Optimal Currency Crises[1]

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Abstract

Flawed government policies have been offered as an explanation for currency crises in most of the previous literature. With few exceptions, the role of the banking system is ignored. Empirical evidence suggests that in recent decades banking crises and currency crises have been linked. A model is developed where the "twin" crises result from low asset returns. Large movements in exchange rates are desirable to the extent that they allow better risk sharing between a country's bank depositors and the international bond market. The rationale for using short-term debt denominated in a foreign reserve currency is also investigated.

1 Introduction

The large movements in exchange rates that occurred in many South East Asian countries in 1997 have revived interest in the topic of currency crises. In many of the early models of currency crises, such as Krugman (1979), currency crises occur because of inconsistent and unsustainable government policies (see Flood and Marion (1998) for a survey of the literature on currency crises). These models were designed to explain the problems experienced by a number of Latin American countries in the 1970's and early 1980's. In the recent South East Asian crises, by contrast, many of the countries which experienced problems had pursued macroeconomic policies that were consistent and sustainable. This characteristic of the recent crises has prompted a re-examination of theoretical models of currency crises.

The other characteristic of the South East Asian crises that has received considerable attention is that the banking systems of these countries also experienced crises. In an important paper, Kaminsky and Reinhart (1999) have investigated the relationship between banking crises and currency crises. They find that in the 1970's, when financial systems were highly regulated in many countries, currency crises were not accompanied by banking crises. However, after the financial liberalization that occurred during the 1980's, currency crises and banking crises became intertwined. The usual sequence of events is that initial problems in the banking sector are followed by a currency crisis and this in turn exacerbates and deepens the banking crisis. Although banking crises typically precede currency crises, the common cause of both is usually a fall in asset returns due to a recession or a weak economy. Often the fall is part of a boom-bust cycle that follows financial liberalization. It appears to be rare that banking and currency crises occur when economic fundamentals are sound.

Despite the apparent inter-relationship between currency crises and banking crises in recent episodes, the literatures on the two topics have for the most part developed separately. Important exceptions are Chang and Velasco (1998a,b). The first paper develops a model of currency and banking crises based on the Diamond and Dybvig (1983) model of bank runs. Chang and Velasco introduce money as an argument in the utility function. A central bank controls the ratio of currency to consumption. Different exchange rate regimes correspond to different rules for regulating the currency-consumption ratio. There is no aggregate uncertainty in these models: banking and currency crises are "sunspot" phenomena. In other words, there are at least two equilibria, a "good" equilibrium in which early consumers receive the proceeds from short-term assets and late consumers receive the proceeds from long-term assets and a "bad" equilibrium in which everybody believes a crisis will occur and these beliefs are self-fulfilling. Chang and Velasco (1998a) shows that the existence of the bad equilibrium depends on the exchange rate regime in force. In some regimes, only the good equilibrium exists; in other regimes there also exists a bad equilibrium in addition to the good equilibrium. The selection of the good or the bad equilibrium is not modeled. In Chang and Velasco (1998b) a similar model is used to consider recent crises in emerging markets. Again there is no aggregate uncertainty and crises are sunspot phenomena.

A number of other recent papers have focused on the possibility of multiple equilibria. These include Flood and Garber (1984), Obstfeld (1986; 1994) and Calvo (1988). In these models governments are unable to commit to policies and this lack of commitment can give rise to multiple equilibria, at least one of which is a self-fulfilling crisis. Again, the selection of equilibrium is problematic. An exception is Morris and Shin (1995) who show that lack of common knowledge among traders about the state of the economy can lead to a unique equilibrium selection.

Kaminsky and Reinhart's (1999) finding that crises are related to economic fundamentals is consistent with work on U.S. financial crises in the nineteenth and early twentieth centuries. Gorton (1988) and Calomiris and Gorton (1991) argue that the evidence is consistent with the hypothesis that banking crises are an essential part of the business cycle rather than a sunspot phenomenon. Allen and Gale (1998) develop a model in which banking crises are generated by aggregate uncertainty about asset returns. Moreover, although equilibrium is not necessarily unique, it can be shown that crises are a feature of all equilibria of the model when asset returns are low.

In the Allen-Gale model, crises can improve risk sharing but they also involve deadweight costs if they cause projects to be prematurely liquidated. A central bank can avoid these deadweight costs and implement an optimal allocation of resources through an appropriate monetary policy. By creating fiat money and lending it to banks, the central bank can prevent the inefficient liquidation of investments while at the same time allowing optimal sharing of risks.

In this paper we extend the model of Allen and Gale (1998) to an international context and study the relationship between banking and currency crises. Section 2 begins by describing a simple one-country version of the model with three dates and two assets. As in Allen and Gale (1998), there is a large number of ex ante identical agents who discover at the intermediate date whether they require liquidity immediately or at the final date. There are two assets, a safe, short-term asset represented by a storage technology and a risky, long-term asset that pays off at the final date. At the intermediate date a leading economic indicator reveals the true return to the risky asset. If the long-term asset is liquidated at the intermediate date, there is a liquidation cost. The optimal allocation is characterized as a planner's problem with state contingent contracts.

In Section 3 it is shown that a banking system in which banks use a nominal non-contingent deposit contract and the central bank gives them access to a line of credit can implement the first-best allocation. Depositors bear risk, but it is allocated optimally between early consumers and late consumers.

Section 4 extends the model by introducing an international bond market in which the domestic country can borrow and lend at a fixed rate. The domestic country is assumed to be small relative to the rest of the world and therefore has no impact on prices in the bond market. Also, since the domestic country is small relative to the global market, lenders are risk neutral.

We begin by studying the optimal allocation implemented by a planner who is allowed to trade state contingent contracts on the international market. Since the market is risk neutral, the optimal allocation requires the (risk-averse) domestic depositors to bear no risk. Instead, their consumption is non-stochastic and all the risk is borne by the international capital market.

Next we consider a decentralized regime with flexible exchange rates and international debt denominated in the domestic currency. It is shown that, despite the absence of any explicitly contingent contracts, a laisser-faire market equilibrium corresponds to the first-best allocation. Banks issue a large amount of bonds denominated in domestic currency and invest the money in bonds denominated in foreign currency. Variations in the return to the risky asset cause the domestic price level and hence the exchange rate to fluctuate in precisely the way required to make the real value of domestic debt optimally contingent. The outcome is that domestic depositors receive a non-stochastic consumption allocation and once again the international market bears all the risk.

This result depends on banks having large offsetting positions in domestically denominated bonds and foreign denominated bonds. This is consistent with the observation that the volume of trading in foreign exchange markets is much higher than can be justified by the needs of world trade.

It is also shown that the use of short-term debt is optimal if the yield curve in the international bond market is flat or upward sloping. The reason is that providing liquidity at the intermediate date by rolling over debt is at least as good as borrowing long-term in these circumstances. This may help to rationalize the otherwise puzzling use of unhedged short-term debt in many emerging markets.

The use of debt denominated in domestic currency presents a risk to foreign investors in the domestic country. After the contracts with foreign bondholders are written, the country has an incentive to inflate its currency and effectively expropriate the bondholders. This means that lenders may be reluctant to hold debt denominated in the domestic currency. Instead, they may demand debt denominated in terms of a foreign (reserve) currency which is not subject to inflation risk. Section 5 considers two variants of the model in which debt denominated in foreign currency is used. In the first, there is no central bank in the domestic country. Both the international bonds and the deposit contracts used by banks are specified in foreign currency (i.e. real) terms. Since all the contracts are in real terms, a banking crisis again becomes a possibility. It is assumed that in the event of bankruptcy, the claims of domestic depositors and foreign bondholders have equal priority. It is shown that if the banks borrow a large amount and invest in the international bonds they can eliminate downside risk for depositors. This does not eliminate inefficient liquidation, however. Nor does it permit optimal risk sharing between depositors with early and late liquidity needs.

In the second variant of the model, a central bank is introduced and allowed to offer a line of credit to banks in terms of domestic currency. By writing nominal contracts in domestic currency, the amount of bankruptcy caused by the foreign denominated debt can be reduced for a given portfolio of bank assets and a given amount of real liabilities. Although risk sharing between early and late consumers is improved, risk sharing between depositors and the international bond market is eliminated. Given these trade-offs, the existence of a central bank and a domestic monetary system may or may not improve welfare when international debt is denominated in foreign currency.

Section 6 discusses the policy implications of the model for the role of the International Monetary Fund (IMF).

2 Optimal Risk Sharing

In this section, we define the risk sharing problem for a closed economy. Later the model will be opened to include an international bond market.

The basic structure of the model is drawn from Allen and Gale (1998). There are three dates $t = 0, 1, 2$. At each date, there is a single good that can be used for consumption and investment. There are two kinds of asset in the domestic economy, a safe asset and a risky asset. The safe asset is modeled as a storage technology: one unit of the good invested at date t produces one unit of the good at date $t + 1$, for $t = 0, 1$. The risky asset takes two periods to mature: x units of the good invested at date 0 yields $Rh(x)$ units of the good at date 2 where $h(x)$ is a neoclassical, decreasing-returns-to-scale production function (increasing, strictly concave, twice continuously differentiable). The random variable R has realization r and a support $[r_0, r_1]$, where $0 \leq r_0 < r_1 < \infty$. The cumulative distribution function $F(r)$ is assumed to be continuous and increasing on the support $[r_0, r_1]$. At date 1 agents observe a signal, which can be thought of as a leading economic indicator. For simplicity, it is assumed that this signal predicts with perfect accuracy the value of r that will be realized at date 2. We begin by considering the planner's problem, in which the optimal allocation is contingent on r. In subsequent sections we consider the case where it is impossible to write explicit contracts contingent on r.

There is a continuum of ex ante identical agents. Each agent has an endowment of one unit of the good at date 0 and none at dates 1 and 2. Agents are subject to a time-preference shock at date 1. A fraction of them will become *early consumers*, who only value consumption at date 1 and the remainder of them will become *late consumers*, who only value consumption at date 2. For simplicity, we assume that there are equal numbers of early and late consumers and that each consumer has an equal chance of belonging to each group. The size of each group is normalized to one. Thus, the agent's utility function can be written as

$$U(c_1, c_2) = u(c_1) + u(c_2)$$

where $c_t \geq 0$ is the agent's consumption at date $t = 1, 2$ and $u(\cdot)$ is a neoclassical utility function (increasing, strictly concave, twice continuously differentiable).

At date 0 all agents are ex ante identical and they hold the same beliefs about the uncertain returns to the assets. Uncertainty is resolved at the beginning of date 1: individual agents learn whether they are early or late consumers and the returns to the risky asset are revealed. A consumer's type is not observable, so late consumers can always imitate early consumers. Therefore, contracts explicitly contingent on this characteristic are not feasible.

Suppose that a planner were given the task of choosing an optimal risk-sharing arrangement. Since all agents are ex ante identical, it is natural for the planner to treat all agents alike and maximize their ex ante expected utility. The optimal consumption allocation will depend only on the aggregate wealth of the economy. Let (x, y) denote the optimal portfolio, where x is the investment in the risky asset and y is the investment in the safe asset. Let $(c_1(r), c_2(r))$ denote the optimal consumption allocation, where $c_t(r)$ is the consumption at date $t = 1, 2$ when r is the realization of the risky return R.

The planner's problem can be defined as follows:

$$\begin{array}{ll} \max & E_R[u(c_1(r)) + u(c_2(r))] \\ \text{s.t.} & x + y \leq 2 \\ & c_1(r) \leq y \\ & c_2(r) \leq rh(x) + y - c_1(r) \\ & c_1(r) \leq c_2(r). \end{array} \tag{1}$$

The first constraint is the budget constraint at date 0, which says that the investment in safe and risky assets must be less than or equal to the endowment. The second constraint is the budget constraint at date 1, which says that consumption at date 1 must be less than or equal to the amount of the safe asset held over from date 0. The third constraint is the budget constraint at date 2, which says that consumption at date 2 must be less than the return from the risky asset $rh(x)$ plus the amount of the safe asset $y - c_1(r)$ left over from date 1. The final constraint is the incentive constraint, which says that the late consumers (weakly) prefer their own allocation to that of the early consumers. If this constraint was violated, the late consumers would pretend to be early consumers, receive $c_1(r)$ at date 1, save it in the form of the safe asset until date 2, and then consume it.

