PHY 475/375
Lecture 14
(May 14, 2012)

The Cosmic Microwave Background (CMB)

In this lecture, we will study the Cosmic Microwave Background (CMB) in more detail, specifically what it tells us about the history of the Universe.

Recall that the CMB is relic radiation from an early stage of the Universe that fills the sky. It can be modeled as radiation from a black body with temperature $T_0 = 2.725$ K, and current energy density

$$\varepsilon_{\gamma,0} = \alpha T_0^4 = 0.261 \text{ MeV m}^{-3} \tag{9.1}$$

This is only 5×10^{-5} times the current critical energy density. However, the number density of CMB photons in the Universe is large:

$$n_{\gamma,0} = 4.11 \times 10^8 \text{ m}^{-3} \tag{9.2}$$

These numbers may be compared to the current values for baryons. With $\Omega_{\text{bary, 0}} \approx 0.04$, we get the current energy density of baryons:

$$\varepsilon_{\text{bary, 0}} = \Omega_{\text{bary, 0}}\, \varepsilon_{c,0} \approx 0.04 \left(5200 \text{ MeV m}^{-3}\right) \approx 210 \text{ MeV m}^{-3} \tag{9.3}$$

and with the rest energy of a proton or neutron being $E_{\text{bary}} \approx 939$ MeV, we get for the current number density of baryons:

$$n_{\text{bary, 0}} = \frac{\varepsilon_{\text{bary, 0}}}{E_{\text{bary}}} = \frac{210 \text{ MeV m}^{-3}}{939 \text{ MeV}} \approx 0.22 \text{ m}^{-3} \tag{9.4}$$

So, the energy density in baryons today is about 800 times the energy density in CMB photons.

On the other hand, the ratio of baryons to photons is

$$\eta = \frac{n_{\text{bary, 0}}}{n_{\gamma,0}} \approx \frac{0.22 \text{ m}^{-3}}{4.11 \times 10^8 \text{ m}^{-3}} \approx 5 \times 10^{-10} \tag{9.5}$$

which means that baryons are vastly outnumbered by photons in the Universe, by a ratio of about two billion to one!

Observing the CMB

Recall that we discussed in an earlier lecture that the CMB was serendipitously discovered by Penzias and Wilson in 1965 with a horn-reflector radio antenna observing at microwave wavelengths, and they received the Nobel Prize for their discovery.

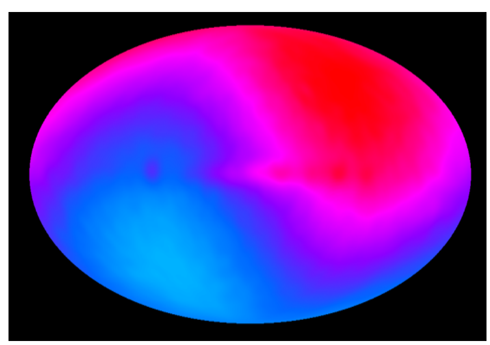
While Penzias and Wilson had observed the CMB at the operating microwave wavelength of their telescope of $\lambda = 7.35$ cm, the CMB spectrum actually reaches its peak near $\lambda \sim 2$ mm. Now, microwaves with wavelengths shorter than $\lambda \sim 3$ cm are strongly absorbed by water molecules in the Earth's atmosphere, so measurement of the CMB spectrum at wavelengths shorter than 3 cm can only be done by observing with high-altitude balloons or from the South Pole (where the combination of cold temperatures and high altitude — the South Pole is nearly 3 km above sea level, keep the atmospheric humidity low). The best way, however, is to go above the Earth's atmosphere. The first measurement of the CMB from space was done by the Cosmic Background Explorer (COBE) satellite.

COBE, launched in 1989 into an orbit 900 km above the Earth's surface, had three instruments on board.

