PHY 475/375
Lecture 15
(May 16, 2012)

CMB Anisotropies (contd.)

Recall that we studied in the last lecture that the CMB is isotropic to a very low level of $\delta T/T$, reflecting the isotropy of the Universe.

At the 10^{-3} level, there is a *dipole anisotropy* caused by the net motion of the COBE satellite with respect to a frame in which the CMB is isotropic.

After removing this dipole anisotropy, there is anisotropy at very low levels of $\delta T/T \sim 10^{-5}$ — this is expected, since there must have been non uniformity in the matter density to cause structure in the Universe today. So photons traveling from denser regions would have had to climb out of a deeper potential well and arrived cooler, whereas photons from less dense regions did less work and arrived warmer. So, detection of this temperature anisotropy is direct evidence for primordial density non uniformity, which explains why George Smoot won the Nobel Prize in 2006 for detecting this anisotropy (he shared it with John Mather, who won for characterizing the black body spectrum).

Note that $\delta T/T \sim 10^{-5}$ is smaller than expected, based on the observed structure of luminous matter, but is resolved by the presence of dark matter. Of course, the density inhomogeneities of dark matter cannot be seen via CMB anisotropy, since dark matter has no electromagnetic interactions. Nevertheless, clumping of dark matter formed the gravitational potential wells into which the baryonic matter fell. This early extra growth of density perturbations would have meant that less baryonic inhomogeneity was needed at recombination to produce the structure seen today. In other words, a smaller $\delta T/T$ is sufficient in the baryonic matter to create the structure seen today, if the bulk of the nonluminous matter is non baryonic. We will set precise limits from the CMB power spectrum below.

Now, recall from the last lecture that:

- We can decompose the fluctuations using spherical harmonics:

$$T_f(\theta, \phi) \equiv \frac{\delta T}{T}(\theta, \phi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} a_{lm} Y_{lm}(\theta, \phi) \qquad (9.45)$$

- Assuming Gaussian random fluctuations in the underlying density (that are responsible for the temperature fluctuations in the CMB), all the information about the CMB fluctuations are contained in the second order angular power spectrum

$$C_l = \left\langle \left| a_{lm} \right|^2 \right\rangle$$

  The connection between the angular power spectrum C_l and the real space correlation function $C(\theta)$ is written in equation (9.47) in the last lecture.

- In general, a particular C_l is a measure of temperature fluctuations between points on the sky separated by an angle $\theta \sim 180^\circ/l$.

  This means that the multipole l is interchangeable, for all practical purposes, with the angular scale θ. Larger values of l correspond to smaller angular scales.

- The $l = 0$ term gives the overall constant, or the average value of T_f over the whole sky.

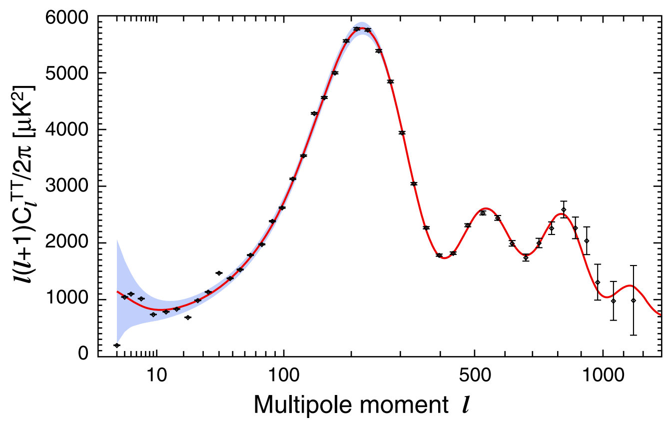
- The $l = 1$ term has an angular scale of $180^\circ \equiv 360^\circ/2$, and is the dipole term which results from the Doppler shift due to our motion through space.

- It is the terms with $l \geq 2$ that are of most interest to cosmologists.

Again, recall that in presenting the results of CMB observations, it is customary to plot the function

$$\Delta_T^2 \equiv \left[\frac{l(l+1)C_l}{2\pi}\right] \langle T \rangle^2$$

along the vertical axis, as shown in the figure on the right (taken from the NASA GSFC WMAP page), or its square root. The data from WMAP are plotted with error bars. The solid red line is a ΛCDM model. The shaded areas show the cosmic variance errors.



