PHY 375
Homework 2
(Due by beginning of class on Wednesday, April 18, 2012)

Submit neat work, with answers or solutions clearly marked by the question number. Unstapled, untidy work will be charged a handling fee of 20% penalty. Writing only an answer without showing the steps used to get to that answer will fetch very few points, even if the answer is correct. Late homework will not be accepted.

1. Suppose you are a two-dimensional being, living on the surface of a sphere with radius R. An object of width $ds \ll R$ is at a distance r from you (remember, all distances are measured on the surface of the sphere).

(a) What angular width $d\theta$ will you measure for the object?

(b) Examine and explain the behavior of $d\theta$ as $r \to \pi R$.

2. The critical mass density of the Universe at the current epoch is given by

$$\rho_{c,0} = \frac{3}{8\pi G} H_0^2$$

where the Hubble parameter at the current epoch is $H_0 = (70 \pm 7)$ km s^{-1} Mpc^{-1}.

(a) Calculate the value of the critical mass density in the current epoch. Show your calculation clearly.

(b) Calculate the uncertainty in the value of the critical mass density in the current epoch. Show all steps clearly.

Hint: If F is a function of variables $X_1, X_2, \ldots$, so that $F \equiv F(X_1, X_2, \ldots)$, then the uncertainty in F is obtained by taking the partial derivative of F with respect to each variable, multiplying it by the uncertainty in that variable, and adding the individual terms in quadrature, that is:

$$\sigma_F = \sqrt{\Big[(\partial F/\partial X_1)\,\sigma_{X_1}\Big]^2 + \Big[(\partial F/\partial X_2)\,\sigma_{X_2}\Big]^2 + \ldots}$$

Check your answers in parts (a) and (b) with equation (4.27) before proceeding.

(c) Since we just got through with March madness, let us suppose the universe is comprised of basketballs, each of mass 0.62 kg, and radius 0.12 m. If the basketballs were divided uniformly throughout the universe, what number density of basketballs would be required to make the mass density of the universe equal to the critical mass density in the current epoch that you calculated in part (a)?

(d) Given the number density of basketballs you found in part (c), how far would you be able to see, on average, before your line of sight intersected a basketball?

(e) Since we can see galaxies at distances of $c/H_0 \sim 4000$ Mpc, comment on whether the transparency of the universe on this length scale (of 4000 Mpc) places any useful limit on the number density of intergalactic basketballs.