- The *Far InfraRed Absolute Spectrophotometer* (FIRAS) was designed to measure the spectrum of the CMB in the range 0.1 mm $< \lambda <$ 10 mm; this range of wavelengths includes the peak in the CMB spectrum.
- The *Differential Microwave Radiometer* (DMR) was designed to make full-sky maps of the CMB at three different wavelengths: $\lambda = 3.3$ mm, 5.7 mm, and 9.6 mm. These wavelengths were chosen to minimize the contamination from Galactic emission.
- The Diffuse InfraRed Background Experiment (DIRBE) was designed to measure radiation at the wavelengths 0.001 mm $< \lambda <$ 0.24 mm, primarily from stars and dust within our own Galaxy.

There were three major results from the analysis of the COBE data.

1. At any angular position (θ, ϕ) on the sky, the spectrum of the CMB is very close to an ideal blackbody, at a level $\Delta\epsilon/\epsilon \approx 10^{-4}$.
2. The CMB exhibits a dipole distortion in temperature, as shown in the figure below.

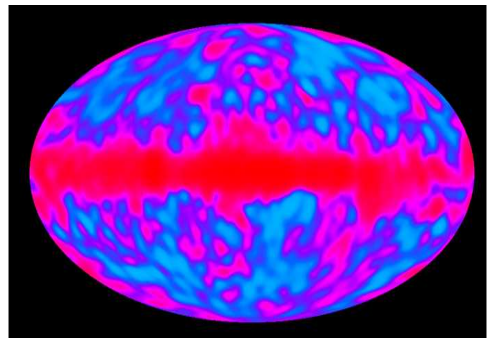
   

   That is, although each point in the sky has a blackbody spectrum, in one half of the sky the spectrum is slightly blueshifted to higher temperatures, and in the other half it is slightly redshifted to lower temperatures. This dipole distortion is a simple Doppler shift, caused by the net motion of the COBE satellite relative to a frame of reference in which the CMB is isotropic. To remove the dipole distortion so we can get at underlying asymmetries, we must correct for several motions.

   We must correct for the orbital motion of COBE around the Earth ($v \sim 8$ km s^{-1}), for the orbital motion of the Earth around the Sun ($v \sim 30$ km s^{-1}), for the orbital motion of the Sun around the Galactic Center ($v \sim 220$ km s^{-1}), and for the orbital motion of our Galaxy relative to the center of mass of the Local Group ($v \sim 80$ km s^{-1}). After correcting for all these motions, one finds that the Local Group is moving in the general direction of the constellation Hydra with a speed $v = 630 \pm 20$ km s^{-1} $= 0.002\,c$, or 0.2% the speed of light.

This is not at all a surprise, and arises merely due to a combination of the gravitational acceleration of the Local Group toward the Virgo cluster, the nearest large cluster to us, and in turn, the acceleration of the Virgo cluster toward the Hydra-Centaurus supercluster, the nearest supercluster to us.

3. After the dipole distortion of the CMB is removed, the remaining temperature fluctuations are small in amplitude, and shown in the figure below. If the temperature of the CMB at a given point on the sky is $T(\theta, \phi)$, then the mean temperature averaging over all locations is



$$\langle T \rangle = \frac{1}{4\pi} \int T(\theta, \phi) \sin\theta \, d\theta \, d\phi = 2.725 \text{ K} \qquad (9.6)$$

The dimensionless temperature fluctuation at a given point on the sky is

$$\frac{\delta T}{T}\Big(\theta, \phi\Big) \equiv \frac{T(\theta, \phi) - \langle T \rangle}{\langle T \rangle} \qquad (9.7)$$

From the maps of the sky made by the DMR instrument aboard COBE, it was found that after subtraction of the Doppler dipole, the root mean square temperature fluctuation was

$$\left\langle \left(\frac{\delta T}{T} \right)^2 \right\rangle^{1/2} = 1.1 \times 10^{-5} \qquad (9.8)$$

The analysics used to derive this value excluded the regions of the sky contaminated by foreground emission from our own Galaxy, that is, the region occupied by the band that runs horizontally from left to right in the figure above. This tells us that the temperature of the CMB varies by only 1.1×10^{-5} (2.725) K, or by only 30 microK across the sky, and is a remarkably close approach to isotropy.