We can separate the CMB power spectrum shown above into 3 regions.

- Region 1 is for $l < 100$ (remember smaller l corresponds to larger angular scales). This flat region at large angular scales, called the *Sachs-Wolfe plateau*, corresponds to oscillations with a period larger than the age of the Universe at the photon decoupling time (this term is defined at the end of this lecture). These waves are essentially frozen in their initial configuration.

- Region 3 is for $l > 1000$. Photon decoupling did not take place instantly, but took about 50,000 yr. So the last scattering surface (also defined at the end of this lecture) had a finite thickness. Photons diffused out from any over-dense region if that region was smaller than the mean free path of the photon (which was increasing as the Universe expanded). The net effect was an exponential damping of the oscillation amplitude on the very small scales (remember larger l corresponds to smaller scales).

- Region 2 ($100 < l < 1000$) is comprised of angular scales where the photon-baryon fluid exhibited the fundamental modes and overtones that we discussed in the last lecture. We will now discuss how the relative heights of the acoustic peaks are related to cosmological parameters.

Before discussing the acoustic peaks, though, let us consider the following.

Recall that, at recombination, the Universe was already dominated by matter, so equation (6.6) from Lecture 10 can be written as

$$\frac{H^2}{H_0^2} = \frac{\Omega_{m,0}}{a^3} = \Omega_{m,0}\Big(1+z\Big)^3 = 0.3\,\Big(1+z\Big)^3$$

so that

$$H(z_{ls}) = \left[\frac{70,000 \text{ m s}^{-1} \text{ Mpc}^{-1}}{3.1 \times 10^{22} \text{ m Mpc}^{-1}}\right]\sqrt{0.3}\Big(1+z\Big)^3 = 1.24 \times 10^{-18} \text{ s}^{-1}\Big(1+z\Big)^{3/2}$$

Consider, then, the following length scale at the time of last scattering (at which $z = 1100$):

$$\frac{c}{H(z_{ls})} \approx \frac{3 \times 10^8 \text{ m s}^{-1}}{1.24 \times 10^{-18} \text{ s}^{-1}(1+1100)^{3/2}} \approx 6.6 \times 10^{21} \text{ m} \approx 0.2 \text{ Mpc}$$

In a flat geometry, a patch of the last scattering surface with this physical size will have an angular size, as seen from Earth of

$$\theta_H = \frac{c/H(z_{ls})}{d_A}$$

We can calculate d_A from equation (7.41):

$$d_a = \frac{d_{\text{hor}}(t_0)}{z} \approx \frac{14,000 \text{ Mpc}}{1100} = 13 \text{ Mpc}$$

So, we get that a patch of the last scattering surface with physical size $c/H(z_{ls})$ will have an angular size, as seen from Earth of

$$\theta_H = \frac{c/H(z_{ls})}{d_A} \approx \frac{0.2 \text{ Mpc}}{13 \text{ Mpc}} \approx 0.015 \text{ rad} \approx 0.9^\circ$$

Now look at the power spectrum.

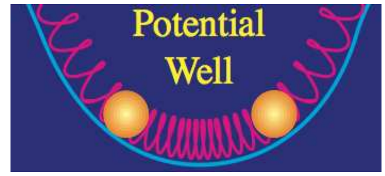
The strongest acoustic peak is near $l \sim 200$, so $\theta \sim \dfrac{180^\circ}{200} \sim 0.9^\circ \approx \theta_{\text{H}}$ — the Universe is very close to being *spatially flat*!!!

How would it be different if the Universe were curved? Recall that in a positively curved universe, two parallel rays will converge. So, if you have two rays that terminate at the observation point at $z = 0$, and end on either side of a region of fixed size at $z \sim 1100$, you should get a larger angle than 0.9°. That is, in a positively curved universe, the first acoustic peak should be seen at $\theta > 0.9^\circ$, i.e., at $l < 200$.