The following assumptions are adopted to ensure the planner's problem has an interior optimum. The preferences and technology are assumed to satisfy the inequalities

$$E[r] > 1 \tag{2}$$

and

$$u'(0) > E[u'(rh(2))rh'(2)]. \tag{3}$$

The first inequality ensures that a positive amount of the risky asset is held while the second ensures a positive amount of the safe asset is held.

In solving the planner's problem, it turns out that we can ignore the incentive constraint. To see this, we drop the constraint and solve the unconstrained problem. From the first-order conditions, we see that a necessary condition for an optimum is that the consumption of the early and late consumers be equal, unless the budget constraint $c_1(r) \leq y$ is binding, in which case it follows from the first order-conditions that $c_1(r) = y \leq c_2(r)$. Thus, the incentive constraint will always be satisfied if we optimize subject to the first three constraints only and the solution to the planner's problem is in fact the first-best allocation.

Proposition 1 *The solution $(x, y, c_1(\cdot), c_2(\cdot))$ to the planner's problem is uniquely characterized by the following conditions:*

$$c_1(r) = c_2(r) = \frac{rh(x) + y}{2} \text{ if } y \geq rh(x),$$

$$c_1(r) = y, c_2(r) = rh(x) \text{ if } y \leq rh(x),$$

$$x + y = 2$$

and

$$E[u'(c_1(r))] = E[u'(c_2(r))rh'(x)].$$

Under the maintained assumptions, the optimal portfolio must satisfy $x > 0$ and $y > 0$. The allocation is first-best efficient.

Proof. See the Appendix.

The optimal allocation is illustrated by Figure 1 which plots consumption at each of the two dates against r. At date 0, the portfolio (x, y) is chosen to equate the expected marginal utilities of early and late consumers. Suppose that at date 1 it is found out that $r = 0$. The only output is from the short-term asset y and so the optimal allocation is to give $y/2$ to the early consumers and carry over $y/2$ in the safe asset for the late consumers. As r increases both early and late consumers can be given more as it is still possible to equate consumption at the two dates by carrying over some of the output to the last date using the safe asset. As r continues to increase there comes a point where $\overline{r}$ such that $\overline{r}h(x) = y$. Now, as r increases, it is no longer possible to equate consumption at the two dates. Most of the output is now produced at date 2 instead of date 1. Whereas it is technologically feasible to carry output forward through time, it is not physically possible to do the reverse. The best that can be done is to give all the output available at date 1, that is, y to the early consumers. The late consumers receive everything produced at date 2, that is, $rh(x)$.

It is a well-known result that in this type of model, the allocation with a classical stock market is inefficient (Jacklin (1987)). In such a market, agents can invest their individual endowments in the long and short assets to provide for consumption at dates 1 and 2; but this provides no insurance against the intertemporal preference shock. If they invest in the short asset to provide consumption at date 1, they miss out on the higher returns from the long asset. If they invest in the long asset to provide consumption at date 2, they run the risk of having to sell the asset at a low price to provide consumption at date 1. The absence of an effective market for insuring individual preference shocks means that the first-best cannot be implemented using the stock market alone.

3 Banking

We next consider the risk sharing that can be achieved through a competitive banking system, in which individual banks purchase assets to provide for the future consumption of depositors. The country is assumed to have a large number (continuum) of banks. Competition among banks leads them to maximize the expected utility of the typical depositor subject to a zero-profit (feasibility) constraint. The agents in the country only have access to banks.

Banks are assumed to take deposits from agents at date 0 and offer them a deposit contract specified in real terms promising $d_1 \geq 0$ units of consumption at date 1 and $d_2 \geq 0$ units of consumption at date 2. It is crucial here, as in all the literature on bank runs, that the deposit contract is not explicitly contingent on the returns to the risky assets. When the returns to the risky assets are low, the banks may not be able to meet their commitments to pay out fixed amounts to their depositors. In that case, what the banks do pay out depends on the rules governing the banks' behavior and the possibility/necessity of liquidating assets. A banking panic may result.

Allen and Gale (1998) show that such banking panics can, in fact, be beneficial when the risky asset is completely illiquid. When the return on the risky asset is low, optimal risk sharing requires that the consumption of both the early and late consumers be reduced. A banking panic achieves this end. Some of the late consumers join the early consumers in withdrawing their deposits. Given the limited amount of liquidity available for those withdrawing at the first date, the amount each agent receives is smaller the greater is the number of premature withdrawals by late consumers. Panics allow deposit contracts to be de facto contingent on r. The optimality of bank runs in this model depends crucially on the assumption that assets cannot be liquidated prematurely. When liquidation is possible and costly, things are not quite so simple.

Suppose that the risky asset can be liquidated at date 1, and that this premature liquidation is costly. Here we simplify the analysis by assuming that premature liquidation costs are a fixed proportion of the return at maturity.[1] More precisely, if the return on the risky asset is r at date 2, x units of the asset can be liquidated at date 1 for a return of $\gamma r h(x)$, where $0 < \gamma < 1$.

If the costs of liquidation are small, it may sometimes be optimal to use the liquidation technology to provide liquidity, rather than holding the short asset. We rule out this possibility by imposing the following restriction:

$$\gamma u'(y) < u'(r_1 h(x)), \tag{4}$$

where r_1 is the upper bound of r and (x, y) is the optimal portfolio from the planner's problem. It is easy to see that this condition is necessary and sufficient to ensure that the planner would never want to liquidate risky assets at date 1. When the risky asset attains its highest value, the consumption profile is

$$c_1(r_1) = y, c_2(r_1) = r_1 h(x).$$

In order to increase the early consumer's consumption by γ units the consumption of the late consumers has to be reduced by 1 unit. Under the assumption above, this will never increase the objective function. The constraint (4) typically requires that γ be small, so the bank will only liquidate in exceptional circumstances.

[1] In Allen and Gale (1998), we show how liquidation values can be endogenized by introducing a market for the risky asset at date 1. The return on the liquidated asset is then determined by the price at which it can be sold at short notice on the asset market.

Next, we specify the bankruptcy rules that govern the bank's behavior if it cannot meet its obligations. If it can pay d_1 to depositors demanding withdrawal at date 1 it must do so, even if that means liquidating its holding of the risky asset at a loss; if it cannot pay d_1 to all the depositors demanding withdrawal at date 1, it must liquidate all its assets and pay out the liquidated value to the depositors at date 1. Obviously, in this last case, there will be nothing left for depositors at date 2, so all depositors, whether early or late consumers, will withdraw at date 1. In other words, there will be a run on the bank.

The assets remaining in the bank at date 2 are paid out to the remaining depositors. Hence, it is optimal for the bank to choose d_2 large enough so that nothing is left over after the late consumers have been paid. Since only premature liquidation is costly there are no deadweight losses from doing this.

As a result of these assumptions, there will be a critical value of r at which the bank is just able to avoid a run. To avoid a run, it must be possible to give both early and late consumers d_1 units of consumption. Given (4) it will never be optimal for the bank to choose $d_1 > y$. It would be better for the bank to increase y and avoid the need to liquidate the long asset. Thus, in equilibrium we have $d_1 \leq y$, that is, liquidation only occurs when there is a run.

The consumption of late consumers in the absence of a run is

$$c_2(r) = rh(x) + y - d_1.$$

Let r^* denote the critical value of r defined by the condition $c_2(r) = d_1$. Then r^* is implicitly defined by

$$d_1 = r^* h(x) + y - d_1$$

or

$$r^* = \frac{2d_1 - y}{h(x)}.$$

For $r \geq r^*$, the early consumers receive d_1 and the late consumers receive

$$c_2(r) = rh(x) + y - d_1.$$

For $r < r^*$, all consumers receive the liquidated value of the assets at date 1:

$$c_1(r) = c_2(r) = \frac{\gamma rh(x) + y}{2}.$$

With these assumptions, the bank's decision problem can be written as follows:

$$\begin{array}{ll} \max & E_R[u(c_1(r)) + u(c_2(r))] \\ \text{s.t.} & x + y \leq 2 \\ & c_1(r) = d_1, \forall r \geq r^* \\ & c_2(r) = rh(x) + y - d_1, \forall r \geq r^* \\ & c_1(r) = c_2(r) = \frac{1}{2}(\gamma rh(x) + y), \forall r < r^* \\ & r^* = (2d_1 - y)/h(x). \end{array}$$

Assuming that the planner does not want to use the liquidation technology at the optimum, we can compare the solution of the planner's problem directly with the solution of the typical banker's problem and conclude that the two are different if there is a positive probability of liquidation.

Proposition 2 *Let* $(x, y, c_1(r), c_2(r))$ *be the solution to the planner's problem and let* $(\hat{x}, \hat{y}, \hat{c}_1(r), \hat{c}_2(r))$ *be the solution to the bank's problem above. If condition (4) is satisfied the solution to the planner's problem does not require premature liquidation of the long asset. Conversely, if* $\Pr[r < r^*] > 0$ *then the solution to the bank's problem does require premature costly liquidation, so it yields depositors a lower ex ante expected utility than they obtain in the first-best allocation.*

Figure 2 illustrates the allocation provided by a banking system with deposit contracts specified in real terms. The optimal allocation has the same form as in Figure 1 with one important difference. For $r < r^*$ there is costly liquidation of the risky asset. There is thus a discontinuity at r^*. As a result of the deadweight loss associated with inefficient liquidation, the amount invested in the risky asset will typically be less than in the first-best case. Also, it is possible that $y < d_1$ so that there is a "buffer" of the safe asset to lower r^*.

3.1 Optimal Monetary Policy

The inefficiency of equilibrium with bank runs arises from the fact that liquidating the risky assets at date 1 is costly. Costly liquidation can be avoided if the central bank follows an appropriate monetary policy. Essentially, the policy consists of setting the price level by being willing to exchange goods for money at an appropriate ratio. In order for this intervention to have the required effect on the choice of portfolio and the allocation of consumption, the deposit contract has to be specified in *nominal* terms. Formally, a deposit contract promises the depositor D_1 units of money if she withdraws in the middle period and D_2 if she withdraws in the final period. (Nominal amounts are denoted by upper case variables.) As before, there is no loss of generality in assuming that D_2 is chosen large enough that the depositors receive whatever assets the representative bank has left in the final period.

Let $p_t(r)$ denote the price level at date $t = 1, 2$ when the return on the risky asset is r. In what follows, it simplifies matters to note that

$$p_1(r) = p_2(r) = p(r).$$

This follows from a no-arbitrage argument. Note that when $p_1(r) = p_2(r)$, the return on holding money between date 1 and date 2 is the same as the return on the safe asset. By contrast, if $p_1(r) > p_2(r)$ then banks will only be willing to hold money while if $p_1(r) > p_2(r)$ they will only be willing to hold goods.

Let $(x, y, c_1(\cdot), c_2(\cdot))$ denote the solution to the planner's problem in Section 2 and suppose that at date 0 the representative bank chooses the portfolio (x, y). The central bank determines the price level $p(r)$ by promising to exchange money for goods at a ratio of $p(r)$. Since r is publicly observable the central bank is able to implement such a policy. The individual banks take $p(r)$ as given. If $p(r)$ is chosen to be inversely proportional to $c_1(r)$, then the banks will choose D_1 so that

$$\frac{D_1}{p(r)} = c_1(r). \tag{5}$$

D_2 is chosen large enough so that it does not constrain the allocation. For example, we could choose the deposit contract (D_1, D_2) so that

$$D_1 = y; D_2 = r_1 x.$$

To show that (x, y, D_1, D_2) is optimal for the bank's decision problem, we simply appeal to the fact that $(x, y, c_1(r), c_2(r))$ solves the planner's problem. Thus, there is no better allocation $(x, y, c_1(r), c_2(r))$ satisfying the constraints of the planner's problem. It is easy to show that anything that is feasible for the bank must also satisfy the planner's constraints. Thus, it cannot do better than the solution to the planner's problem.

