The Hardy-Lorentz spaces $H^{p,q}(\mathbb{R}^n)$

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The Hady-Lorentz spaces $H^{p,q}(\mathbb{R}^n)$ are the intermediate spaces between the Hardy spaces $H^{p_1}(\mathbb{R}^n)$ and $L^\infty(\mathbb{R}^n)$. I will prove that there is an atomic decomposition for the distributions in these spaces . Using the atomic decomposition, one can recover the identification of $H^{p,q}(\mathbb{R}^n)$ as an intermediate spaces. The spaces $H^{1,q}(\mathbb{R}^n)$ are of particular interest because their intersection with $L^1(\mathbb{R}^n)$ form a nested family of subspaces of $L^1(\mathbb{R}^n)$ that contains all the functions in $L^1(\mathbb{R}^n)\backslash H^1(\mathbb{R}^n)$. I will describe the intermediate spaces between $H^1(\mathbb{R}^n)$ and weak $H^1(\mathbb{R}^n)$. To better understand the functions in $L^1(\mathbb{R}^n)\backslash H^1(\mathbb{R}^n)$, I will discuss families of spaces that covers the gap between $L^1(\mathbb{R}^n)$ and $H^1(\mathbb{R}^n)$, and study the behavior of maximal functions and Calderon-Zygmund singular integrals acting on them.