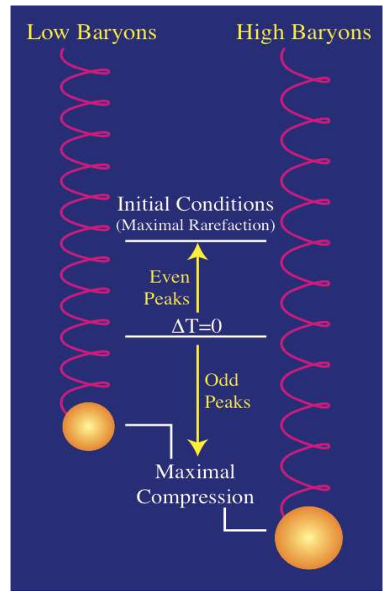
On the other hand, parallel rays diverge in a negatively curved universe, so with the same construction described above, the first acoustic peak should be seen at $\theta < 0.9^\circ$, i.e., at $l > 200$.

To understand the second acoustic peak, we consider a model of masses and springs, where the masses are analogous to the baryonic matter, and the springs are analogous to the radiation in the Universe.

They are located inside a potential well, as shown the figure on the right (taken from Wayne Hu's website). The potential wells themselves are the result of the same fields that were responsible for inflation.



More baryons add inertia to the photon-baryon fluid, which is equivalent to adding more mass on the spring, as shown the figure on the right (also taken from Wayne Hu's website). So adding more baryonic matter means greater compression at the bottom of the potential well, but maximum rarefaction is not impacted because it does not depend on the masses. Now, think of these compressions and rarefactions in analogy to sound waves in an open tube, like we did in the previous lecture. The first peak is then the result of the mode that just compresses, whereas the second peak is the mode that compresses, then rarefies. Likewise, we can think of all odd-numbered peaks as compression peaks, and all even-numbered peaks as rarefaction peaks. This means that if we were to fix everything and increase only the baryonic content, the odd-numbered compression peaks would be increased in height compared to the even-numbered rarefaction peaks. Helpful simulations illustrating the above are available on Wayne Hu's website: http://background.uchicago.edu/∼whu/metaanim.html.



Note added after lecture: When I saw first saw this analogy, I jumped on it because it related a potentially complicated concept to a well known model. However, after thinking about it a little more, I've decided it's not a very good analogy, because the springs are being made to do something no spring I've worked with has ever done (i.e., have the same equilibrium position and asymmetric amplitude!) — my cautionary note from the beginning of this course endures — analogies for complicated phenomena such as these are rarely satisfactory.

In any case, getting back to the CMB power spectrum, the ratio of the amplitudes of the 2nd peak to the amplitude of the 1st peak can be used to set the baryon fraction $\Omega_{\rm b}$.

From 7-yr WMAP data, $\Omega_b = 0.046 \pm 0.001$, meaning that 4.6% of our Universe is composed of baryonic matter. This is more than detected in actual counts (which, as we discussed in a previous lecture, we tend to undercount), but is consistent with inferences from nucleosynthesis.

Measuring the first 3 acoustic peaks helps to also constrain Ω_m, the total matter content of the Universe (including luminous and non luminous matter). From 7-yr WMAP data, $\Omega_{\rm DM} = 0.233 \pm 0.013$, meaning that 23.3% of our Universe is composed of non baryonic dark matter.

The Early Universe

To understand the early Universe, we must take note of the following fact. Each particle of mass m has associated with it a characteristic threshold temperature T_{th}, where $kT_{th} = mc^2$, so that if $T > T_{th}$, the particles will be relativistic. Also, above T_{th}, particle/anti-particle pairs can be continually created and destroyed, e.g.,

$$pa + \overline{pa} \Leftrightarrow \gamma + \gamma$$

where $pa \equiv$ particle, $\overline{pa} \equiv$ antiparticle (e.g., pa could be protons, or neutrons).

As T falls below T_{th}, creation of new particle/anti-particle pairs will stop, and existing particle/anti-particle pairs should annihilate, unless there was a small excess of particles over anti-particles (which there must have been, otherwise we wouldn't be here!). Note that the energy from these annihilations goes into the photon gas, raises its temperature, and is redistributed among those species that are still coupled to the photons.

It is worth remembering that T_{th} can be calculated if m is given in MeV from

$$T_{th} = \frac{m}{\text{MeV}} \times 10^{10} \text{ K} \tag{10.a}$$

e.g., for protons with $m = 1.6 \times 10^{-27}$ kg, we get

$$E = mc^2 = 1.5 \times 10^{-10} \text{ J} = \left(\frac{1.5 \times 10^{-10} \text{ J}}{1.6 \times 10^{-19} \text{ J/eV}} \right) 10^{-6} \text{ MeV/eV} = 939 \text{ MeV}$$

so that, for protons $T_{th} \sim 10^{13}$ K; in similar manner, for electrons, $T_{th} \sim 5 \times 10^9$ K.