The observations that the CMB has a nearly perfect blackbody spectrum and that it is nearly isotropic (once the Doppler dipole is removed) provide strong support for the Hot Big Bang model of the Universe. In fact, it was the physicist George Gamow who predicted in the late 1940's that if the Universe was once much hotter and denser that it is now, a typical photon would scatter many times on a trip across the Universe. This would keep the radiation and matter in thermal equilibrium, and result in the radiation having a blackbody spectrum. Then, when neutral atoms formed and the radiation and matter decoupled, the photons would be free to stream across the Universe unimpeded, except that their wavelengths would stretch ($\lambda \propto a$), and so their temperature would decrease linearly with the scale factor ($T \propto a^{-1}$). A background of nearly isotropic blackbody radiation is natural if the universe was once hot, dense, opaque, and nearly homogeneous, as it was in the Hot Big Bang model.

As transformational as the COBE results were, COBE had a coarse resolution of ∼10 degrees in the sky. To improve upon this, NASA launched the Wilkinson Microwave Anisotropy Probe (WMAP) in 2001 to make full-sky maps of the CMB at 0.2 degree resolution. WMAP has produced spectacular results, but in order to understand them, we need to familiarize ourselves with what must have transpired in the very early moments of the Universe.

Temperature Fluctuations in the CMB

In this section, we will try and understand why there are temperature fluctuations in the CMB. This is important to know, since these fluctuations are the seeds of the large-scale structures we see today. In the following discussion, I've drawn freely from a masterful discussion by Wayne Hu (University of Chicago) and Martin White (UC, Berkeley) in Scientific American (*The Cosmic Symphony*, Feb 2004); in many places, I will quote Hu & White directly, although I might avoid putting quotes in some places to maintain the flow of the narrative.

The first point to realize is that the early Universe would have contained *sound waves* (as did James E. Peebles of Princeton University and graduate student Jer Yu, and at almost the same time, Yakov B. Zeldovich and Rashid A. Sunyaev of the Moscow Institute of Applied Mathematics in the late 1960's).

- Recall that when radiation and matter were in temperature equilibrium, we would have had a tightly coupled system of photons, electrons, and ions behaving like a single gas, with photons scattering off electrons.
- Just as in the air around us, a small disturbance in the density of this photon-electron-ion gas would have propagated as a sound wave with alternate compressions and rarefactions. One might wonder what caused these disturbances; they can be attributed to the same quantum fluctuations in the field responsible for inflation in the early Universe, about which we will study later.
- You will recall from thermodynamics that compressing a gas heats it, while expansion results in cooling. Therefore, the disturbances in the early universe would have resulted in a dynamic pattern of temperature fluctuations.
- At recombination (protons latching on to electrons to form neutral atoms), photons became free to stream through the universe without being scattered off electrons. The photons released from hotter, denser regions were more energetic than photons emitted from the colder, rarefied regions, "so the pattern of hot and cold spots induced by the sound waves was frozen into the CMB" (Hu & White). Simultaneously, freed of the radiation pressure of the photons that had prevented the compressed regions from collapsing, the denser regions became stars and galaxies.

The sound waves in the early Universe locked information into the CMB that enables us to understand the nature of the early Universe. To understand why, consider the following. If you create a sound wave in open air, pretty much any frequency can propagate. However, if you create sound in a space with boundaries, the characteristics are very different; some special frequencies will gain much higher amplitudes than others (which explains why you sound so nice while singing in the shower, even if you don't have any musical inclinations). In fact, this is the principle behind musical instruments — both stringed instruments and wind instruments. Blowing into a pipe open at both ends creates a sound wave whose fundamental frequency corresponds to a wave with maximum air displacement at either end and minimum displacement in the middle (or opposite in a pressure representation). The wavelength of this fundamental mode is twice the length of the pipe. Moreover, the sound has a series of overtones with frequencies equal to $2f_1, 3f_1, 4f_1, \ldots$, where f_1 is the fundamental frequency.