Proposition 3 *If the central bank chooses the appropriate price level and banks use nominal contracts, the first-best allocation can be implemented as an equilibrium.*

The price level that implements the first-best allocation is

$$p(r) = \begin{cases} 1 & \text{for } r \geq \overline{r} \\ = \frac{2D_1}{rh(x)+y} & \text{for } r < \overline{r}. \end{cases}$$

This is illustrated in Figure 3. For $r \geq \overline{r}$, the central bank fixes the price level at 1 by promising to exchange money for goods at this ratio. For $r < \overline{r}$, the central bank sets the price level equal to the ratio of the nominal claims on the banking system $2D_1$ to the real output from the banking system's assets $rh(x) + y$. This policy ensures that the banks are always able to meet their nominal commitments so runs and costly liquidation are prevented. In order to implement this policy the central bank must observe r and set the price level appropriately by standing ready to exchange money for goods at the appropriate rate. For $r < \overline{r}$, the lower is r the higher is $p(r)$ and this has the effect of lowering consumption. In this way the central bank and the banking system through its effect on the nominal price level at each date implements optimal risk sharing between the early and late consumers.

In the equilibrium described above crises are avoided through variations in the nominal price level and the only role of money is as a unit of account. In Allen and Gale (1998) we show how the same allocation can be implemented in a different way with money playing a more important role when $r < \overline{r}$. For these values of r there is a run on banks at date 1. In order to prevent costly liquidation the central bank gives the representative bank a loan of M units of money. The bank gives depositors a combination of money and goods equal to the value of the deposit contract D_1. Since early consumers want to consume their entire wealth at date 1, they exchange the money for consumption goods with the early-withdrawing late consumers. The price level adjusts so that the early consumers end up with the first-best consumption level and the early-withdrawing late consumers end up holding all the money. At date 2, the representative bank has to repay its loan. For simplicity, we assume the loan bears zero interest. The money now held by late consumers is just enough to allow the bank to repay its loan and the bank has just enough consumption from its remaining assets to give the early-withdrawing late consumers the first-best consumption level. In this version of the model the central bank does not need to fix the price level when $r < \overline{r}$. The central bank "suspends convertibility" and the price level is determined by the ratio of the exchange of money for goods in the market. Money acts as a store of value for the early-withdrawing late consumers as well as a unit of account. See Allen and Gale (1998) for details. In what follows, for ease of exposition the model is described in terms of the central bank setting the price level for $r < \overline{r}$ and avoiding crises but it is important to realize this alternative interpretation is also possible.

4 International Finance

The closed economy can be opened by assuming the existence of an international bond market. Initially, we assume that short-term bonds can be used. The introduction of long-term bonds is considered later. The risk-free return on bonds is fixed at $\rho \geq 1$ (we assume that the small open economy has no effect on international interest rates). This means that one unit of the good at date t can be exchanged for ρ units at date $t+1$ for $t = 0, 1$. Of course, the risk of default will be reflected in the face value of any debt that is issued by the banks of the small country. We assume that because the country is small, the international bond market is risk neutral in the sense that, when default can occur, the loan is priced so that the expected return is ρ. The banks of the small country can also invest in the international bond market. International bonds now replace the storage technology as the safe asset.

To guarantee there is positive investment in the risky asset, it is necessary that (2) be replaced by

$$E[rh'(0)] > \rho^2.$$

Access to international capital markets is potentially valuable for three reasons.

- First, it means that when r is high, liquidity can be obtained for early consumers by borrowing at date 1.
- Second, it may be possible to make a profit by borrowing short and investing long in the risky asset at date 0 because the return on bonds is lower than the expected return on the risky asset.
- Third, it may be possible to transfer the small country's asset return risk to lenders in the international bond market.

An important issue concerns who has access to what markets. In the closed-economy version of the model, we assumed that individuals had access to the storage technology. Here we assume that they have access to the international bond market. They can buy or sell bonds or, in other words, lend or borrow at the rate ρ.

4.1 Optimal Risk Sharing

As usual we start by characterizing the first-best allocation assuming the planner can use contracts which are contingent on r. If a planner in the small country can write such state-contingent contracts with the international capital market it would be possible for risk to be transferred to the foreigners and consumers at each date would consume a constant amount independent of r. Let $I(r)$ be the transfer from the international capital market contingent on r. The planner's problem is

$$\begin{array}{ll} \max & E_R[u(c_1(r)) + u(c_2(r))] \\ \text{s.t.} & x + y \leq 2 \\ & c_1(r) + c_2(r)/\rho \leq rh(x)/\rho + \rho y + I(r) \\ & \int_0^\infty I(r)dF = 0 \\ & \rho c_1(r) \leq c_2(r). \end{array}$$

The first constraint is the budget constraint at date 0, which says that the investment in safe and risky assets must be less than or equal to the endowment. The second constraint is the budget constraint in terms of date 1 resources. It says that the present value of consumption at date 1 must be less than or equal to the present value of asset returns plus the state contingent transfer from the international capital market. The third constraint ensures that on average the state contingent transfer is zero. In other words the international capital market is simply providing insurance. The final constraint is the incentive constraint. Since all consumers have access to the international capital market the planner must satisfy this constraint. If it were not satisfied the late consumers could withdraw $c_1(r)$ and invest it in the international bond market and earn a return ρ. The constraint is an inequality because the early consumers are unable to leave there money in the bank and borrow against it.

Since consumption must be non-negative, a feasible transfer function must satisfy $rh(x)/\rho + \rho y + I(r) \geq 0$. It will be seen that this condition is automatically satisfied by the solution to the planner's problem. The first-order conditions for this problem are

$$u'(c_1(r)) \geq \rho u'(c_2(r)) \text{ with equality if } \rho c_1(r) < c_2(r).$$

$$\int_0^\infty u'(c_1(r)) \left(\frac{r}{\rho}h'(x) - \rho\right) dF = 0$$

$$u'(c_1(r)) = \lambda,$$

where λ is the Lagrange multiplier on the third constraint.

The third condition ensures that $c_1(r)$ is a constant, independently of r. The following result then follows directly.

Proposition 4 *The incentive-efficient allocation with access to complete, risk-neutral international capital markets has a consumption allocation*

$$(c_1(r), c_2(r)) = (\bar{c}_1, \bar{c}_2)$$

where the ordered pair $(\bar{c}_1, \bar{c}_2)$ *solves the problem*

$$\begin{array}{ll} \max & u(\bar{c}_1) + u(\bar{c}_2) \\ s.t. & \bar{c}_1 + \bar{c}_2/\rho = E[r]h(x)/\rho + \rho(2-x) \\ & \bar{c}_1 \leq \bar{c}_2/\rho \end{array}$$

and the amount $x > 0$ *invested in the risky asset satisfies*

$$E[r]h'(x) = \rho^2.$$

Since the international capital market is risk neutral and domestic depositors are risk averse, if state-contingent contracts are possible the optimal allocation involves investors bearing no risk at all. They simply receive a share of expected output. The international capital market bears all the risk. The investment in the risky asset is such that the expected marginal product is equal to the opportunity cost in the bond market.

4.2 Domestic Currency Debt

We next consider the allocation of resources when the international capital market is a debt market and there is a domestic banking system that issues deposits denominated in the domestic currency. For the moment we assume that domestic banks can issue debt denominated in the domestic currency on the international capital market. A bond issued at date 0 promises one unit of the domestic currency to the holder at date 1. Let q denote the price of one domestic currency bond and let B denote the number of bonds issued at date 0. The benefits of borrowing at a low rate in the international bond market are passed on to the depositors in the form of a more attractive deposit contract $(c_1(r), c_2(r))$. We assume, as before, that competition in the banking sector ensures that each bank seeks to maximize the expected utility of the typical depositor.

For simplicity, we assume that the nominal domestic interest rate is 0. Arbitrage between international reserve currency bonds and domestic currency ensures that

$$p_1(r) = \rho p_2(r) \tag{6}$$

for every value of r, where $p_t(r)$ is the domestic price level at date $t = 1, 2$. In what follows we write $p(r)$ for the date 1 price level and set $p_2(r) \equiv p(r)/\rho$.[2]

Suppose the representative bank chooses a portfolio (x, y) at date 0, where x is the investment in the risky asset and y is the investment in the safe asset (debt denominated in the international reserve currency). The bank takes in a deposit of 1 from each of the consumers and in return gives each a nominal claim of D_1 at date 1 and D_2 at date 2. As usual, we can assume without loss of generality that D_2 is chosen so large that the late withdrawers get whatever assets are left over at date 2. In what follows, we write D in place of D_1 and assume that $D_2 = \infty$.

The bank borrows qB of the consumption good on the international capital market so the budget constraint at date 0 is

$$x + y \leq 2 + qB. \tag{7}$$

At date 1, r is observed and investors learn whether they are early or late consumers. The bank has promised early withdrawers D units of the domestic currency and the international bond holders B. The real values of these claims are $D/p(r)$ and $B/p(r)$, respectively. Since the bank can borrow and lend at the rate ρ (now that uncertainty has been resolved) the present value budget constraint at dates 1 and 2 can be written as

$$c_1(r) + \frac{c_2(r)}{\rho} + \frac{B}{p(r)} = \frac{rh(x)}{\rho} + \rho y.$$

The bank must also satisfy the incentive constraint

$$\rho c_1(r) \leq c_2(r).$$

[2] With the domestic interest rate normalized to 0, the domestic price level must fall between date 1 and date 2 in order to satisfy the covered interest arbitrage condition (6). A different normalization would imply a different rate of inflation. For example, if we set the domestic interest rate equal to $\rho - 1$ then the domestic inflation rate would be 0.

Otherwise, late consumers can withdraw D units of currency at date 1 and spend it on $D/p_2(r) = \rho D/p(r) = \rho c_1(r)$ units of goods at date 2, thus increasing their utility. Using the definition of $c_1(r) = D/p(r)$ and the budget constraint, we can rewrite the incentive constraint as

$$\frac{D}{p(r)} \leq \frac{c_2(r)}{\rho} = \frac{rh(x)}{\rho} + \rho y - \frac{D+B}{p(r)}$$

or

$$\frac{2D+B}{p(r)} \leq \frac{rh(x)}{\rho} + \rho y. \tag{8}$$

Conversely, if this constraint is satisfied, then the budget constraint and the incentive constraint can also be satisfied with $c_1(r) = D/p(r)$.

If it is possible to satisfy all of these constraints at date 1 then the bank is solvent, and there is no need to liquidate the risky asset. (We are assuming that there are no unnecessary runs - if there is one equilibrium with runs and one without we focus on the no-run equilibrium). However, if it is not possible to satisfy the budget constraint and the incentive constraint with $c_1(r) = D/p(r)$ so that (8) is not satisfied, then the bank must declare bankruptcy. All assets must be liquidated to meet the claims of the domestic depositors and the international bondholders. The liquidated value of all the assets is distributed in proportion to the creditors claims. Thus, the depositors each receive a fraction $D/(2D+B)$ of the asset value and the international bondholders receive the rest. Then

$$c_1(r) = c_2(r)/\rho = \frac{D}{2D+B}\left(\gamma rh(x) + y\right).$$

Note that, although early and late consumers receive equal shares of the liquidated assets of the bank, the late consumers can invest in the international bond market, so their share of the liquidated assets yields them a higher consumption level at date 2.

The bank takes prices as given and chooses a portfolio (x, y, B) and a deposit contract D to maximize the expected utility of the typical depositor. Thus, the bank's decision problem is

$$\begin{array}{ll} \max & E_R[u(c_1(r)) + u(c_2(r))] \\ \text{s.t.} & x + y \leq 2 + qB \\ & \rho c_1(r) \leq c_2(r), \forall r, \end{array} \tag{9}$$

where the consumption functions of the early and late consumers are given by the equations

$$c_1(r) = \begin{cases} \frac{D}{p(r)} & \text{if } \frac{2D+B}{p(r)} \le \frac{rh(x)}{\rho} + \rho y \\ \frac{D}{2D+B}\left(\gamma r h(x) + y\right) & \text{otherwise.} \end{cases}$$

and

$$\frac{c_2(r)}{\rho} = \begin{cases} rh(x)/\rho + \rho y - \frac{D+B}{p(r)} & \text{if } \frac{2D+B}{p(r)} \le \frac{rh(x)}{\rho} + \rho y \\ \frac{D}{2D+B}\left(\gamma r h(x) + y\right) & \text{otherwise,} \end{cases}$$

respectively. As explained above, if the incentive constraint can be satisfied, there is no run, the early consumers receive the promised payment D and the late consumers are the residual claimants. If the incentive constraint cannot be satisfied, there is a run and the early and late consumers share the liquidated assets of the bank equally.