Therefore, subsequent to the Big Bang, the decreasing temperature in the Universe allowed for the existence at any particular epoch of various groups of elementary particles. When the falling thermal energy kT could no longer produce a certain type of particle-antiparticle pairs, the lack of a fresh supply of anti-particles resulted in their disappearance from equilibrium states.

Now, to determine what kinds of particle reactions would be taking place at any given time in the early Universe, we must take into account the reaction rate. All reactions in the Universe will have an associated *reaction rate* Γ and, its inverse, a characteristic *reaction timescale*. The reaction rate is given by

$$\Gamma = n\sigma v \tag{10.b}$$

where n is the number density of the reactant particles, σ is the reaction cross section, and $v =$ relative velocity of the particles.

For a reaction to occur, its reaction rate Γ must be faster than the expansion rate of the Universe as measured by the contemporary Hubble parameter H:

$$\Gamma \geq H \tag{10.c}$$

Now, you may have realized already that the reaction rate in equation (10.c) is linked to the prevalent temperature through the velocity v. This is because the velocity distribution is determined by the thermal energy of the system.

So, as the Universe cooled, the reaction rate would fall until it became slower than the expansion rate of the Universe. At this moment, the reactions would effectively cease, and the numbers of reactants on either side of the reaction would become fixed with the value they had at this moment (when the reaction rate and expansion rate of the Universe had crossed). This situation is called *freeze-out*. In essence, the particles have thermodynamically decoupled from those that are still undergoing reactions.

Let us now look in more detail at some epochs in the early Universe.

Epoch of Thermal Energy ≥ 1 GeV ($t < 10^{-5}$ s)

A convenient reference point to begin is when the temperature of the Universe was $T = 10^{13}$ K, from which we get

$$\text{Thermal Energy} \sim kT = \left(1.38 \times 10^{-23}\ \text{J K}^{-1}\right)10^{13}\ \text{K} = \frac{1.38 \times 10^{-10}\ \text{J}}{1.6 \times 10^{-19}\ \text{J/eV}} \approx 10^{9}\ \text{eV} = 1\ \text{GeV}$$

This corresponds to a cosmic time $t \sim 10^{-5}$ s.

This is a convenient reference point because, as we said moments ago, the threshold temperature for freeze-out of protons is 10^{13} K. So, at temperatures $> 10^{13}$ K, there is complete thermodynamic equilibrium between protons and anti-protons, neutrons and anti-neutrons, electrons and positrons, photons, and neutrinos, and we have approximately equal numbers of each species.

Most of the physics of this era is highly uncertain. Therefore, we will ignore this epoch, and regard its end at $t \sim 10^{-5}$ s, when the individual nucleons (i.e., protons and neutrons) became non-relativistic, as the start of the *standard model* of the Big Bang.

Before we leave this epoch, though, we can entertain ourselves with some of the incredible things that must have happened during it.

- At some point above $T > 10^{13}$ K ($t < 10^{-5}$ s), it is believed that individual nucleons (i.e., protons and neutrons) likely gave way to a quark-gluon plasma.
- Around 100 Gev ($T \sim 10^{15}$ K), it is believed that the electromagnetic and weak forces were unified into an electroweak force (i.e., one would have measured 3 fundamental forces at this time, instead of 4 like we have today).
- At much greater energies, perhaps around 10^{25} eV, it is believed (based on extrapolations of coupling constants) that the strong force and electroweak forces were united into one force. Of course, this is highly speculative.
- Finally, at much higher energies (perhaps at the Planck energy of 10^{28} eV, it is believed that gravity was also unified with all the other forces, so instead of four fundamental forces, you had one unified force.

There are many puzzles left over from this epoch. One of the most prominent is what caused the asymmetry that resulted in an excess of matter over antimatter. More generally, how were baryons produced? Were primordial black holes formed during any of these early phases?

