer together in the past. We can run the expansion movie backwards and eventually find that the Universe must have started from a very dense (and hot) state. This initial dense state from which the Universe began to evolve is what is known as the Big Bang. It is an unfortunate name, because it conjures up in every mind the idea of an explosion. There was no such thing. In fact, the term Big Bang was coined as an insult against the idea of a hot dense beginning by Fred Hoyle who believed in a "*Steady State*" theory of continuous creation and destruction of matter; it was an attractive name, and it stuck!

Consider next a pair of galaxies currently separated by a distance r, with a velocity $v = H_0\, r$ relative to each other. If there are no forces acting to accelerate or decelerate their relative motion, then their velocity is constant, and the time that has elapsed since they were very close together is

$$t_0 = \frac{r}{v} = \frac{r}{H_0\, r} = \frac{1}{H_0} \equiv H_0^{-1} \tag{2.16}$$

independent of the current separation r.

The time H_0^{-1} is referred to as the *Hubble time.*

For $H_0 = 70 \pm 7$ km s^{-1} Mpc^{-1}, the Hubble time is $H_0^{-1} = 14.0 \pm 1.4$ Gyr
(where Gyr stands for gigayear, equal to 10^9 yr).

The importance of the Hubble time H_0^{-1} is that it gives us an idea of the approximate age of the Universe.

- If the attractive force of gravity acting on the largest scales has slowed down the expansion of the Universe over time, so that the Universe was expanding more rapidly in the past than it is now, then the Universe is younger than H_0^{-1}.
- On the other hand, if the expansion of the Universe is speeding up (as seems likely to be the case, which we will learn when we discuss dark energy in more detail), the Universe may be older than H_0^{-1}.

Just as the Hubble time provides a natural time scale for our Universe, the Hubble distance $c/H_0 = 4300 \pm 400$ Mpc provides a natural distance scale. Just as the age of the Universe is equal to H_0^{-1} with the exact value depending on the expansion history of the Universe, as discussed above, so too the horizon distance, which is the greatest distance a photon can travel during the age of the Universe is equal to c/H_0, with the exact details again depending on the expansion history.

Note how Hubble's Law ties in with Olbers' Paradox. If the Universe has a finite age, then the night sky can be dark even if the Universe is infinitely large, because light from distant galaxies has not yet had the time to reach us.

The Universe contains different types of particles

The objects that we observe around us daily are made up of protons, neutrons, and electrons (at the sub-atomic level).

Your text gives details about these particles in Chapter 2, and you should read them. Here, we will be concerned only with a few of these details.

Just as the history of cosmology has involved searching for the largest scale structures, the history of atomic and nuclear physics has often involved going smaller and smaller to find the smallest building blocks of matter. At this time, *quarks* and *leptons* are considered the most "elementary particles," that is, no smaller units are considered to exist from which quarks and leptons can be built up.

Starting from these quarks, particle physicists have constructed a standard model that can account for all other known particles (over 200). No isolated quark has ever been seen, however, so the quark model is only indirectly verifiable by its predictions of the properties of particles built up from these quarks. In the standard model, there are six "flavors" (i.e., kinds) of quarks:

up, *down*, *charm*, *strange*, *top*, & *bottom*

The up, charm, and top quarks each have a charge of $+2/3$, whereas the down, strange, and bottom quarks each have a charge of $-1/3$. Also, for each type of quark, there is a corresponding antiquark.

According to the standard model, quarks occur only in groups, and are never found alone. Composite particles made of quarks are called *hadrons*, and while individual quarks have fractional electric charges, they combine so that hadrons only have a net integer electric charge.

There are two classes of hadronic particles.

- Particles comprised of 3 quarks are called *baryons*.
- Particles comprised of a quark and an antiquark are called *mesons*.

Two of the most well known examples of baryons are *protons* and *neutrons*.

