PHY 375
Homework 5
(Due by schedule posted below)

If you want this graded and returned in class on Wed (5/30), you must submit by Tue (5/29). ***The final deadline is noon on Thu (5/31), unless*** *you are present at our Physics colloquium on Wed, in which case this may be turned in latest by noon on Fri (6/1).*

1. Answer the following questions.

(a) Why is there a dipole anisotropy in the Cosmic Microwave Background (CMB)?

(b) Differentiate between the epoch of recombination, the epoch of photon decoupling, and the epoch of last scattering.

(c) Smarty P. Antz tells you that recombination took place when the mean energy per CMB photon fell below the ionization energy of hydrogen (13.6 eV). A quick calculation shows you that this would correspond to a temperature of 60,000 K. Yet we know that recombination took place long afterward, when the Universe had cooled down to 3800 K. Why was this the case (i.e., what is wrong with Smarty P. Antz's reasoning)?

(d) Read Section 11.3 of Ryden, and state (in a very concise form — a few sentences, at most) what is meant by the magnetic monopole problem. Then, explain how inflation takes care of this problem (Section 11.4).

2. When the Universe was fully ionized, photons interacted primarily with electrons via Thomson scattering, for which the cross-section is $\sigma_e = 6.65 \times 10^{-29}$ m^2.

(a) Find the average distance traveled by a photon between collisions, also known as the mean free path: $\lambda_{\rm mfp} = 1/n_e\sigma_e$, at the time of radiation-matter equality when $a_{\rm rm} \approx 3 \times 10^{-4}$.

Hint: To find n_e, consider that when the universe was fully ionized, $n_e = n_p = n_{\rm bary}$; you know the current $n_{\rm bary,\,0} = 0.22$ m^{-3}, and also that $n_{\rm bary} \propto 1/a^3$.

(b) The Friedmann equation for a radiation-dominated flat universe is

$$\frac{H^2}{H_0^2} = \frac{\Omega_{r,0}}{a^4}$$

Use this to find the Hubble parameter at the time of radiation-matter equality when $a_{\rm rm} \approx 3 \times 10^{-4}$. Take the value of $\Omega_{r,0}$ from the Benchmark model.

(c) The photons remain coupled to the electrons as long as their mean free path $\lambda_{\rm mfp}$ is shorter than the Hubble distance c/H. Verify that they are coupled at the time of radiation-matter equality.

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3. Given that the Universe is described by the Benchmark model, and the redshift of the last scattering surface is $z_{\rm ls} = 1100$,

(a) Find the angular diameter distance to the surface of last scattering.

(b) Find the luminosity distance to the surface of last scattering.

(c) Find the proper distance $d_p(t_0)$ to the surface of last scattering.

4. Big Bang Nucleosynthesis of ^4He was a race against time to bind neutrons before they decayed to protons.

(a) First, prove that the maximum possible value of the primordial ^4He fraction is

$$\Big[Y_p\Big]_{\rm max} = \frac{2f}{1+f}$$

where $f = n_n/n_p \leq 1$ is the neutron-proton ratio at the time of nucleosynthesis.

(b) Assuming that the neutron-proton ratio remained constant at $f = n_n/n_p = 1/5$ after freeze-out, and that all available neutrons were incorporated into ^4He, find the value of $[Y_p]_{\rm max}$.

(c) In reality, $[Y_p]_{\rm max}$ was less than this value because some of the free neutrons decayed into protons, thereby decreasing the number of neutrons available to combine with protons to form ^4He.

Given that if you start out with a population of free neutrons n_{ni}, the number of free neutrons remaining after time t will be

$$n_n = n_{ni}\,\exp\left(-\,\frac{t}{\tau_n}\right)$$

where τ_n is the decay time of the neutron,

Show that if nucleosynthesis starts after a time delay of $t_{\rm nuc}$, neutron decay makes the neutron-to-proton ratio decrease from its freeze-out value of $n_n/n_p = 1/5$ to

$$\frac{n_{nf}}{n_{pf}} = \frac{\exp\,(-t_{\rm nuc}/\tau_n)}{5 + \Big[1 - \exp\,(-t_{\rm nuc}/\tau_n)\Big]}$$

(d) As a result of the process described in part (c), find the neutron-proton ratio $f_{\rm new} = n_{nf}/n_{pf}$, and the corresponding value of $[Y_p]_{\rm max}$, if nucleosynthesis starts after a delay of $t_{\rm nuc} = 200$ s, and the decay time of the neutron is $\tau_n = 890$ s. Assume that all available neutrons (that haven't decayed) are incorporated into ^4He nuclei.

(e) Suppose, instead, that the neutron decay time were $\tau_n = 89$ s, with all other physical parameters unchanged (including that we still have $t_{\rm nuc} = 200$ s). Calculate $[Y_p]_{\rm max}$, again assuming that all available neutrons are incorporated into ^4He nuclei.

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5. Consider the inflaton field

$$V(\phi) = A\phi^4$$

Equation (11.53) can be used to find when the slow roll condition breaks down by setting

$$\left(\frac{E_P}{V}\frac{dV}{d\phi}\right)^2 \sim 1$$

where $E_P = (\hbar c^5/G)^{1/2}$ is the Planck energy.

(a) Use the limiting condition given above to find the value of ϕ at which the slow roll conditions break down; this marks the end of the period of inflation. *Leave your answer as an expression, don't substitute numerical values.*

(b) Find the number of e-foldings N.

Hints: Since $a(t_f)/a(t_i) = e^N$, you could do $N = \ln[a(t_f)/a(t_i)]$, except that the expressions for $a(t)$ are extremely messy. Instead, it is better to use the relation of the scale factor to the Hubble parameter and evaluate

$$N = \ln\left[\frac{a(t_f)}{a(t_i)}\right] = \int_{t_i}^{t_f} H(t)\,dt$$

However, we are not concerned with the time-dependence of H, so recast this as an integral over $d\phi$ by using equation (11.51):

$$H = \left(\frac{8\pi GV}{3c^2}\right)^{1/2}$$

and equation (11.48):

$$3H\dot{\phi} = -\hbar c^3\frac{dV}{d\phi}$$

Of course, you should resist the temptation to use equation (11.59), which is written only for the potential shown in Figure 11.3.

Submit neat work, with answers or solutions clearly marked by the question number. Unstapled, untidy work will be charged a handling fee of 20% penalty. Writing only an answer without showing the steps used to get to that answer will fetch very few points, even if the answer is correct. Late homework will not be accepted.