1. If 61.5 liters (symbol L) of oxygen at 18.0°C and an absolute pressure of 2.45 atmospheres are compressed to 48.8 L and at the same time the temperature is raised to 50.0°C, what will the new pressure be?

**Solution:**

If you use $PV = nRT$, you will have to express everything in SI units. However, if you set up ratios, you can get by using liters and atmospheres. We will choose to do the latter here, to save on time.

Given

\[ V_A = 61.5 \text{ L}, \quad T_A = 18^\circ C = 18 + 273 = 291 \text{ K}, \quad P_A = 2.45 \text{ atm} \]

\[ V_B = 48.8 \text{ L}, \quad T_B = 50^\circ C = 50 + 273 = 323 \text{ K}, \]

we have from the ideal gas law that $P_A V_A = n R T_A$ and $P_B V_B = n R T_B$

Dividing one equation by the other, we end up with

\[ \frac{P_A V_A}{T_A} = \frac{P_B V_B}{T_B} \]

so that

\[ P_B = \frac{P_A V_A}{T_A} \left(\frac{T_B}{V_B}\right) \]

\[ = \left(\frac{2.45 \text{ atm}}{291 \text{ K}}\right) \left(\frac{61.5 \text{ L}}{48.8 \text{ L}}\right) \left(\frac{323 \text{ K}}{48.8 \text{ L}}\right) \]

\[ \Rightarrow P_B = 3.43 \text{ atm} \]
2. An automobile tire has a volume of $1.64 \times 10^{-2}$ m$^3$ and contains air at a gauge pressure of 165 kPa when the temperature is 0.00°C. What is the gauge pressure of the air in the tires when its temperature rises to 27.0°C and its volume increases to $1.67 \times 10^{-2}$ m$^3$? Assume atmospheric pressure is $1.00 \times 10^5$ Pa.

*Make sure you understand the distinction between gauge pressure and absolute pressure, and what value you need to use in this problem.*

**Solution:**

Given

$$V_A = 1.64 \times 10^{-2} \text{ m}^3, \quad T_A = 0^\circ C = 0 + 273 = 273 \text{ K},$$

$$V_B = 1.67 \times 10^{-2} \text{ m}^3, \quad T_A = 27^\circ C = 27 + 273 = 300 \text{ K},$$

and, given $(P_{\text{gauge}})_A = 165 \text{ kPa}$, which implies that $(P_{\text{tot}})_A = (165 \times 10^3 + 1 \times 10^5) \text{ Pa}$

Once again, from the ideal gas law that $P_AV_A = nRT_A$ and $P_BV_B = nRT_B$

Dividing one equation by the other, we end up with

$$\frac{P_AV_A}{T_A} = \frac{P_BV_B}{T_B}$$

so that

$$(P_{\text{tot}})_B = \frac{P_AV_A}{T_A} \left( \frac{T_B}{V_B} \right)$$

$$= \left[ (165 \times 10^3 + 1 \times 10^5) \text{ Pa} \right] \left( \frac{1.64 \times 10^{-2} \text{ m}^3}{273 \text{ K}} \left( \frac{300 \text{ K}}{1.67 \times 10^{-2} \text{ m}^3} \right) \right)$$

$$\Rightarrow (P_{\text{tot}})_B = 2.86 \times 10^5 \text{ Pa}$$

Then

$$(P_{\text{gauge}})_B = (P_{\text{tot}})_B - (P_{\text{tot}})_{\text{atm}}$$

$$= 2.86 \times 10^5 \text{ Pa} - 1 \times 10^5 \text{ Pa}$$

$$= 1.86 \times 10^5 \text{ Pa}$$

$$\Rightarrow (P_{\text{gauge}})_B = \boxed{186 \text{ kPa}}$$
3. 22 particles have speeds as follows ($N_i$ represents the number of particles with speeds $v_i$):

<table>
<thead>
<tr>
<th>$N_i$</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_i$ (cm/s)</td>
<td>1.0</td>
<td>2.0</td>
<td>3.0</td>
<td>4.0</td>
<td>5.0</td>
</tr>
</tbody>
</table>

(a) Compute their average speed $\langle v \rangle$.