Aside: This is a good time to introduce the Planck quantities, which are obtained by appropriate combinations of G, c, and $\hbar$ $(= h/2\pi)$. We are interested in the Planck energy:

$$E_{\rm P} = M_P\, c^2 = \left(\frac{\hbar c}{G}\right)^{1/2} c^2 = \left(\frac{6.626 \times 10^{-34}\ {\rm J.s}\ (3 \times 10^8\ {\rm m\ s^{-1}})^5}{6.67 \times 10^{-11}\ {\rm m^3\ kg^{-1}\ s^{-2}}}\right)^{1/2} = 1.96 \times 10^9\ {\rm J}$$

so that we get

$$E_{\rm P} = \frac{1.96 \times 10^9\ {\rm J}}{1.6 \times 10^{-19}\, {\rm J/eV}} = 1.2 \times 10^{28}\ {\rm eV}$$

Try the others on your own: Planck length, Planck time, Planck temperature. Check your answers with equations (1.1)-(1.5) in your text.

Epoch of Thermal Energy 1 GeV - 0.1 MeV (10^{-5} s $< t <$ 10 s)

As we enter this epoch, recall that freeze-out of protons and neutrons has taken place. That is, with T of the Universe now below T_{th} of protons and neutrons, these particles are no longer relativistic, and as such, proton/anti-proton and neutron/anti-neutron pairs can no longer be spontaneously created and destroyed. At freeze-out, there was a slight excess of matter over anti-matter due to some as yet unexplained reason, so we have protons and neutrons left over (that is, not all protons and neutrons were annihilated by their anti-particles).

The energy is therefore now shared between the photons and the leptons: electrons (e^-), muons (μ^-), and neutrinos (ν_e, ν_μ, ν_τ), and their corresponding anti-particles ($e^+, \mu^+, \overline{\nu}_e, \overline{\nu}_\mu, \overline{\nu}_\tau$); the tau-particles ($\tau^+$ and τ^-) have already annihilated. Muons and anti-muons also freeze out quickly around 10^{-4} s.

Although the muons and tau-particles have decayed, the much lighter μ-neutrinos (ν_μ) and τ-neutrinos (ν_τ) are still around (as are the electron-neutrinos, ν_e), as well as their corresponding anti-neutrinos ($\overline{\nu}_e, \overline{\nu}_\mu, \overline{\nu}_\tau$), so they continue to interact with electrons through the so-called neutral current weak interactions:

$$e^+ + e^- \Leftrightarrow \nu_i + \overline{\nu}_i \qquad (10.d)$$

where $i = e, \mu$, or τ, for e-neutrinos, μ-neutrinos, and τ-neutrinos respectively, and their anti-neutrino counterparts.

Note that the reactions in equation (10.d) involve the weak force and not the electromagnetic force that is usually involved in the annihilation of charged particles into photons. Since electrons are still closely coupled to the photons, the reactions in equation (10.d) keep the neutrinos coupled to the photons also.

Calculations show that the mu-neutrinos and tau-neutrinos freeze out at around 5×10^{10} K. After this point, ν_μ and ν_τ neutrinos do not interact with each other or with the rest of the Universe, except gravitationally.

The electron neutrinos continue to interact with electrons (e^-) and positrons (e^+) via the so-called charged current weak interactions:

$$p + e^- \Leftrightarrow n + \nu_e$$
$$n + e^+ \Leftrightarrow p + \overline{\nu}_e \tag{10.e}$$

These reactions have a slightly shorter reaction timescale than the neutral current weak interactions, so electron-neutrinos (and $\overline{\nu}_e$) freeze out only after the temperature has fallen to about 10^{10} K. After this, ν_e and $\overline{\nu}_e$ neutrinos do not interact with each other or with the rest of the Universe, except gravitationally.

The temperature of the neutrinos continues to fall with the expansion of the Universe as $T \propto a^{-1}$ for as long as they are relativistic, which they always will be if they have zero rest mass. Recall that we obtained $T \propto a^{-1}$ for CMB photons from thermodynamics in Lecture 3; this applies for any relativistic particle, like neutrinos.

Also note that we continue to have neutrinos and anti-neutrinos, i.e., they do not annihilate (unlike p and n). This is true even if neutrinos have significant mass and become non-relativistic at some later epoch. Pair annihilation will still not occur at their threshold temperature because the neutral current weak interactions (10.d) required for neutrino-antineutrino annihilation have a reaction timescale that greatly exceeds the timescale of the age of the Universe.