- A proton is made up of 2 up quarks and 1 down quark, so its electrical charge is

$$2\left(\frac{2}{3}\right)+1\left(-\frac{1}{3}\right)=+1$$

- A neutron is made up of 1 up quark and 2 down quarks, so its electrical charge is

$$1\left(\frac{2}{3}\right)+2\left(-\frac{1}{3}\right)=0$$

The other type of fundamental particles of matter are the *leptons*. Unlike the sociable quarks that exist only with other quarks in composite particles, leptons are solitary creatures.

There are 6 leptons. Three of these have electrical charge, and three do not.

The best known lepton is the *electron* (e), which has a negative charge. The other two charged leptons are the *muon* (μ) and the *tau* (τ); both also have negative charges, but are much more massive than the electron. The other 3 leptons are the three types of *neutrinos* that have no electrical charge.

If youre interested in knowing more, you could look at several sites. Most of the materials in this lecture related to quarks and leptons have been taken from the Berkeley Lab website at

`http://www.particleadventure.org/quarks_leptons.html`

Since the mass of protons and neutrons is much greater than that of an electron, the ordinary matter that we see around us in the form of molecules, atoms, and ions is usually referred to as baryonic matter (i.e., in spite of it containing leptons, the name ignores this fact, because the mass contributed by them is so much smaller than that contributed by the baryons). About 3/4 of the baryonic matter (by mass) in the Universe is currently in the form of hydrogen, while most of the rest is primarily helium. Heavier elements only contribute a small percentage of the mass in the Universe.

What is important to understand here is that any cosmological theory must be able to account for this composition.

The Universe is filled with a uniform background radiation

Electromagnetic radiation (visible light, infrared, radio, UV, X-rays, gamma rays), in general, can be modeled either as a wave or a stream of particles called photons. Photons are considered as massless particles that interact readily with matter.

When it is described in terms of waves, electromagnetic radiation is usually characterized by its frequency (f) or wavelength (λ), with $c = f\lambda$.

Meanwhile, when it is described in terms of photons, electromagnetic radiation is usually characterized by its energy $E = hf$, where h is the Planck's constant.

A feature that is common to all stars is that the radiation emitted from their surfaces can be characterized by their surface temperature only (measured in kelvin, K). This is also true of a gas cloud, as it is of an iron poker or any other object. An alternative way of putting this is that every object in the Universe radiates electromagnetic radiation due to its temperature. The total energy radiated, and the wavelength at which the peak intensity is emitted depends on the temperature of the object only. This is called black body radiation. More properly, most objects are only approximate black bodies, and an emissivity term varying between 0 and 1 is needed to express this fact, but it isn't relevant for most objects we will discuss, so we will consider all of them to be black bodies.

The energy density of black body photons in the frequency range f to $(f + df)$ is given by

$$\varepsilon(f)\, df = \frac{8\pi h}{c^3} \frac{f^3\, df}{\exp{(hf/kT)} - 1} \tag{2.25}$$

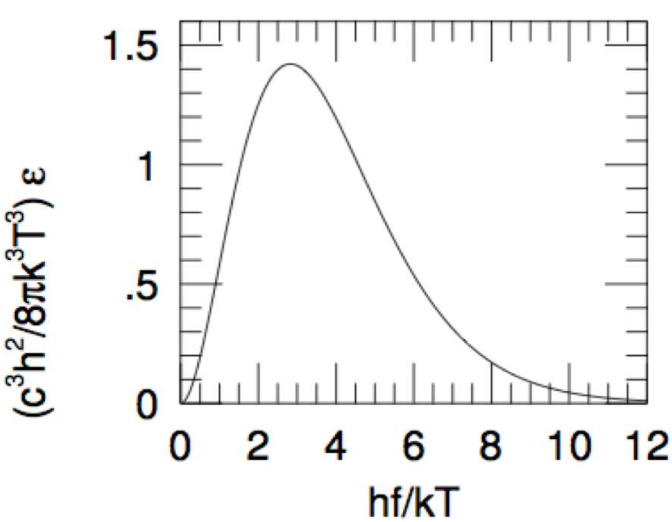
If we plot equation (2.25), with hf/kT along the horizontal axis, and $\varepsilon(f)$, scaled by the quantities shown along the vertical axis, we will get a plot like that shown below.