**Solution:** Given that $N_i$ represents the number of particles that have speed $v_i$.

The average speed is

$$\langle v \rangle = \frac{\sum N_i v_i}{\sum N_i} = \frac{2(1.0) + 4(2.0) + 6(3.0) + 8(4.0) + 2(5.0)}{2 + 4 + 6 + 8 + 2} = \boxed{3.2 \text{ cm/s}}$$

(b) Compute their root mean square speed $v_{rms}$.

**Solution:**

The rms speed is

$$v_{rms} = \sqrt{\frac{\sum N_i v_i^2}{\sum N_i}} = \sqrt{\frac{2(1.0)^2 + 4(2.0)^2 + 6(3.0)^2 + 8(4.0)^2 + 2(5.0)^2}{2 + 4 + 6 + 8 + 2}} = \boxed{3.4 \text{ cm/s}}$$

(c) Of the five speeds shown, which is the most probable speed $v_p$?

**Solution:**

The most probable speed $v_p$ is the speed at the peak of the distribution. So

$$v_p = \boxed{4.0 \text{ cm/s}}$$

But wait!

Didn’t we say in lecture that $v_p < \langle v \rangle < v_{rms}$? That is not what we get here.

Ah, but what we have here is **not** a Maxwell-Boltzmann distribution. That is why, in this case, $v_p$ can be greater than $\langle v \rangle$, and $v_{rms}$.
4. At 0.00°C and 1.00 × 10⁻² atm, the density of a gas is 1.24 × 10⁻⁵ g/cm³. Find \( v_{\text{rms}} \) for the gas molecules.

**Solution:**

Given that the density of a gas is
\[
\rho = 1.24 \times 10^{-5} \text{ g/cm}^3 \equiv 1.24 \times 10^{-2} \text{ kg/m}^3
\]
at \( T = 273 \text{ K} \), and
\[
P = 1.00 \times 10^{-2} \text{ atm} \equiv \left(1.00 \times 10^{-2} \text{ atm}\right)\left(1 \times 10^5 \text{ Pa/atm}\right) = 1000 \text{ Pa}
\]
The rms speed is given by
\[
v_{\text{rms}} = \sqrt{\frac{3k_B T}{m}}
\]
where \( m \) is the mass of one molecule.

In order to express \( m \) in terms of some of the quantities given above, consider that the density is given by
\[
\rho = \frac{\text{mass of all the gas} \ (M_{\text{gas}})}{V} = \frac{N m}{V}
\]
where \( N \) is the number of molecules, and \( m \) is the mass of one molecule, already stated above.

Rearranging terms, \( m = \frac{\rho V}{N} \)

Then
\[
v_{\text{rms}} = \sqrt{\frac{3k_B T}{m}} = \sqrt{\frac{3k_B T}{\rho V/N}} = \sqrt{\frac{3Nk_B T}{\rho V}} = \sqrt{\frac{3PV}{\rho V}}
\]
where we have replaced \( Nk_B T \) in the numerator with \( PV \) using the ideal gas law (\( PV = nRT \)).

Therefore, we get
\[
v_{\text{rms}} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3\left(1000\right)}{1.24 \times 10^{-2}}} = 491.9 \text{ m/s}
\]

Therefore, we get the rms velocity as
\[
v_{\text{rms}} = \boxed{492 \text{ m/s}}
\]
5. The figure shows a thermodynamic process carried out on 120 mg of helium.

(a) Determine the pressure (in atm), the temperature (in °C), and volume (in cm³) of the gas at points 1, 2, and 3.

Solution: We will need the number of moles, so

\[ n = \frac{120 \times 10^{-3} \text{ g}}{4 \text{ g/mol}} = 0.030 \text{ mol} \]

At point 1, we know \( V_1 = 1000 \times 10^{-6} \text{ m}^3 = 10^{-3} \text{ m}^3 \), and \( T_1 = 133^\circ \text{C} = 406 \text{ K} \), so we can use the ideal gas law \( PV = nRT \) to find \( P_1 \):

\[ P_1 = \frac{nRT_1}{V_1} = \frac{(0.030 \text{ mol})(8.31 \text{ J/mol.K})(406 \text{ K})}{10^{-3} \text{ m}^3} = 101215.8 \text{ Pa} \]

Once again, I will retain many more digits than significant to avoid rounding errors, and adjust for significant digits only at the end.