4.3 Optimal Exchange Rate Policy as Security Design

Similarly to Section 3.1, we assume that the price level $p(r)$, and hence the exchange rate $1/p(r)$, is controlled by the central bank. The central bank's exchange rate policy determines the real values of the deposit contract and the domestic-currency bond at date 1. By adopting an optimal exchange rate policy, the central bank is effectively designing an optimal security for depositors. In this section, we incorporate this security design element into the analysis of equilibrium risk sharing. Before embarking on a formal analysis of risk sharing, we offer some heuristics to explain how the risk associated with investment in domestic assets can be shifted from domestic investors to the international capital market.

The important point is that the exchange rate is correlated with the returns on the banking sector's assets. This means the banks can hedge the risk of their assets. When r is low the exchange rate $1/p(r)$ is weak. By borrowing in domestic currency and investing in bonds denominated in foreign currency it is possible for banks to make a profit when asset returns and the exchange rate are low. When r is high and $1/p(r)$ is high this portfolio makes a loss. Since domestic and international bonds are both fairly priced, the expected return on the portfolio is zero. The effect of the strategy is therefore to transfer returns from states with high r to states with low r. This is precisely what's needed to hedge the risk associated with the domestic risky asset and transfer it from domestic banks to the international bond market.

Suppose that no runs occur in equilibrium. In that case, the real securities held by international bondholders and depositors are proportional to $1/p(r)$. The equilibrium price q at which domestic-currency bonds are issued must on average allow the lenders to recoup ρq on each bond. Then the fair-pricing condition is

$$q = \frac{1}{\rho} \int_0^\infty \frac{1}{p(r)} dF. \tag{10}$$

From the first-period budget constraint,

$$y = 2 + qB - x,$$

which indicates that, for a fixed investment x in risky assets, the more the domestic-currency debt the bank issues, the more it can invest in international bonds. In effect, it is exchanging domestic-currency debt with a real return $1/p(r)$ for international debt with a safe return of ρ.

Substituting the expression for y into the third-period budget constraint and rearranging, we get the following expression for $c_2(r)$:

$$c_2(r) = rh(x) - \rho^2 x + 2\rho^2 - \frac{\rho D}{p(r)} + \rho \left(E\left[\frac{1}{p(r)}\right] - \frac{1}{p(r)} \right) B. \tag{11}$$

The term in parentheses on the right hand side has zero mean and variance proportional to B^2. For fixed values of x and D, an increase in B will change the variance of $c_2(r)$ without changing the mean. If $p(r)$ and $c_2(r)$ are positively correlated, then increasing B will reduce the variance of $c_2(r)$. This suggests that, if the exchange rate $1/p(r)$ is chosen appropriately, simultaneous borrowing in terms of the domestic currency and lending in terms of the international reserve currency can reduce the risk borne by domestic investors. In effect, the banks are shorting domestic currency which is risky in real terms in order to go long in the foreign asset which is riskless in real terms.

In certain special cases, we can show explicitly how all the risk of the domestic assets can be shifted to the international capital market. Consider an equilibrium in which there are no runs and the incentive-constraint $\rho c_1(r) = c_2(r)$ is binding for every r. For simplicity, normalize the face value of the nominal deposit to $D = 1$ and let $\bar{B}$ be the number of bonds issued by the representative bank. Then, subsituting from the budget constraints at date 1 and date 2 into the incentive constraint, we get

$$\begin{aligned} \rho\frac{D}{p(r)} &= \rho c_1(r) \\ &= c_2(r) \\ &= rh(x) + \rho^2(2 + qB - x) - \rho\frac{D + B}{p(r)} \end{aligned}$$

or using the fact that D was normalized to 1 and $\overline{B}$ bonds are issued by the representative bank this simplifies to

$$\rho\frac{2 + \overline{B}}{p(r)} = rh(x) + \rho^2(2 + q\overline{B} - x)$$

This equation can be solved to give an expression for the real value of a domestic-currency bond

$$\frac{1}{p(r)} = \frac{rh(x)/\rho + \rho(2 + q\bar{B} - x)}{2 + \bar{B}}. \tag{12}$$

The price level is determined by the ratio of the nominal claims $2 + \bar{B}$ to the assets of the representative bank $rh(x) + \rho(2 + q\bar{B} - x)$. In particular, note that the real return to a domestic-currency bond is an affine function of the risky return $rh(x)$.

Since the risky asset returns are positively correlated with the exchange rate, by shorting domestic-currency bonds and simultaneously going long in the reserve-currency bonds it is possible to hedge the risk. In fact, in this case a single bank can eliminate the risk faced by the late consumers entirely. To see this, substitute from equation (12) above into (11):

$$\begin{aligned} c_2(r) &= rh(x) - \rho^2 x + 2\rho^2 - \rho\frac{1}{2 + \bar{B}}\left(rh(x)/\rho + \rho(2 + q\bar{B} - x)\right) \\ &\quad + \rho^2 qB - \rho\frac{B}{2 + \bar{B}}\left(rh(x)/\rho + \rho(2 + q\bar{B} - x)\right). \end{aligned}$$

To eliminate $rh(x)$ from this equation, just collect the like terms and set the coefficient to 0:

$$1 - \frac{1}{2 + \bar{B}} - \frac{B}{2 + \bar{B}} = 0. \tag{13}$$

Since the expected value of $c_2(r)$ is independent of B (this follows from the fact that bonds are fairly priced), eliminating the risk from their consumption allocation must maximize the expected utility of the late consumers. The consumption risk and the expected utility of the early consumers are unaffected by this operation, since early consumers receive $D/p(r)$ independently of B.

In general, it is not possible for the bank to hedge all the risk in this way. First, the incentive-constraint may not be binding for some or all values of r, in which case the real returns to the domestic-currency bonds will not be an affine function of the risky return $rh(x)$ and shorting bonds will not provide a perfect hedge for this risk. Secondly, if we solve equation (13) above, we find that the number of bonds that needs to be issued in order to achieve a perfect hedge is

$$B = \bar{B} + 1.$$

In other words, in order to hedge the risk perfectly, each bank has to issue more bonds than the other banks. This alerts us to the fact that existence of equilibrium may be problematical, unless we find some way to limit the issue of domestic currency bonds.

We adopt the following strategy for analyzing "equilibrium" in the limiting case where B becomes very large. First, we restrict banks to choose (x, y, B, D) so that runs never occur. Secondly, we restrict the borrowing of the representative bank so that $B \leq \bar{B}$ where $\bar{B}$ is an exogenously imposed bound on borrowing in terms of the domestic currency. Then the banks' modified decision problem is:

$$\begin{array}{ll} \max & E_R[u(c_1(r)) + u(c_2(r))] \\ \text{s.t.} & x + y \leq 2 + qB \leq 2 + q\bar{B} \\ & c_1(r) = D/p(r) \\ & c_2(r) = rh(x) + \rho^2 y - \rho(D+B)/p(r) \\ & \rho c_1(r) \leq c_2(r). \end{array} \tag{14}$$

It can be seen that banks are constrained to choose policies such that runs never occur by substituting $c_1(r)$ and $c_2(r)$ into the incentive constraint to give (8). This problem has a concave objective function and a convex feasible set for any price function $p(r)$.

Define a *pseudo-equilibrium* to be an array $(x, y, B, D, q, p(\cdot))$ such that (x, y, B, D) solves the problem (14) for the given values of $(q, p(\cdot))$ and q satisfies the condition (10). Since runs are not allowed in a pseudo-equilibrium, the condition (10) is the appropriate fair-pricing condition for equilibrium. The representative bank is maximizing the expected utility of the investors, as required in an ordinary equilibrium, subject to two additional constraints, one being the limit on domestic-currency borrowing and the other being the no-runs condition. The first of these we can treat as a regulatory requirement for the moment. The no-runs condition will later be shown to be optimal when the borrowing limit $\bar{B}$ is sufficiently large.

First, we note that a pseudo-equilibrium exists for each possible borrowing limit $\bar{B} > 0$.

Proposition 5 *For any value of $\bar{B} > 0$, there exists a pseudo-equilibrium $(x, y, B, D, q, p(\cdot))$ such that $B = \bar{B}$, $D = 1$, and for each value of r the consumption allocation $(c_1(r), c_2(r))$ solves the problem:*

$$\begin{array}{ll} \max & u(c_1(r)) + u(c_2(r)) \\ s.t. & c_1(r) + c_2(r)/\rho = rh(x)/\rho + \rho(2 + qB - x) - \frac{B}{p(r)} \\ & c_1(r) \leq c_2(r)/\rho. \end{array}$$

Proof. See the appendix. ■

The pseudo-equilibrium described in Proposition 5 has three special features:

- the consumption allocation satisfies the conditions analogous to those in Proposition 4;
- the nominal value of a deposit is normalized to 1;
- every bank borrows the maximum on the international bond market.

The fact that the consumption allocation satisfies necessary conditions for incentive-efficiency, given the other choices of the bank, reflects the way in which prices are chosen, that is, the exchange rate policy attributed to the central bank. In order for a feasible (incentive-compatible) consumption allocation $(c_1(r), c_2(r))$ to solve the maximization problem in the proposition, the following conditions are necessary and sufficient:

$$u'(c_1(r)) \geq \rho u'(c_2(r)),$$

and

$$u'(c_1(r)) = \rho u'(c_2(r)) \text{ if } \rho c_1(r) < c_2(r).$$

Another way of expressing this is to say that

$$c_2(r) = \max\{\rho c_1(r), \varphi(c_1(r))\},$$

where $\varphi(\cdot)$ is defined implicitly by the equation $u'(z) = \rho u'(\varphi(z))$. To ensure that these conditions are satisfied in equilibrium, we choose the price function $p(r)$ so that

$$\max\left\{\rho\frac{D}{p(r)}, \varphi\left(\frac{D}{p(r)}\right)\right\} = rh(x) + \rho^2(2 + q\bar{B} - x) - \rho\frac{1 + \bar{B}}{p(r)}.$$

This equation determines the price function $p(r)$ uniquely for any values of x and q.

This policy is not necessarily optimal and it is certainly not the only policy that the central bank could have chosen. We adopt it here because it is salient (suggested by Proposition 4) and because it is consistent with an incentive-efficient outcome in the limit, as the next proposition shows.

Normalizing the face value of the deposit to 1 is equivalent to normalizing prices. It ensures that the nominal constraint $B \leq \bar{B}$ on borrowing is a real constraint (equi-proportionate changes in D, B, and $p(\cdot)$ leave the pseudo-equilibrium conditions unchanged).

As a result, the fact that banks borrow the maximum amount $B = \bar{B}$ has real content: banks want to shift the maximum amount of risk to the international market and in fact would like to borrow more if they were allowed to do so.

The next proposition shows that, as the borrowing limit $\bar{B}$ increases, all risk is shifted from the domestic economy to the international capital market.

Proposition 6 *Let $\{\bar{B}^k\}$ be an increasing sequence of bounds such that $\bar{B}^k \to \infty$ and let $(x^k, y^k, 1, \bar{B}^k, q^k, p^k(\cdot))$ be the corresponding pseudo-equilibrium described in Proposition 5. Then for all values of r,*

$$\bar{p} = \lim_{k\to\infty} p^k(r),$$
$$(\bar{c}_1, \bar{c}_2) = \lim_{k\to\infty} (c_1^k(r), c_2^k(r)),$$
$$\bar{x} = \lim_{k\to\infty} x^k,$$

where $(\bar{c}_1, \bar{c}_2)$ is the incentive-efficient consumption allocation from Proposition 4 and $\bar{x}$ is the efficient investment in the risky asset.

Proof. See the appendix. ■

In effect, what is happening as $\bar{B}^k \to \infty$ is that the individual banks construct portfolios consisting of a large investment in riskless reserve-currency debt y^k and a small investment x^k in the risky asset. Most of this portfolio is "owned" by the foreign bondholders, who hold the outstanding domestic-currency bonds $\bar{B}^k$, so the domestic investors receive a relatively small share of the returns. As a result, they bear a relatively small share of the risk generated by the returns from the risky asset. Most of the risk is transmitted to the foreign market and, in the limit, it all goes to the international bond market. The change in the price level for the case where the incentive constraint binds is shown in Figure 4.