Notice that the function peaks at a specific value of frequency given by

$$hf_{\text{peak}} \approx 2.82\, kT$$

When written in terms of wavelength, this is the well known Wien's Law:

$$\lambda_{\text{peak}}(\text{in nm}) \;=\frac{3\times 10^6}{T(\text{in K})}$$



Note that "peak" here does not refer to peak wavelength, but rather the *frequency or wavelength at which the peak intensity is emitted.*

Wien's Law tells us that a hotter object emits its peak intensity at higher frequencies, or equivalently, shorter wavelengths. You may recall the example from introductory physics in the chapter on heat, where an iron poker first feels hot (infrared radiation), then red hot, and eventually would become white hot if heated to a sufficiently high temperature.

Now, if we integrate equation (2.25) over all frequencies, we will get a total energy density for blackbody radiation of

$$\varepsilon_\gamma = \alpha T^4 \tag{2.26}$$

where

$$\alpha = \frac{\pi^2}{15} \frac{k^4}{\hbar^3 c^3} = 7.56 \times 10^{-16} \text{ J m}^{-3} \text{ K}^{-4} \tag{2.27}$$

Be careful that you write T only in kelvins in equation (2.26).

In equation (2.27), k is Boltzmann's constant; $k = 1.38 \times 10^{-23}$ m^2 kg s^{-2} K^{-1}

$\hbar = h/2\pi$, where h is Planck's constant; $h = 6.626 \times 10^{-34}$ m^2 kg s^{-1}

Meanwhile, the number density of photons in black body radiation can be found from equation (2.25) as

$$n_\gamma = \beta T^3 \tag{2.28}$$

where

$$\beta = \frac{2.404}{\pi^2} \frac{k^3}{\hbar^3 c^3} = 2.03 \times 10^7 \text{ m}^{-3} \text{ K}^{-3} \tag{2.29}$$

As examples, you radiate an approximate black body spectrum. Since your body temperature $\sim 37°\text{C} \equiv 310$ K, its peak can be found from Wien's law to be at about

$$\frac{3 \times 10^6}{310K} = 9677 \text{ nm} \approx 10^4 \text{ nm} = 10 \text{ micron}$$

This peak will be in the infrared region of the electromagnetic spectrum.

The Sun also radiates an approximate black body spectrum. Since its surface temperature is about 5800 K, the peak will occur at about

$$\frac{3 \times 10^6}{5800K} = 517 \text{ nm} \approx 10^4 \text{ nm}$$

which falls in the green region of the spectrum; the curve is pretty flat over the entire visible spectrum, which is why the Sun looks yellowish-white. Note that the actual visible Solar spectrum is not a smooth curve, but has numerous absorption dips called Fraunhofer absorption lines.

The material on the following pages was covered in class on W (4/4), but is included here for continuity.

Discovery of the CMB

In 1964, Arno Penzias and Robert Wilson, two radio astronomers working at Bell Labs, were carrying out observations with a horn antenna. This antenna had originally been built by Bell Laboratories to communicate with satellites, but had been rendered obsolete by a system built by a rival company, and had therefore been released for research. Penzias and Wilson decided to use it to study radio signals from the Milky Way and galaxies. During the testing stage, they kept finding a persistent signal that manifested itself regardless of the direction in which the antenna was pointing. They were completely mystified, and even went to the extent of shooing out pigeons from the antenna and cleaning out their droppings, but the signal remained. From discussions with Robert Dicke and Jim Peebles of Princeton, it soon became clear to them that this might be the "relic radiation" left over from the evolution of the early Universe that had been predicted years earlier in 1948 by George Gamow, Ralph Alpher, and Robert Herman. In fact, Dicke and Peebles had been planning to build a telescope to try and detect this radiation. Penzias and Wilson coordinated the publication of their discovery in the Astrophysical Journal in 1965 with Dicke and Peebles, who published a separate paper explaining Penzias and Wilson's results. Penzias and Wilson received the Nobel Prize in 1978 for their discovery.