The sound waves in the early Universe are similar to the sound waves in a pipe open at both ends, except we must consider the waves oscillating in time instead of space. That is, one end of the "pipe" is a very early moment in the universe and the other end is the moment of recombination when photons and matter decoupled.

- Quantum fluctuations in the early Universe would have caused sound waves at all frequencies simultaneously, with roughly the same amplitude. This would have given rise to compressions and rarefactions on all scales, and therefore, temperature variations on all scales.

- If you inject a continuum of frequencies into a tube open at both ends, only a select few (corresponding to the fundamental frequency of the tube, and its integer multiples) will resonate. In the same manner, only some of the frequencies generated in the early Universe would have high amplitude.

- Consider a certain region of space that has a maximum positive displacement (and, hence, maximum temperature) at the early moment of the Universe. As the sound waves propagate, the region will head toward minimum displacement (average temperature), and then toward maximum negative displacement (minimum temperature). The wave that causes the region to reach maximum negative displacement exactly at recombination is the fundamental wave of the early Universe.

- Likewise, oscillating with twice, thrice, or more times the frequency of the fundamental wave, the overtones would cause smaller regions of space to reach maximum displacement at recombination.

- The first overtone, however, has a lower amplitude than the fundamental. To understand this, recall that both ordinary matter and dark matter supply mass to the primordial gas and enhance its gravitational pull, but only ordinary matter undergoes acoustic compressions and rarefactions. At recombination, the fundamental wave is in a phase where gravity enhances its compression of the denser regions of gas. On the other hand, the first overtone with half the fundamental wavelength is caught in the opposite phase in which gravity tries to compress the plasma while gas pressure tries to expand it. This makes the temperature variation caused by this overtone to be less pronounced that that of the fundamental wave, and explains why the second peak in the power spectrum is lower than the first. For more details, see the graphic on page 52 of the Hu & White Scientific American article.

- At higher harmonics, amplitudes would be even smaller because photons can stream out of smaller regions more easily, thereby smoothing out the fluctuations in temperature.

We will now analyze these acoustic oscillations in greater detail by looking at its *power spectrum*. First, though, what is a power spectrum? Recall that if you have a one-dimensional function $f(x)$ defined in a finite region of length L, you can always write the function as a Fourier series:

$$f(x) = \sum_{n=0}^{\infty} A_n \cos\left(\frac{2n\pi x}{L} + \phi_n\right)$$

This means that all of the information about $f(x)$ is now encoded in A_n and ϕ_n. In particular, the smaller-scale structures of the function will be modeled by harmonics with larger n-values.

We can do the same Fourier decomposition for the temperature fluctuations like we did for the 1-D function $f(x)$ above, except that for the temperature functions, which are defined on the surface of a celestial sphere, we have a two-dimensional function $T_f\,(\theta,\phi)$, where I have written $T_f \equiv \delta T/T$ that is in your text, for convenience.

So, to decompose $T_f(\theta,\phi)$, we need an appropriate basis of 2-D functions, and one that we have readily available since it finds wide applications in physics are the spherical harmonics $Y_{lm}(\theta,\phi)$. So we can write

$$T_f(\theta,\phi) \equiv \frac{\delta T}{T}(\theta,\phi) = \sum_{l=0}^{\infty}\sum_{m=-l}^{l} a_{lm}\, Y_{lm}\,(\theta,\phi) \tag{9.45}$$

Cosmologists are not concerned with the exact pattern of hot and cold spots in the sky, but rather their statistical properties. Therefore, they are mostly interested in the *correlation function* $C(\theta)$.