The mechanism by which risk is transferred is rather subtle. In the limit, prices are constant at $\bar{p}$ and so the domestic-currency debt is riskless: it pays $1/\bar{p}$ for every realization of R. However, if the banks were to issue a real bond, that is, a bond denominated in the reserve currency, none of the risk could be transferred to the international market. In order to transfer risk, the real returns to the two assets, domestic-currency bonds and international bonds, must be different. This requires variability in the exchange rate $1/p(r)$. As $\bar{B}^k \to \infty$, the degree of price variability required to transfer the risk to the international market shrinks in proportion. But for each finite value of k, the early consumers who receive $c_1^k(r) = D/p^k(r)$ must bear some of the risk.

There are thus two reasons why borrowing using domestic-currency bonds must be limited. One is to ensure existence and the other is to ensure a sufficient difference in the real returns to foreign and domestic bonds. There is a similarity between this problem and the non-existence of equilibrium with incomplete markets studied by Hart (1975). Hart gave an example of an economy with two states of nature, two goods, and two assets with returns represented by a basket of goods. When markets are complete, the monetary returns to the two assets are collinear and the assets cannot be used to span the entire commodity space. When markets are incomplete, the monetary returns to the two assets are not collinear and the assets can be used to span the entire commodity space. Thus, markets can neither be complete or incomplete: an equilibrium does not exist. Placing an arbitrary bound on trades in the two assets can resolve the non-existence problem. As the bound is relaxed, the monetary returns of the two assets will become more nearly collinear and, in order to span the entire commodity space, the trades in the two assets will grow larger as well. In the limit, even infinite trades in the assets will not suffice to make markets complete because the asset returns have become perfectly collinear. While Hart's example suffices to give substance to the non-existence problem, we are not aware of any practical application before now. It has also been noted that Hart's example, which relies on an exogenously specified matrix of asset returns, is non-generic (see Duffie and Schaefer (1985, 1986)). The asset returns in our model are endogenous.

The role of the central bank in maintaining an optimal exchange rate rule is critical. If the central bank did not choose the price function $p^k(r)$ in the manner prescribed, the banks would not necessarily choose $B^k = \bar{B}^k$, and the risk borne by the domestic investors would not necessarily disappear even in the limit as $\bar{B}^k \to \infty$.

So far we have only considered pseudo-equilibria, in which banks are constrained to choose their portfolios and deposit contracts so that runs do not occur. However, for $\bar{B}^k$ sufficiently large, we can show that this is in fact an optimal choice. A pseudo-equilibrium $(x, y, B, D, q, p(\cdot))$ relative to the borrowing constraint $\bar{B}$ is called an equilibrium if (x, y, B, D) solves the maximization problem (9) as well as the maximization problem (14).

Proposition 7 *For all k sufficiently large, the pseudo-equilibrium* $(x^k, y^k, \bar{B}^k, 1, q^k, p^k(\cdot))$ *described in Proposition 6 is an equilibrium relative to the bound* $\bar{B}^k$.

Proof. See the appendix. ■

This is not surprising, since when we get very close to the incentive-efficient allocation, there is no gain from violating the incentive constraint because risk is almost eliminated (Proposition 4). The only effect of having a run is to cause costly liquidation and this can't be optimal.

In contrast to the first-best case in the closed economy where depositors bear the risk associated with domestic risky investment, here depositors bear no risk. All the risk is shifted to the international bond markets and is borne by the risk neutral foreign lenders.

It is interesting to note that when implementing the first best allocation the representative bank must simultaneously borrow large amounts in domestic currency and then invest in foreign bonds. This is consistent with the puzzling observation that the volume of trade in foreign exchange is many times the magnitude that would be needed to finance world trade.

4.4 Long-Term versus Short-Term Debt

We have considered short-term debt and have so far excluded the case of long-term debt. Suppose next that instead of borrowing qB at date 0 and repaying B at date 1 the representative bank borrows $q_L L$ at date 0 and repays L at date 2. The opportunity cost in real terms for lenders in the international bond market between dates 0 and 1 is ρ_{2L}. Thus the counterpart to (10) is

$$q_L = \frac{1}{\rho_{2L}} \int_0^\infty \frac{1}{p_2(r)} dF. \tag{15}$$

The other changes are that the date 0 budget constraint becomes

$$x + y \leq 2 + q_L L$$

and the date 1 budget constraint becomes

$$c_1(r) + \frac{c_2(r)}{\rho} + \frac{L}{\rho p_2(r)} = \frac{rh(x)}{\rho} + \rho y.$$

It is easiest to start by considering the case with a flat yield curve so that

$$\rho_{2L} = \rho^2.$$

Substituting this into (15) and using the fact that $\rho p_2(r) = p(r)$ it can be seen that $q_L = q$. Then the two budget constraints are identical to before since we can choose $L = B$. Hence it does not matter whether short or long term debt is used. This is not very surprising given that all uncertainty is resolved at date 1. It doesn't matter whether debt is rolled over or repaid at the final date.

If $\rho_{2L} > \rho^2$ so that the yield curve is upward sloping then clearly long-term borrowing will be undesirable relative to short-term debt, other things equal, because it is more expensive. Of course, if the yield curve is downward sloping so $\rho_{2L} < \rho^2$, long-term debt will be superior but this is not often the empirically relevant case. This gives the following result.

Proposition 8 *When the yield curve is flat there is no difference between long-term borrowing and short-term borrowing. When it is upward (downward) sloping short-term debt is strictly preferred (inferior) to long-term borrowing.*

5 Foreign Currency Debt

The analysis in the previous section suggests that the combination of a flexible exchange rate and international debt denominated in domestic currency can lead to a first best allocation of resources. In the advanced industrial countries such as the U.S., U.K., Japan, Germany and France it is possible for banks to borrow in the domestic currency and invest in foreign currency bonds. The results in the previous section should be thought as being applicable to them. In contrast, in emerging economies foreign debt is usually denominated in dollars (i.e. in real terms) rather than in domestic currency. How can this be understood in the context of the current model? The problem is that the large amounts of domestic-currency debt held by foreigners creates a temptation for the government to adopt inflationary policies after debt contracts have been signed. This "inflation tax" has the effect of reallocating resources to the government from the domestic depositors and foreign bondholders. The government may be able to return some of these resources to the domestic depositors so that the net effect of such inflation is to expropriate foreign bondholders. If political constraints or the desire to create a reputation for fiscal rectitude limit inflationary policies then the foreign lenders will be able to reflect their expectations in the interest rate they charge. If political constraints are lax or the desire to form a reputation are low then the "inflation premium" foreign lenders demand may be substantial. They will also only be willing to lend short term because this reduces the inflation risk they are exposed to. In extreme cases foreign lenders may not be willing to lend in the country's domestic currency at all. When there is a significant inflation premium for the country banks will find it preferable to borrow using debt denominated in a foreign currency or in other words in real terms.

Denominating international debt in terms of foreign currency avoids the inflation premium but it also introduces other problems. The ability to avoid costly liquidation can be lost and the degree of risk sharing that can be obtained may also be reduced. The benefits that the central bank can generate are reduced. We start by considering what happens in the absence of a central bank that issues domestic currency and then consider what benefits the central bank can bring.

5.1 The Dollarized Economy

The dollarized economy is essentially a real economy. It is closed apart from access to the international bond market. There is no interaction between the banks. The rate at which each bank can borrow on the international market depends on the amount that it borrows and the portfolio of bonds and risky investments it chooses. Each bank has a distinct contracting problem and each bank's decisions can be analyzed separately.

As in the case of domestic currency debt, there is scope for shifting risk to the international capital market by simultaneously borrowing and lending large amounts. When there is no variation in the price level and no borrowing, the bank's optimal choice will be as in Section 3.1 and is illustrated in Figure 2. Both early and late consumers bear risk for $r < r^*$. Moreover, the liquidation costs mean that there is a discontinuity and both groups consumption drops as r falls below r^*. By borrowing and lending a large amount in the international bond market the bank can make the domestic risky assets small relative to the total size of the portfolio. In the event of bankruptcy the foreign lenders receive a large proportion of the bank's assets and bear most of the risk while the domestic depositors receive a small proportion and bear little of the risk. There are two aspects to this reduction in risk. First, the variation in returns on the total portfolio becomes small so in bankruptcy depositors do not bear much risk. Second, the difference in the value of the depositors claims in bankrupt states compared to solvent states is reduced. The risk is shifted to the lenders as the proportion of the risky assets they receive in bankruptcy is increased.

With a continuum of states simultaneously borrowing and lending in the international bond market alters r^* and this makes the welfare effects on depositors complex. To simplify the analysis, we assume that the random variable R has a two-point support $\{r_l, r_h\}$, where $0 < r_l < r_h < \infty$. Let $0 < \pi_i < 1$ denote the probability that $R = r_i$ for $i = l, h$. The analysis of the representative bank's decision problem can be broken down into three cases.

5.1.1 No default

The bank chooses a consumption allocation $\{c_1(r_i), c_2(r_i)\}$ that is determined by a portfolio (x, y), a level of borrowing b, and a deposit contract d. Because there is no risk of default, the price of the bank's debt is $q = 1/\rho$. The bank's decision problem is

$$\begin{array}{ll} \max & \sum_i \pi_i \{u(c_1(r_i)) + u(c_2(r_i))\} \\ \text{s.t.} & x + y \leq 2 + b/\rho \\ & c_1(r_i) = d, i = 1, 2 \\ & c_2(r_i) \leq r_i h(x) - \rho(d + b), i = 1, 2 \\ & \rho c_1(r_i) \leq c_2(r_i), i = 1, 2. \end{array}$$

The first three constraints are budget constraints corresponding to dates 0, 1, and 2, respectively and the last is the incentive constraint. Also, notice that if the incentive constraint is satisfied for r_l it is automatically satisfied for r_h.

Since domestic debt and international debt are perfect substitutes, we can assume without essential loss of generality that the bank will not simultaneously borrow and lend on the international market. There are two cases of interest. In the first the bank does not borrow in the international market but does invest in it so $b = 0$ and $y > 0$. Here access to the international bond market does not make any difference to the allocation and investment in the risky asset is the same as in the closed economy. In the second borrowing is positive $b > 0$ and hence $y = 0$. Here the investment in the domestic risky asset is greater than the endowment and the welfare of depositors is greater than in the closed economy.

The avoidance of default is costly in several ways. First, the depositors bear the risk of the returns on the long asset. Secondly, there will be an intertemporal distortion because the early consumers do not bear any of the risk and receive a low average consumption. The larger the uncertainty about asset returns and the more risk averse the depositors, the greater these costs will be. It will only be worth bearing these costs if they are more than offset by the costs of premature liquidation. This allocation will thus dominate the ones with default when uncertainty is low, the risk aversion of depositors is low and the costs of premature liquidation are high.

5.1.2 Default in one state

Suppose that bankruptcy occurs only in the state where asset returns are low at r_l. The international bond holders receive the face value of the debt b if asset returns are high at r_h and a fraction $\beta \equiv b/(2d + b)$ of the bank's assets if asset returns are low at r_l. The maximum amount the bank can borrow at date 0 is the expected present value of this stream:

$$qb = \frac{1}{\rho}\left\{\pi_l \beta\left(\gamma r_l h(x) + \rho y\right) + \pi_h b\right\}.$$

Substituting this into the first-period budget constraint, we get

$$x + y = 2 + \frac{1}{\rho}\left\{\pi_l \beta\left(\gamma r_l h(x) + \rho y\right) + \pi_h b\right\}.$$

The bank's maximization problem can be written as follows

$$\begin{array}{ll} \max & \sum_i \pi_i \left\{u(c_1(r_i)) + u(c_2(r_i))\right\} \\ \text{s.t.} & x + y \leq 2 + \frac{1}{\rho}\left\{\pi_l \beta\left(\gamma r_l h(x) + \rho y\right) + \pi_h b\right\} \\ & c_1(r_l) = c_2(r_l)/\rho = \beta\left(\gamma r_l h(x) + \rho y\right) \\ & c_1(r_h) = d \\ & c_2(r_h)/\rho \leq r_h h(x)/\rho + \rho y - (d+b) \\ & c_1(r_h) \leq c_2(r_h)/\rho. \end{array} \tag{16}$$

Unfortunately, this problem turns out not to have a solution. The "optimum" requires unbounded values of b and y. Rather than analyze the problem (16) directly, we adopt a two-step strategy. First, we set up an artificial problem in order to define a consumption allocation benchmark. The artificial problem has the same objective function as the original problem (16). The constraints are such that any solution to the original problem is also a solution to the artificial problem. Thus the solution to the artificial problem (the consumption allocation benchmark), must be at least as good as the solution to the original problem. The second step is to show that the benchmark can be approximated by a choice of (b, d, x, y) that satisfy the feasibility constraints of the original problem (16). As the amount borrowed and invested in the international bond market becomes larger so $b \to \infty$, $y \to \infty$ and the allocation tends to the benchmark.

