Double universal series by Walsh-Paley system in weighted $L^1_\mu[0,1]^2$ spaces

Sergo A. Episkoposian

Department of Physics, State University of Yerevan, Alex Manukian 1, 375049 Yerevan, Armenia

Let $\{W_n(x)\}_{n=0}^{\infty}$ be a Walsh-Paley system and $\mu(x)$,$0 < \mu(x) \le 1, x \in [0,1]$ be a measurable on $[0,1]$ function and let $L^1_\mu[0,1]$ be a space of real measurable functions $f(x)$, $x \in [0,1]$ with $\int_0^1 |f(x)|\mu(x)dx < \infty$.

In [1] and [2] the following results are proved.

Theorem. There exists a series of the form

$$\sum_{k=1}^{\infty} c_k W_k(x) \quad with \quad \sum_{k=1}^{\infty} |c_k|^q < \infty, \quad \forall q > 2 \tag{1}$$

such that for any number $\epsilon > 0$ a weighted function $\mu(x)$ with $0 < \mu(x) \le 1, |\{x \in [0,1] : \mu(x) \ne 1\}| < \epsilon$ can be constructed, so that the series (1) is universal in $L^1_\mu[0,1]$ with respect to rearrangements (concerning subseries).

We proved that these Theorems can be transferred from one-dimensional case to two-dimensional one.

Moreover, the following statements are true.

Theorem 1. There exists a double series of the form

$$\sum_{n,k=1}^{\infty} c_{n,k} W_n(x) W_k(y) \quad with \quad \sum_{n,k=1}^{\infty} |c_{n,k}|^q < \infty, \quad \forall q > 2 \tag{2}$$

with the following property: for any number $\epsilon > 0$ a weighted function $\mu(x,y)$, $0 < \mu(x,y) \le 1, \left|\{(x,y) \in T = [0,1]^2 : \mu(x,y) \ne 1\}\right| < \epsilon$ can be constructed so that the series (2) is universal in $L^1_\mu(T)$ concerning rearrangements (subseries) with respect to convergence in the sense of both spherical and rectangular partial sums.

REFERENCES

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