Thus the number density of neutrinos and anti-neutrinos remains comparable to that of the photons. Recall that we discussed in an earlier lecture that in the present epoch, the Universe should be filled everywhere with these primordial neutrinos and anti-neutrinos with a thermal spectrum ($T_{\nu,0} \approx 1.9$ K). We have not detected them yet, because they lie below the detection limits of our current technology.

As the temperature reaches near $T \sim 5 \times 10^9$ K (thermal energy $kT \sim 0.5$ MeV), the electron (e^-) and positron (e^+) pairs annihilate. Thus, positrons disappear from the Universe (the baryon/anti-baryon asymmetry can be extended to electrons-positrons because the Universe is electrically neutral; because there were more electrons than positrons, all the positrons would have been annihilated).

All the energy from the electron-positron annihilation goes into the photons only, because the freeze out of all forms of neutrinos has already occurred.

It is because the photons receive this extra energy that they are predicted to have a slightly higher temperature than the neutrinos. Calculations show that this should be given by $T_\nu = T_\gamma/1.4$. This is how we find that today, the temperature of the neutrinos should be $T_{\nu,0} = 1.9$ K, as discussed above.

Stated another way, the temperature difference between photons and neutrinos today tells us that neutrinos decoupled thermodynamically prior to electron annihilation.

During all the time that we have been following the Universe (i.e., from $T = 10^{13}$ K), there would have been a tendency for protons and neutrons to bind through strong interactions into nuclear bound states. However, the energetic photons present would have blasted apart these nuclei almost immediately (photodissociation). When the thermal energy fell to $kT \sim 0.7$ eV ($T \sim 8 \times 10^9$ K), however, the photons were no longer energetic enough to photodissociate the bound nuclei. This would eventually set the ^{4}He abundance, as we will learn when we look at Big Bang Nucleosynthesis in more detail.

Epoch of Thermal Energy 0.1 MeV - 1 eV ($10 \text{ s} < t < 400,000$ yr)

With the free electrons left behind after electron-positron annihilation (due to an excess of electrons over positrons), there is enough scattering between photons and free electrons that matter and radiation are still strongly coupled, and remain at essentially the same temperature.

When the temperature falls to about $T \sim 4000$ K, we have the epoch of *recombination*, when the electrons and protons are able to to form neutral hydrogen. This occurred at $t \sim 200,000$ yr. Of course, the word recombination isn't quite correct, since they had never combined before. Note that one might naively have expected recombination to take place at $T \sim 100,000$ K, corresponding to the ionization potential of hydrogen (13.6 eV). However, recombination occurs at a much lower temperature because there are many more photons than particles, and there is a substantial high energy tail to the frequency distribution of photons at a given temperature, along with some other subtle details — all of this is covered in exquisite detail in your text (*Ryden*, Section 9.3), if you're interested.

Also, to avoid confusion, it is worth distinguishing among three closely related, but not identical, moments:

- The epoch of *recombination* is when the baryonic component of the Universe goes from being ionized to being neutral. Numerically, one might define it as the moment when the number density of ions is equal to the number density of neutral atoms. This occurred at about $T \sim 4000$ K, corresponding to $t \sim 200,000$ yr ($z \sim 1400$ by setting $\Omega = H$).
- The epoch of *photon decoupling* (or freeze-out, as we've called it above) is when the rate at which photons scatter from electrons becomes smaller than the contemporary Hubble parameter. When photons decouple, they cease to interact with the electrons and the Universe becomes transparent. This occurred at about $T \sim 3000$ K, corresponding to $t \sim 300,000$ yr ($z \sim 1100$).
- The epoch of *last scattering* is the time when a typical CMB photon underwent its last scattering from an electron. Recall that we discussed this in Lecture 3 early in the quarter, when we defined the last scattering surface surrounding every observer from which the CMB photons have been streaming freely with no further scattering by electrons. The probability that a photon will scatter from an electron is very small once the expansion rate of the universe is faster than the scattering rate, so the epoch of last scattering is very close to the epoch of photon decoupling: $T \sim 3000$ K, corresponding to $t \sim 300,000$ yr ($z \sim 1100$).

The values of t quoted above are approximate and, of course, dependent on the cosmological model assumed. After WMAP, one tends to quote the epoch of last scattering as 380,000 yr.