The artificial problem is defined as follows:

$$\begin{array}{ll} \max & \sum_i \pi_i \left\{u(c_1(r_i)) + u(c_2(r_i))\right\} \\ \text{s.t.} & \sum_i \pi_i \left(c_1(r_i) + c_2(r_i)/\rho\right) \leq \rho(2-x) + \left(\pi_l \gamma r_l + \frac{\pi_h r_h}{\rho}\right) h(x) \\ & c_1(r_l) = c_2(r_l)/\rho \\ & c_1(r_h) \leq c_2(r_h)/\rho \\ & c_1(r_l) \leq c_1(r_h). \end{array} \tag{17}$$

The first constraint is a present value budget constraint in terms of consumption, endowments, and profits. The second and third constraints are incentive constraints. The final constraint is added because the rules for bankruptcy imply that $c_1(r_l) \leq c_1(r_h)$. It is straightforward to check that a solution of the bank's problem must satisfy the constraints of the artificial problem.

Proposition 9 *Suppose that* $(\{c_1(r_i), c_2(r_i)\}, b, d, x, y)$ *satisfies the feasibility constraints associated with the problem (16). Then* $(\{c_1(r_i), c_2(r_i)\}, x)$ *satisfies the feasibility constraints associated with the problem (17). The solution to (17) must be at least as good as the solution to (16).*

Proof. See the appendix. ■

The way that the proof proceeds is to show that the budget constraints for date 1 and date 2 from (16) imply that $(\{c_1(r_i), c_2(r_i)\}, x)$ satisfies the budget constraint for (17). The incentive constraints are the same in both problems and the final constraint in (17) follows from the fact that there is bankruptcy in state r_l. Since the objective functions are the same the solution to (17) must be at least as good as the solution to (16).

The next step is to characterize the solution to (17) and show that borrowing and lending large amounts allows the benchmark solution to be approximated.

Proposition 10 *Suppose that* $(\{\hat{c}_1(r_i), \hat{c}_2(r_i), \hat{x})$ *is a solution to the artificial problem (17). Then*

$$\hat{c}_1(r_l) = \hat{c}_1(r_h),$$

$$\hat{c}_2(r_l) = \rho\hat{c}_1(r_l)$$

and

$$\hat{c}_2(r_h) = \max\{\rho\hat{c}_1(r_h), \phi(\hat{c}_1(r_h))\},$$

where $\phi(c)$ *is defined implicitly by the equation*

$$(\pi_l + \pi_h)u'(c) + \pi_l\rho u'(\rho c) \equiv (2\pi_l + \pi_h)\rho u'(\phi(c)), \forall c.$$

For any $\varepsilon > 0$ *we can find a feasible choice* $(\{c_1(r_i), c_2(r_i)\}, x, y, b, d)$ *for the original problem (16) such that*

$$\sum_i \pi_i \{u(c_1(r_i)) + u(c_2(r_i))\} \geq \sum_i \pi_i \{u(\hat{c}_1(r_i)) + u(\hat{c}_2(r_i))\} - \varepsilon.$$

As $b \to \infty, y \to \infty$ *and* $\varepsilon \to 0$.

Proof. See the appendix. ■

The reason that simultaneously borrowing and lending large amounts improves the allocation is that the operation of the bankruptcy mechanism allows the risk to be shifted to the lenders in the international bond market as explained at the beginning of the section. It will be optimal to default in one state when the costs of premature liquidation are moderate and the benefits of shifting risk to the international bond market outweigh these.

5.1.3 Default in both states

Bankruptcy in both states implies that

$$c_1(r_i) = \frac{c_2(r_i)}{\rho} = \frac{1}{2}(1-\beta)\left(\gamma r_i h(x) + \rho y\right) \tag{18}$$

for $i = l, h$ where $\beta \equiv b/(2d+b)$and the first-period budget constraint can be written

$$x + y \leq 2 + \frac{\beta}{\rho}\left\{\gamma \bar{r} h(x) + \rho y\right\},$$

where $\bar{r} \equiv \pi_l r_l + \pi_h r_h$. The problem is solved by choosing β, x and y to maximize the usual objective function subject to the budget constraint.

Inspection of (18) suggests that as $b \to \infty, y \to \infty$ and risk is eliminated from the allocation. This result is in fact more general than the two state case and is considered further below. This allocation will be optimal when the gains from eliminating risk are large and when the costs of premature liquidation are small.

5.2 Bankruptcy and Risk Elimination

In this section we return to the case with a continuum of states and show that if bankruptcy occurs in every state then all risk can be eliminated from the allocation. Let $\beta \equiv b/(2d+b)$ denote the fraction of the assets that goes to the international bond holders in the usual way. Choose b and d so that the bank is just bankrupt in the highest state r_1 at date 1, that is,

$$b = \beta\left(\frac{r_1 h(x)}{\rho} + \rho y\right)$$

and

$$2d = (1-\beta)\left(\frac{r_1 h(x)}{\rho} + \rho y\right).$$

From the budget constraint at date 1,

$$y(1-\beta) = 2 - x + E\left[\beta\frac{rh(x)}{\rho^2}\right],$$

so y is uniquely determined by the choice of β. Thus, for each choice of β we have a feasible choice of (b, d, x, y) and a corresponding consumption allocation $\{c_1(r), c_2(r)\}$. Then using the budget constraint above,

$$\begin{aligned} c_1(r) + \frac{c_2(r)}{\rho} &= (1-\beta)\left(\frac{rh(x)}{\rho} + \rho y\right) \\ &= (1-\beta)\left(\frac{rh(x)}{\rho}\right) + \rho(2-x) + \rho E\left[\beta\frac{rh(x)}{\rho^2}\right] \\ &\to \rho(2-x) + \rho E\left[\beta\frac{rh(x)}{\rho^2}\right] \text{ as } \beta \to 1. \end{aligned}$$

Hence it is possible to entirely eliminate risk by borrowing and investing large amounts in the international bond market. The reason is the usual one that this makes the domestic risky asset a small part of the bank's portfolio and in the case of bankruptcy shifts the risk to the international bond market. Although risk is eliminated, allowing bankruptcy in every state will only be optimal in certain special cases.

One case where this is true is where bankruptcy is costless and the incentive constraints always bind at the first best. Suppose that $\gamma = 1/\rho$ and let $\{\hat{c}_1(r), \hat{c}_2(r)\}$ denote the first-best consumption allocation and let $x = \hat{x}$ be the first best investment in the risky asset. Then if $\hat{c}_2(r) = \rho\hat{c}_1(r)$ for every r the bank can approximate the first best by borrowing and investing a large amount in the international bond market and declaring bankruptcy in every state to eliminate risk as above. In some other cases where the costs of premature liquidation are low and the benefits from shifting risk are large this allocation may also be optimal.

One point worth noting is that when default does occur with probability one both the domestic depositors and international bond holders are essentially holding shares in the domestic bank. When bankruptcy is costly, default is something to be avoided. In that case, the use of equity contracts can avoid default while allowing some beneficial risk sharing.

5.3 Long-Term versus Short-Term Debt

So far we have concentrated on short-term debt. A result similar to Proposition 8 can also be proved in this context. As was the case there, initially suppose $\rho_{2L} = \rho^2$ so that the yield curve is flat. In bankruptcy it is assumed that assets are split in proportion to the present value of the claims. If $q_l l$ is borrowed at date 0 with anticipated repayment l at date 2 then bankruptcy at date 1 will result in a share $\beta_l = (\frac{l/\rho}{2d+l/\rho})$ of the liquidation proceeds. This will replace $\beta = \left(\frac{b}{2d+b}\right)$in the constraints in (16). It can be checked that the transformed problem for the bank is exactly the same as (16) except that β_l replaces β. Hence when the yield curve is flat long and short-term borrowing are equivalent. If the yield curve is not flat then as before long-term debt will be inferior in the standard case where the yield curve is upward sloping or superior if it is downward sloping. This gives the counterpart to Proposition 8.

Proposition 11 *Suppose debt and deposits are denominated in foreign currency and there is no central bank. When the yield curve is flat there is no difference between long-term borrowing and short-term borrowing. When it is upward (downward) sloping short-term debt is strictly preferred (inferior) to long-term borrowing.*

5.4 Foreign-Currency Loans and Domestic-Currency Deposits

Next consider what a central bank can achieve. By adjusting the domestic price level it can potentially prevent some inefficient liquidation. It cannot eliminate it, however, since the debt is in foreign currency terms it may not be feasible to pay it in full. If there is bankruptcy it is assumed initially that the foreign debtholders have priority and obtain the full proceeds of liquidation. Other possibilities are discussed further below. In addition to reducing inefficient liquidation the introduction of domestic currency may allow risk sharing between early and late consumers. Since the debt is denominated in foreign currency there cannot be risk sharing with the foreign bondholders.

The analysis of the equilibrium at date 1 is similar to the previous section. Let qb be the amount borrowed at date 0 and b be the amount repaid at date 1. The bank will go bankrupt when the output available is insufficient to pay the foreign debt. Hence r^* is given by

$$\frac{r^*h(x)}{\rho} + \rho y = b. \tag{19}$$

The value of q will be set so that the foreign bondholders obtain their opportunity cost,

$$\int_0^{r^*} (\gamma r h(x) + \rho y) dF + \int_{r^*}^{\infty} b dF = \rho q b. \tag{20}$$

For simplicity we again focus on the case where the incentive constraint $\rho c_1(r) \leq c_2(r)$ binds. Given this and $p_1(r) = \rho p_2(r)$ it follows that $D_1 = D_2 = D$ is optimal. For $r < r^*$ the foreign bondholders receive everything at date 1 and the domestic depositors receive nothing. For $r \geq r^*$ the price level is given by the ratio of nominal claims to output when the incentive constraint binds

$$p_1(r) = \rho p_2(r) = \frac{2D}{rh(x)/\rho + \rho(2-x) + (\rho q - 1)b}$$

$$c_1(r) = c_2(r)/\rho = \frac{1}{2}(rh(x)/\rho + \rho(2-x) + (\rho q - 1)b). \tag{21}$$

If the counterpart of (3) is satisfied so that

$$u'(0)\rho^2 > E[u'(rh(2))rh'(2)], \tag{22}$$

an interior solution in the sense that $x < 2$ is assured. Now if $r^* \leq r_0$, then $\rho q = 1$ and the level of b is irrelevant. If $r^* > r_l$, then $\rho q < 1$ and the representative bank's optimal choice given its objective is to maximize the expected utility of the representative consumer involves $b = 0$.

Although there will be no borrowing at date 0 there will of course be borrowing at date 1 to smooth consumption between periods. Apart from that the outcome is similar to the case where there is no international finance. In particular there will be no bankruptcy or inefficient liquidation.

It can be seen that the introduction of a central bank is a mixed blessing. It does allow risk sharing between the early and late consumers for all values of r. However, there is no risk sharing with the international bond market. As demonstrated above, when there is no central bank so all contracts are in foreign currency terms there can be risk sharing with the international bond market. It is therefore not immediate whether a central bank and independent monetary policy is desirable. It will depend on the parameter values. For $\gamma = 1/\rho$, using foreign currency denominated debt and deposits will be optimal since in that case $r^* = r_1$ and the first best can be implemented. At the other extreme if γ is very small and $E[r]$ is sufficiently large, a system with a central bank will do better.