Since \( 1 \rightarrow 2 \) is a constant volume process, we know the following at point 2:

\[ V_2 = V_1 = 10^{-3} \text{ m}^3, \quad P_2 = 5P_1 = 506079 \text{ Pa} \]

Again, we can use the ideal gas law, this time to find \( T_2 \):

\[ T_2 = \frac{P_2V_2}{nR} = \frac{(506079 \text{ Pa})(10^{-3} \text{ m}^3)}{(0.030 \text{ mol})(8.31 \text{ J/mol.K})} = 2030 \text{ K} \]

Next, since \( 2 \rightarrow 3 \) is an isothermal process, we know the following at point 3:

\[ T_3 = T_2 = 2030 \text{ K}, \quad P_3 = P_1 = 101215.8 \text{ Pa} \]

and yet again, using the ideal gas law, we find that

\[ V_3 = \frac{nRT_3}{P_3} = \frac{(0.030 \text{ mol})(8.31 \text{ J/mol.K})(2030 \text{ K})}{101215.8 \text{ Pa}} = 0.005 \text{ m}^3 \]

Next, divide the pressure values by \( 1 \times 10^5 \text{ Pa} \) to convert to atm, multiply the volume by \( 10^6 \) to convert to cm³, and subtract 273 K from the temperature values to convert to °C, as directed in the statement of the problem. The results are shown in a table on the next page.
Problem 5 (a) — continued from previous page

The table of results for $P$, $T$, and $V$ at points 1, 2, and 3 is shown below

<table>
<thead>
<tr>
<th>Point</th>
<th>Pressure $(\text{atm})$</th>
<th>Temperature $(\degree \text{C})$</th>
<th>Volume $(\text{cm}^3)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.01</td>
<td>133</td>
<td>1000</td>
</tr>
<tr>
<td>2</td>
<td>5.05</td>
<td>1757</td>
<td>1000</td>
</tr>
<tr>
<td>3</td>
<td>1.01</td>
<td>1757</td>
<td>5000</td>
</tr>
</tbody>
</table>

(b) How much work is done on the gas during each of the 3 segments (1 → 2, 2 → 3, and 3 → 1 respectively)?

**Solution:**

Since 1 → 2 is a constant volume process, the work done in this segment is zero.

For 2 → 3, an isothermal process, we must calculate the work done by carrying out the integration. Use the ideal gas law to replace $P = nRT/V$, and take $nRT$ outside the integral (because $T$ is a constant for an isothermal process, the number of moles $n$ is assumed to remain constant during the entire cycle, and $R$ is a constant).

$$W_{2→3} = \int_{V_2}^{V_3} P \, dV = \int_{V_2}^{V_3} \left(\frac{nRT}{V}\right) \, dV = nRT \int_{V_2}^{V_3} \frac{dV}{V}$$

Integrating

$$W_{2→3} = nRT \ln \left(\frac{V_3}{V_2}\right)$$

Substituting values

$$W_{2→3} = \left(0.030 \text{ mol}\right) \left(8.31 \text{ J/mol.K}\right) \left(2030 \text{ K}\right) \ln \left(\frac{0.005 \text{ m}^3}{10^{-3} \text{ m}^3}\right) = 814.5 \text{ J}$$

Finally, since the pressure is constant in the 3 → 1 process, we can take $P$ outside the integral:

$$W_{3→1} = \int_{V_3}^{V_1} P \, dV = P \int_{V_3}^{V_1} dV = P \left(V_1 - V_3\right)$$

so that

$$W_{3→1} = \left(101215.8 \text{ Pa}\right) \left[10^{-3} \text{ m}^3 - 0.005 \text{ m}^3\right] = -404.9 \text{ J}$$

Therefore, the work done during the 3 segments is:

$$W_{1→2} = 0, \quad W_{2→3} = \boxed{815 \text{ J}}, \quad W_{3→1} = \boxed{-405 \text{ J}}$$

In 2 → 3, the work is done by the gas (which makes sense, because it is expanding in this stage), and in 3 → 1, the work is done on the gas (which makes sense, because it is being compressed in this stage).