To understand the correlation function, consider two points on the CMB surface that are in the directions defined by the unit vectors $\hat{n}$ and $\hat{n}'$, separated by an angle θ given by

$$\cos\theta = \hat{n}\cdot\hat{n}'$$

In principle, $C(\theta)$ may be found by multiplying $\delta T/T$ at these two points, and averaging the product over all points separated by the angle θ, that is:

$$C(\theta) = \left\langle \frac{\delta T}{T}\,(\hat{n})\,\frac{\delta T}{T}\,(\hat{n}')\right\rangle_{\hat{n}\cdot\hat{n}'=\cos\theta} \tag{9.46}$$

In practice, doing this in real space would be impossible, since the CMB measurements contain information over only a limited range of angular scales.

This is where the expansion in equation (9.45) comes to our rescue. As long as the underlying density fluctuations (that are responsible for the temperature fluctuations in the CMB) are described by a Gaussian random process (as is currently predicted to be the case in the inflation scenario), all the information about the CMB fluctuations will be contained in the second order angular power spectrum

$$C_l = \left\langle \left|a_{lm}\right|^2 \right\rangle$$

where the angled brackets indicate the average over all observers in the Universe; the absence of a preferred direction in the Universe (i.e., isotropy) implies that $\langle |a_{lm}|^2\rangle$ is independent of m. Note that there are models that predict nongaussian density fluctuations, in which case we would need higher order correlations, because they would contain additional information. We will, however, stick with the most widely accepted (inflation) model in this course.

The connection between the angular power spectrum C_l and the real space correlation function $C(\theta)$ is then

$$C(\theta) = \frac{1}{4\pi}\sum_{l=0}^{\infty}(2l+1)\,C_l\,P_l(\cos\theta) \tag{9.47}$$

where $P_l(x)$ are the usual Legendre polynomials: $P_0(x) = 1; P_1(x) = x; P_2(x) = (3x^2-1)/2$, and so on.

In general, a particular C_l is a measure of temperature fluctuations between points on the sky separated by an angle $\theta \sim 180°/l$.

This means that the multipole l is interchangeable, for all practical purposes, with the angular scale θ. Larger values of l correspond to smaller angular scales.

- The $l = 0$ term gives the overall constant, or the average value of T_f over the whole sky.
- The $l = 1$ term has an angular scale of $180° \equiv 360°/2$, and is the dipole term which results from the Doppler shift due to our motion through space.
- It is the terms with $l \geq 2$ that are of most interest to cosmologists, as we shall see shortly.

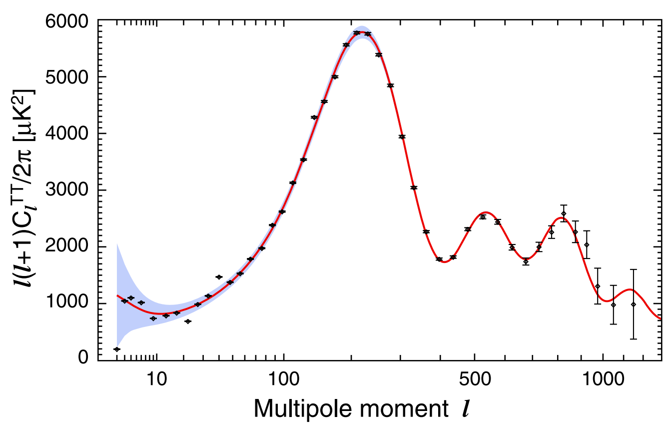
Note that for each multipole l, we have $(2l + 1)$ values of m.

- Consider a small value of l, e.g., $l = 2$.
- This gives us only $(2l + 1) = 5$ independent measures to average. So for smaller l values, we would have fewer independent values to average to find C_l, and potentially larger errors in C_l. This is called *cosmic variance*.

In presenting the results of CMB observations, it is customary to plot the function

$$\Delta_T^2 \equiv \left[\frac{l(l+1)C_l}{2\pi}\right] \langle T \rangle^2$$

along the vertical axis, as shown in the figure on the right (taken from the NASA GSFC WMAP page), or its square root. The data from WMAP are plotted with error bars. The solid red line is a ΛCDM model. The shaded areas show the cosmic variance errors.



In the next lecture, we will discuss this spectrum in more detail.