It was assumed in the analysis above that the foreign debtholders had priority in the event of bankruptcy. Given this it was optimal not to borrow at date 0. With different priorities this may no longer be the case. There is a significant problem with introducing any other priority, however. It follows from (19) and (21) that at $r^*, c_1(r) = c_2(r) = 0$. If depositors receive anything in bankruptcy then there will be an incentive for them to falsely declare bankruptcy even though the bank is in fact solvent. This makes the administration of anything other than full priority to foreign bondholders problematic. However, the lenders can take into account this aspect and adjust the interest rate on the debt appropriately.

At the opposite extreme from the foreigners obtaining all the proceeds of the liquidation in the event of default is where the domestic depositors obtain everything. The analysis for this case is similar. The main difference will be that in (20) there will be no term for $r < r^*$. As a result the interest rate charged will be higher. Given that the international bond market is risk neutral and depositors are risk averse, the effective transfer from high income states to low income states that a higher interest rate involves will lead to an increase in welfare compared to the case where foreign bondholders receive the liquidation proceeds.

In addition to the two extremes where one party or the other receives all the liquidation proceeds there are also many intermediate cases where they both receive a portion. The problem with analyzing these cases is to specify the precise way in which the liquidation proceeds are split between the two groups. As more and more of the economy's output goes to pay the foreign debtholders less is left for domestic depositors and the domestic price level becomes very high. In other words as $r \to r^*, p_1(r) \to \infty$. This means that for $r < r^*$ there is no sense in which there is a domestic price level. It is then problematic to define any sharing rule for liquidation proceeds based on the value of claims in domestic currency. One possibility is that there is ex post bargaining. Another is that the priority rules are specified ex ante based on the proportionate claims at date 0. If the priority rules are specified as a proportion of the liquidation proceeds then the analysis is determinate and can be undertaken as above. The range of possibilities is clearly large. The possible outcomes are likely to reflect the principles illustrated by the results above.

6 Policy Implications

The events in South East Asia in recent years have sparked a debate about the role of the International Monetary Fund (IMF) in dealing with international crises. One part of the debate revolves around the appropriateness of IMF actions in particular countries (see Corsetti, Pesenti, Roubini (1998a, b, 1999) for a detailed discussion of these issues). Another part of the debate focuses on the broader issue of whether the IMF has a role to play in such crises and if so what the rationale for such intervention is.

There is widespread acceptance of the need for a lender of last resort in a domestic context. Krugman (1998) and Fischer (1999) and others have argued that the IMF should act as an international lender of last resort by analogy. At the other end of the spectrum, Friedman (1998) and Schwartz (1998) have argued that when the IMF intervenes it distorts markets and leads to inefficiency. In between these extremes a number of authors such as Sachs (1995) and Feldstein (1998) have suggested the IMF has some role to play but criticize many of their actions. Sachs argues for the need for an international bankruptcy court while Feldstein emphasizes that its actions should be more closely related to overcoming market failures.

Chari and Kehoe (1999) have suggested that the role of the IMF should be limited to cases where there is a clear collective action problem. They argue that in recent international crises liquidity has been adequately provided by the U.S. Federal Reserve and other major central banks. This suggests there is not a collective action problem between them and so the IMF has no role to play in this regard. Chari and Kehoe do suggest that there are important collective action problems with regard to creditor coordination. In the domestic context bankruptcy laws and institutions are designed to overcome these problems but in an international context there is no equivalent. They argue the IMF has an important role to play as an international bankruptcy court. One of the major activities the IMF currently undertakes is the provision of information about the economic situation in member countries. Chari and Kehoe suggest this is valuable for the international capital markets and should continue. Finally, they argue it could provide a currency which countries could peg their exchange rate to.

The model in this paper presents another perspective in this debate. The starting point in our analysis was a domestic economy prone to banking crises. Equity markets do not result in efficiency because they do not provide insurance against liquidity shocks. Banks can provide such insurance by supplying deposit contracts. The disadvantage of these is that they result in financial crises with inefficient liquidation. A central bank allows such crises to be avoided by adopting an appropriate monetary policy. The first-best efficient allocation can then be implemented.

An international bond market was introduced and the differences this made were considered. We did not explicitly consider a complete international financial system but our analysis can be extended to this case. To fix ideas consider a model where there are many small countries with i.i.d. shocks (i.e. each country's R is i.i.d.). First, consider the regime where banks can borrow in domestic currency and there is a central bank which provides a domestic currency line of credit. Risk will be effectively shared between the countries by each country's banks borrowing a very large amount in their domestic currency and investing in a safe basket of foreign assets. In this way all countries can diversify away their risk so their domestic depositors bear no risk and there are no inefficient liquidations.

Although it has many desirable welfare characteristics, this risk sharing equilibrium is inherently *unstable*. Each government has the temptation to finance expenditure by printing money and inflating the amount of currency. This will expropriate the foreign bondholders and can lead to a net gain for the country. Since the amount of foreign debt is so large the temptation to do this is significant. If one government starts to inflate then other governments may be tempted to do the same. Any attempt at a preemptive move is likely to spread. There is a severe collective action problem. The IMF potentially has a role to ensure that countries do not inflate in this way. To the extent a high price level occurs as a risk sharing mechanism it is desirable but to the extent it occurs as a result of financing government expenditure it is undesirable. In an ideal world the IMF should try to prevent the latter.

Many governments, particularly in emerging economies are unable to borrow in domestic currency because of inflation concerns and instead must borrow in a foreign currency, usually U. S. dollars. This case was considered in Section 5. With no central bank, foreign currency denominated debt and deposits, and equal priority in bankruptcy, it was shown there was again an advantage to banks to borrowing a large amount and investing it in foreign assets. There will be extensive interlinkages between banks in different countries. Since the debts are denominated in foreign currency terms when asset returns are low there can be bankruptcy and inefficient liquidation. Allen and Gale (2000) show in the context of a related model that such interlinkages can lead to severe problems of *contagion*. In their analysis there is a free rider problem. The contagion can be prevented if a small amount of liquidity is supplied by all banks but every bank has an incentive to wait for other banks to supply the liquidity. As a result there is a meltdown. In this kind of situation the IMF can help solve the coordination problem and has an important role as a lender of last resort.

When borrowing is denominated in foreign currency and a central bank exists costly liquidation can again occur. If priority is given to foreigner debtholders there will only be borrowing at date 1. For other priorities there may or may not be interlinkages and costly liquidation. When these features are present contagion may again be a problem that the IMF can eliminate.

In summary, our analysis shows that the IMF may have an important role to play. In the case where debt is denominated in domestic currency the role is to prevent governments raising their price levels to expropriate foreign lenders by printing money to finance government expenditure. When debt is denominated in foreign currency there are issues of contagion and inefficient liquidation which the IMF can again help to prevent.

Appendix

Proof of Proposition 1

If we ignore the incentive-compatibility constraint, the optimal risk-sharing problem (1) becomes:

$$\begin{array}{lll} \max & & \mathrm{E}_R[u(c_1(r)) + u(c_2(r))] \\ \text{s.t.} & \text{(i)} & y + x \leq 2; \\ & \text{(ii)} & c_1(r) \leq y; \\ & \text{(iii)} & c_2(r) \leq rh(x) + y - c_1(r). \end{array} \tag{23}$$

A necessary condition for a solution to (23) is that, for each value of r, the consumption levels $c_1(r)$ and $c_2(r)$ solve the problem

$$\begin{array}{lll} \max & & u(c_1(r)) + u(c_2(r)) \\ \text{s.t.} & \text{(ii)} & c_1(r) \leq y; \\ & \text{(iii)} & c_2(r) \leq rh(x) + y - c_1(r). \end{array}$$

The necessary Kuhn-Tucker conditions imply

$$u'(c_1(r)) \geq u'(c_2(r)),$$

with strict equality if $c_1(r) < y$. In any case, this implies that $c_1(r) \leq c_2(r)$, with strict equality if $c_1(r) < y$, so the incentive constraints (iv) will be satisfied automatically. Thus, a solution to (23) is also a solution to the original problem (1).

Since we know that $c_1(r) = c_2(r)$ whenever $c_1(r) < y$, there are two regimes to be considered. Either $c_1(r) = y$ and (hence) $c_2(r) = rh(x)$ or $c_1(r) = c_2(r) = \frac{1}{2}(rh(x) + y)$. The first case can occur if and only if $y \leq rh(x)$, so the optimal risk-sharing allocation must satisfy

$$c_1(r) = c_2(r) = \frac{1}{2}(rh(x) + y) \ \text{ if } y \geq rh(x),$$

and

$$c_1(r) = y, c_2(r) = rh(x) \text{ if } y \leq rh(x).$$

This allows us to write the optimal risk-sharing problem more compactly as follows:

$$\max \int_0^{\overline{r}} 2u\left(\frac{rh(x) + y}{2}\right) dF + \int_{\overline{r}}^{\infty} (u(y) + u(rh(x)))\, dF$$

$$\text{s.t. } y + x \leq 2,$$

where $\overline{r} \equiv y/h(x)$ is the value of the return on the risky asset at which the liquidity constraint begins to bind. Note that so far we have not established that the critical value of $\overline{r}$ belongs to the support of R.

It remains to characterize the optimal portfolio. We first rule out two extreme cases. Suppose that $x = 0$. Then it is clear that $c_1(r) = c_2(r) = 1$ and $\overline{r} = \infty$. This will be optimal only if $y = 2$ maximizes

$$u(y/2) + \mathrm{E}[u(rh(2-y)) + y/2)],$$

and the first-order condition for this is

$$u'(2/2)/2 + u'(2/2)(\frac{1}{2} - \mathrm{E}[r]) \geq 0,$$

which implies $\mathrm{E}[r] \leq 1$, contradicting one of our maintained assumptions.

Next suppose that $y = 0$. Then $c_1(r) = 0 \leq c_2(r) = rh(2)$. For this to be an optimal choice, it must be the case that $x = 2$ maximizes

$$u(2-x) + \mathrm{E}[u(rh(x))],$$

and the necessary first-order condition for this is

$$u'(0) \leq \mathrm{E}[u'(h(r2))rh'(2)],$$

which contradicts another of our maintained assumptions. Thus any optimal portfolio must satisfy $y > 0$ and $x > 0$.

Returning to the compact form of the risk-sharing problem above, we see necessary conditions for an interior solution are:

$$\int u'(c_1(r))dF = \lambda$$

and

$$\int u'(c_2(r))rh'(x)dF = \lambda,$$

where λ is the Lagrange multiplier of the constraint $y + x = 2$. Under the strict concavity of $u(\cdot)$, these first-order conditions uniquely determine the optimal values of y and x, which in turn determine $\overline{r}$, $c_1(r)$, and $c_2(r)$ through the relationships described above. ■

Proof of Proposition 5

Set $B = \bar{B}$ and $D = 1$. From the date 1 budget constraint we have

$$c_1(r) = \frac{1}{p(r)}. \tag{24}$$

Use the date 0 budget constraint $y = 2 + q\bar{B} - x$ to eliminate y from the date 2 budget constraint:

$$c_2(r) = rh(x) - \rho^2 x + 2\rho^2 - \rho c_1(r) + \rho \left(\rho q - c_1(r)\right) \bar{B}. \tag{25}$$

Thus, consumption at each date is expressed in terms of the parameters x, q, and $c_1(r)$. In order for the consumption allocation $(c_1(r), c_2(r))$ to solve the maximization problem in the proposition, it is necessary and sufficient that $u'(c_1(r)) \geq \rho u'(c_2(r))$, $\rho c_1(r) \leq c_2(r)$, and $u'(c_1(r)) = \rho u'(c_2(r))$ if $\rho c_1(r) < c_2(r)$. Another way of expressing this is to say that

$$c_2(r) = \max\{\rho c_1(r), \varphi(c_1(r))\} \tag{26}$$

where $\varphi(\cdot)$ is defined implicitly by the equation $u'(z) = \rho u'(\varphi(z))$.

