Integral Representations of Positive Definite Generalized Toeplitz Kernels

Miron Bekker

A Generalized Toeplitz kernel is a matrix-valued kernel $\mathcal{K}(s, t)$ of the following structure

$$\mathcal{K}(s, t) = \begin{bmatrix} T_1(s - t) & \Gamma^*(s + t) \\ \Gamma(s + t) & T_2(s - t) \end{bmatrix}$$

We show that a continuous matrix-valued kernel $\mathcal{K}(s, t)$ defined either on a finite interval Δ or on the positive semi-axis is a positive definite Generalized Toeplitz if and only if it admits a representation as a Fourier-Stieltjes integral of some non-decreasing and bounded matrix-valued function. We discuss uniqueness of such representation.In case of non-uniqueness we give description of all such representations.