The Cosmic Microwave Background Explorer (COBE) satellite measured this microwave background radiation very precisely, and found that it was well fitted by a black body spectrum with temperature

$$T_{\rm CMB} = 2.725 \pm 0.001 \text{ K} \tag{2.30}$$

The energy density of the CMB can be found from equation (2.26) to be

$$\varepsilon_\gamma = \alpha\, T^4 = \left[7.56 \times 10^{-16}\right] \left(2.725\right)^4 = 4.17 \times 10^{-14} \text{ J m}^{-3} \tag{2.31}$$

Converting to a popular energy unit known as electron volts (eV), where 1 eV $= 1.602 \times 10^{-19}$ J, this is roughly equivalent to a quarter of an MeV per m^3 of space.

From equation (2.28), the number density of CMB photons is

$$n_\gamma = \beta\, T^3 = \left[2.03 \times 10^7\right] \left(2.725\right)^3 = 4.11 \times 10^8 \text{ m}^{-3} \tag{2.32}$$

which means that there are about 411 CMB photons per cm^3 of the Universe at the present day.

The mean energy of CMB photons, however, is quite low, given by

$$E_{\rm mean} = \frac{\varepsilon_\gamma}{n_\gamma} = \frac{4.17 \times 10^{-14} \text{ J m}^{-3}}{4.11 \times 10^8 \text{ m}^{-3}} = 1.014 \times 10^{-22} \text{ J}$$

which when expressed in eV, by dividing by 1.602×10^{-19} J/eV, gives

$$E = 6.34 \times 10^{-4} \text{ eV} \tag{2.33}$$

While the energy is very low, yet the presence of CMB photons is a critical clue to the early conditions in the Universe, and indicates that the Universe must have begun in a hot, dense state.

At the high temperatures of the early Universe (> 10,000 K), the baryonic matter would have been totally ionized.

> Such hot ionized matter creates black body radiation, so there would have been lots of photons flying around in the early Universe.
>
> Moreover, free electrons interact very strongly with photons. Since there were many free electrons available in the early Universe, a photon could not have traveled very far before encountering another free electron and being scattered again. Thus, the early Universe was opaque.
>
> The constant interaction of matter and photons also kept them in thermal equilibrium with each other, so that imprinted on the photons was the prevailing temperature of the Universe at that time.

As the Universe expanded, though, it cooled. When the temperature had dropped to 3000 K, ions and electrons would have combined to form neutral atoms. Since there were no longer a significant number of free electrons, the black body photons would have started streaming freely throughout the Universe. These photons are what we observe today as the Cosmic Microwave Background (CMB).

One might well ask, though, why we see the CMB at 2.7 K today rather than the 3000 K that would have been imprinted on these photons at the time of the last scattering with free electrons, after which the photons started to stream freely through the Universe? The answer is that this is due to the expansion of the Universe.

We will look at this quantitatively in a minute, but it is important to realize why the CMB seems to be coming to us equally from all directions, and with the same temperature (of 2.725 K).

> It is best to look at this from an Earth-centered perspective by drawing a sphere designated as the *surface of last scattering.*
>
> When the Universe had cooled to 3000 K and formed neutral atoms, we would have had the last scattering event, and the photons produced would have had the prevailing temperature of the Universe at that time (3000 K) imprinted on them. Since there were no more free electrons to scatter them, these photons would then have started to stream through the Universe.
>
> So, if we imagine a sphere centered on the Earth, with the surface of the sphere being the surface of last scattering just as the Universe had reached 3000 K, photons from all the points on that sphere would have reached Earth now, except their wavelengths would be stretched by the expansion of the Universe, so that the black body would appear to be at a lower temperature at the present time (which, we know, is equal to 2.725 K).

Let us now look at this quantitatively.

Consider a region of volume V, which expands at the same rate as the Universe, so that $V \propto a(t)^3$, where $a(t)$ is the scale factor that we defined earlier.

The black body radiation in this volume can be thought of as a gas of photons with energy density $\varepsilon_\gamma = \alpha T^4$ (from equation 2.26).

