MAPPING PROPERTIES FOR OSCILLATORY INTEGRALS IN D-DIMENSIONS

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Abstract. For $a_j, b_j \geq 1, j = 1, 2, \cdots, d$, we prove that the operator $Kf(x) = \int_{\mathbb{R}^d_+} k(x,y)f(y)dy$ maps $L^p(\mathbb{R}^d_+)$ into itself for $p = 1 + \frac{1}{r}$, where $r = \frac{a_1}{b_1} = \cdots = \frac{a_d}{b_d}$, and the kernel $k(x,y) = \varphi(x,y)e^{ig(x,y)}, \varphi(x,y)$ satisfies for $|x-y| > 0$

$$|\partial_x^\alpha \partial_y^\beta \varphi(x,y)| \leq C_{\alpha\beta}|x-y|^{-|\alpha|-|\beta|}, \forall \alpha, \beta \in \mathbb{N}^d, \mathbb{N} = \{0, 1, \cdots\}.$$

(e.g. $\varphi(x,y) = |x-y|^{i\tau}, \tau$ real) and the phase $g(x,y) = x^a \cdot y^b = x_1^{a_1}y_1^{b_1} + \cdots + x_d^{a_d}y_d^{b_d}$.

We also obtain $L^p(\mathbb{R}^d_+)$ mapping properties for the operators with more general real-valued phases

$$g(x,y) = x^a \cdot y^b + \mu_{\bar{1}}(x)\mu_{\bar{1}}(y)\Phi(x^a, y^b),$$

where

$$|\partial_x^\alpha \partial_y^\beta \Phi(x,y)| \leq C_{\alpha\beta} \text{ for } x, y \geq \bar{1}, \forall \alpha, \beta \in \mathbb{N}^d, \sum_{j=1}^d (\alpha_j + \beta_j) \geq 1.$$

but in case $a_j \geq b_j$ for $j = 1, \cdots, d$, we need to assume that $b_1 \cdot b_2 \cdots b_d > 1$. Also we set $\mu_{\bar{1}}(x) = \mu_1(x_1)\cdots\mu_1(x_d)$ with $\mu_1(t) = 1$ for $t \geq 2, \mu_1(t) = 0$ for $0 \leq t \leq 1, 0 \leq \mu_1(t) \leq 1$ and $\mu_1(t) \in C^\infty(\mathbb{R}_+)$. Examples of $\Phi's$ are $log(\sum_{j=1}^d x_j + y_j)$ or $(\sum_{j=1}^d x_j + y_j)^l, 0 \leq l \leq 1$.