Moreover, since the photons in the gas also have momentum, the photon gas also has a pressure, given by $P_\gamma = \epsilon_\gamma/3$.

Such a photon gas must obey the laws of thermodynamics. Let us write the first law of thermodynamics:

$$dQ = dE + P\,dV \tag{2.34}$$

where dQ is the amount of heat flowing into or out of the photon gas in the volume V, dE is the change in the internal energy of the gas, and the work done by the photon gas is the pressure (P) times the change in volume (dV).

In a homogenous universe, there cannot be any net flow of heat (since everything must be at the same temperature), so we can write $dQ = 0$. Therefore, equation (2.34) can be written with the passage of time as

$$\frac{dE}{dt} = -P(t)\,\frac{dV}{dt} \tag{2.35}$$

Now, since the total energy can be written as the product of the energy density times the volume:

$$E = \varepsilon_\gamma\,V = \alpha T^4\,V$$

and the pressure is given by

$$P = P_\gamma = \frac{\varepsilon_\gamma}{3} = \frac{\alpha T^4}{3}$$

equation (2.35) can be rewritten as

$$\frac{d}{dt}\left(\alpha T^4\,V\right) = -\left(\frac{\alpha T^4}{3}\right)\frac{dV}{dt}$$

from which we obtain

$$\alpha\left[4T^3\left(\frac{dT}{dt}\right)V + T^4\left(\frac{dV}{dt}\right)\right] = -\left(\frac{\alpha T^4}{3}\right)\frac{dV}{dt} \tag{2.36}$$

Dividing by $\alpha\,T^4\,V$ on both sides, and rearranging terms as below

$$\left(\frac{4}{T}\right)\frac{dT}{dt} = -\left(\frac{1}{3V}\right)\frac{dV}{dt} - \left(\frac{1}{V}\right)\frac{dV}{dt} = -\left(\frac{1}{3V} + \frac{1}{V}\right)\frac{dV}{dt} = -\left(\frac{1+3}{3V}\right)\frac{dV}{dt}$$

we obtain

$$\left(\frac{4}{T}\right)\frac{dT}{dt} = -\left(\frac{4}{3V}\right)\frac{dV}{dt}$$

Therefore, we get finally

$$\frac{1}{T}\frac{dT}{dt} = -\left(\frac{1}{3V}\right)\frac{dV}{dt} \tag{2.37}$$

Now, recall that

$$\frac{d}{dt}\Big[\ln Y\Big] = \frac{1}{Y}\frac{dY}{dt}$$

Applying this in reverse, equation (2.37) becomes

$$\frac{d}{dt}\Big[\ln\, T\Big] = -\left(\frac{1}{3}\right)\frac{d}{dt}\Big[\ln\, V\Big]$$

But note that as the Universe expands, we said $V \propto a(t)^3$, so the above equation becomes

$$\frac{d}{dt}\Big[\ln\, T\Big] \propto -\left(\frac{1}{3}\right)\frac{d}{dt}\Big[\ln\, a(t)^3\Big]$$

Now, since $\ln\,(a^n) = n\,\ln\,a$, the above equation modifies to

$$\frac{d}{dt}\Big[\ln\, T\Big] \propto -\left(\frac{1}{\cancel{3}}\right)\cancel{3}\,\frac{d}{dt}\Big[\ln\, a(t)\Big]$$

and we are left with

$$\frac{d}{dt}\Big[\ln\, T\Big] \propto -\frac{d}{dt}\Big[\ln\, a(t)\Big]$$

Applying $n\,\ln\,a = \ln\,(a^n)$, we get

$$\frac{d}{dt}\Big[\ln\, T\Big] \propto \frac{d}{dt}\Big[\ln\, a(t)^{-1}\Big] \tag{2.38}$$

which implies that

$$T \propto a(t)^{-1} \tag{2.39}$$

Therefore, we have demonstrated that as the Universe expands, its temperature drops linearly with the scale factor of the expansion $a(t)$. Since the temperature has dropped by a factor of 3000 K/2.7 K = 1100 since the Universe became transparent, the scale factor $a(t)$ has increased by a factor of 1100 since then.