Substituting (24) and (25) into (26), we obtain the following:

$$\max\left\{\rho c_1(r), \varphi\left(c_1(r)\right)\right\} = rh(x) + \rho^2(2 + q\bar{B} - x) - \rho\left(1 + \bar{B}\right) c_1(r). \tag{27}$$

This equation determines the consumption function $c_1(r)$ uniquely in terms of x and q. To ensure that these conditions are satisfied in equilibrium, we choose the consumption function $c_1(r)$ to satisfy (27). More precisely, let

$$K \equiv \{(q, x) \in \mathbf{R}_+ \times \mathbf{R}_+ | q \leq \bar{q}, r_0 h(x) + 2 + q\bar{B} - x \geq 0\}.$$

Lemma 12 *For every $(q, x) \in K$ there exists a function $\Phi(\cdot; q, x) : \mathbf{R}_+ \rightarrow \mathbf{R}_{++}$ such that $\Phi(r; q, x)$ satisfies equation (27) for every value of r. Moreover, Φ is continuous.*

Proof. To see that $\Phi(\cdot; q, r)$ is well defined, note that φ is an increasing function. Thus, the left hand side of (27) is an increasing function of $c_1(r)$. The right hand side of (27) is a decreasing function of $c_1(r)$ so there is at most one solution $c_1(r)$ for any pair (q, x). To see that a solution exists, note that both sides are continuous in $c_1(r)$. The left hand side approaches 0 as $c_1(r) \rightarrow 0$ and ∞ as $c_1(r) \rightarrow \infty$. The right hand side approaches $rh(x) + \rho^2(2 + q\bar{B} - x) \geq 0$ as $c_1(r) \rightarrow 0$ and $-\infty$ as $c_1(r) \rightarrow \infty$. Thus, there must be at least one value $c_1(r)$ that satisfies the equation.

By the same argument, the solution value of $c_1(r)$ must be finite and non-negative. Continuity of Φ follows from the implicit function theorem. ■

Construct a mapping from the set K to itself as follows. Given (q,x), the consumption function $c_1(r) = \Phi(r;q,x)$ is well defined, and we can define q' by putting

$$q' = \min\left\{\bar{q}, \int_{r_0}^{r_1} \frac{\Phi(r;q,x)}{\rho} dF\right\}.$$

We choose x' to maximize

$$E\left[u\left(rh(x') + \rho^2(2 + q'\bar{B} - x') - \rho(1+\bar{B})\Phi(r;q,x)\right)\right]$$

subject to the non-negativity constraint $r_0 h(x') + 2 + q'\bar{B} - x' \geq 0$. The set of values of x' that solve the maximization problem is convex and non-empty. Let $Z(q,x) \in K$ denote the set of points (q',x') constructed in this way. Standard arguments suffice to show that Z has a closed graph, so by the Kakutani theorem Z has a fixed point $(q^*,x^*) \in Z(q^*,x^*)$.

We claim that (q^*,x^*) defines the desired pseudo-equilibrium. By construction, the consumption allocations corresponding to (q^*,x^*) solve the maximization problem in the proposition and q^* satisfies the pricing equation (10) as long as $\bar{q}$ is chosen large enough. To see this, it is enough to show that $E[c_1(r)]$ is bounded. From (27) we have

$$\rho c_1(r) = rh(x) + \rho^2(2 + q\bar{B} - x) - \rho\left(1+\bar{B}\right)c_1(r)$$

and by construction $\rho q \leq E[c_1(r)]$ so taking expectations and substituting

$$E[\rho c_1(r)] \leq E[rh(x) + \rho^2(2-x) - \rho c_1(r)],$$

or

$$E[2\rho c_1(r)] \leq E[rh(x) + \rho^2(2-x)].$$

This shows that $E[c_1(r)]$ is bounded independently of $\bar{q}$ and $\bar{B}$, so choosing $\bar{q}$ large enough, we will have $q^* = E[c_1(r)/\rho] < \bar{q}$ at the fixed point.

Since the bank's maximization problem (14) is a convex problem, it is sufficient to show that the Kuhn-Tucker first-order conditions are satisfied. This must be true by construction for all the variables except D and B, since they are chosen optimally. To show that $D = 1$ and $B = \bar{B}$ are optimal for the bank, we have show that the first-order conditions for the maximum problem are satisfied. From (14) we can see that the first-order condition for D is

$$E_R\left[\frac{u'(c_1(r))}{p(r)} - \frac{\rho u'(c_2(r))}{p(r)}\right] \geq 0$$

because we cannot increase D if the incentive constraints bind within the support of R. This condition must be satisfied because we know that $u'(c_1(r) - \rho u'(c_2(r)) \geq 0$ for all r with strict equality if the incentive constraint is not binding.

Similarly, the first-order condition for B is

$$E_R\left[u'(c_2(r))\left(\rho^2 q - \frac{\rho}{p(r)}\right)\right] \geq 0$$

because the constraint $B \leq \bar{B}$ is binding. Then substituting the condition for q from (10) gives us

$$E_R\left[u'(c_2(r))\left(E_R\left[\frac{\rho}{p(r)}\right] - \frac{\rho}{p(r)}\right)\right] \geq 0.$$

Since $u'(c_2(r))$ is decreasing in r and $p(r)$ is decreasing in r

$$E_R\left[u'(c_2(r))\left(E_R\left[\frac{\rho}{p(r)}\right] - \frac{\rho}{p(r)}\right)\right] \geq E_R\left[u'(c_2(r))\right] E_R\left(E_R\left[\frac{\rho}{p(r)}\right] - \frac{\rho}{p(r)}\right) = 0,$$

as required. ■

Proof of Proposition 6
From equation (27), for a fixed but arbitrary r and for each k,

$$\max\left\{\rho c_1^k(r), \varphi\left(c_1^k(r)\right)\right\} = rh(x^k) + \rho^2(2 + q^k\bar{B}^k - x^k) - \rho(1 + \bar{B}^k)c_1^k(r_1).$$

The left hand side is bounded below, so

$$r_1 h(x^k) + \rho^2(2 + q^k\bar{B}^k - x^k) - \rho(1 + \bar{B}^k)c_1^k(r_1) \geq 0, \tag{28}$$

for all k. From the pricing equation (10) we know that $\rho q^k = E[c_1^k(r)]$, and (27) implies that $c_1^k(r)$ is non-decreasing in r, so $\rho q^k \leq c_1^k(r_1)$. Then (28) implies that the sequence $\left\{\left(\rho q^k - c_1^k(r_1)\right)\bar{B}^k\right\}$ is bounded below, which implies that

$$\lim_{k\to\infty} \rho q^k - c_1^k(r_1) \to 0. \tag{29}$$

The pricing equation (10) together with (29) implies that $c_1^k(r)$ converges to a constant almost surely. Then $(c^k(r), c_2^k(r))$ converges to a constant $(\bar{c}_1, \bar{c}_2)$ almost surely, so the first-order condition for x^k

$$E_R\left[u'(c_2^k(r))(rh'(x^k) - \rho^2)\right] \geq 0$$

becomes

$$E_R\left[(rh'(\bar{x}) - \rho^2)\right] = 0$$

in the limit (assuming the bankruptcy constraint is not binding because $\bar{c} = \lim_k c_1^k(r)$ is positive. ■

Proof of Proposition 7
The one condition that needs to be checked is whether the bank will want to violate the no bankruptcy condition for large k. Violating the no-bankruptcy constraint involves a loss of output and a possible distortion in the allocation of consumption, but may improve risk sharing. Because the consumption allocations and prices are becoming approximately constant as $k \to \infty$, there is no gain from risk sharing. ■

Proof of Proposition 9
Using the budget constraints for date 1 and date 2 from (16),

$$\begin{aligned} & \pi_l \left\{ c_1(r_l) + \frac{c_2(r_l)}{\rho} \right\} + \pi_h \left\{ c_1(r_h) + \frac{c_2(r_h)}{\rho} \right\} \\ \leq \; & \pi_l \left\{ (1-\beta)\left(\gamma r_l h(x) + \rho y\right) \right\} + \pi_h \left\{ r_h h(x)/\rho + \rho y - b \right\} \\ = \; & \pi_l \left(\gamma r_l h(x) + \rho y\right) + \pi_h \left(r_h h(x)/\rho + \rho y\right) - \pi_l \beta \left(\gamma r_l h(x) + \rho y\right) - \pi_h b. \end{aligned}$$

From the first-period budget constraint in (16),

$$\rho(x + y - 2) = \pi_l \beta \left(\gamma r_l h(x) + \rho y\right) - \pi_h b,$$

so

$$\begin{aligned} & \pi_l \left\{ c_1(r_l) + \frac{c_2(r_l)}{\rho} \right\} + \pi_h \left\{ c_1(r_h) + \frac{c_2(r_h)}{\rho} \right\} \\ = \; & \pi_l \left(\gamma r_l h(x) + \rho y\right) + \pi_h \left(r_h h(x)/\rho + \rho y\right) - \rho(x + y - 2) \\ = \; & \pi_l \gamma r_l h(x) + \pi_h r_h h(x)/\rho - \rho(x - 2). \end{aligned}$$

So $(\{c_1(r_i), c_2(r_i)\}, x)$ satisfies the budget constraint in (17).

The incentive constraints are the same in both problems, so it remains to show that $c_1(r_l) \le c_1(r_2)$. But this follows from the fact that there is bankruptcy in state r_l, which requires that

$$2d + b > \gamma r_l h(x) + \rho y,$$

so

$$c_1(r_l) = \frac{d}{2d+b}\left(\gamma r_l h(x) + \rho y\right) < d = c_1(r_h).$$

Since the objective functions for the two problems are the same and any feasible solution of the original problem is a feasible for the artificial problem the solution to the artificial problem must be at least as good as the solution to the original problem. ■

Proof of Proposition 10

The first part of the proposition is concerned with characterizing the solution to (17). Since bankruptcy in state r_l implies $c_2(r_l) = \rho c_1(r_l)$, any solution to the problem (17) must solve

$$\begin{array}{ll} \max & \pi_l\left(u(c_1(r_l)) + u(\rho c_1(r_l))\right) + \pi_h\left(u(c_1(r_h)) + u(c_2(r_h))\right) \\ \text{s.t.} & 2\pi_l c_1(r_l) + \pi_h\left(c_1(r_h) + c_2(r_h)/\rho\right) \le \hat{w} \\ & c_1(r_l) \le c_1(r_h) \le c_2(r_h)/\rho, \end{array}$$

where $\hat{w}$ is the value of consumption in the optimum. Suppose that $c_1(r_l) < c_1(r_h)$. From the first-order conditions,

$$\begin{aligned} u'(c_1(r_l)) + \rho u'(\rho c_1(r_l)) &= \lambda \\ u(c_1(r_h)) &= \lambda + \mu \\ u(c_2(r_h)) &= (\lambda + \mu)/\rho. \end{aligned}$$

But $c_1(r_l) < c_1(r_h)$ implies that $c_2(r_h) \ge \rho c_1(r_h) > \rho c_1(r_l)$, so

$$u'(c_1(r_l)) + \rho u'(\rho c_1(r_l)) > u'(c_1(r_h)) + \rho u'(c_2(r_h)),$$

contradicting the first-order conditions. This shows that $c_1(r_l) = c_1(r_h)$, as claimed.

Then any solution to the problem (17) must solve

$$\begin{array}{ll} \max & (\pi_l + \pi_h)u(c_1(r_l)) + \pi_l u(\rho c_1(r_l)) + \pi_h u(c_2(r_h)) \\ \text{s.t.} & (2\pi_l + \pi_h)c_1(r_l) + \pi_h c_2(r_h)/\rho \le \hat{w} \\ & c_1(r_h) \le c_2(r_h)/\rho. \end{array}$$

The first-order conditions are

$$\begin{aligned}(\pi_l+\pi_h)u'(c_1(r_l))+\pi_l\rho u'(\rho c_1(r_l)) &\geq \lambda(2\pi_l+\pi_h)\\ u'(c_2(r_h)) &\leq \lambda/\rho\end{aligned}$$

with strict equality if $\hat{c}_1(r_h)<\rho\hat{c}_2(r_h)$. On the one hand, if the incentive constraint is binding, i.e., $\hat{c}_2(r_h)=\rho\hat{c}_1(r_h)=\rho\hat{c}_1(r_l)$, then

$$\pi_l\rho u'(\rho\hat{c}_1(r_l))\leq\lambda\pi_l.$$

Substituting this into the first-order conditions implies that

$$u'(c_1(r_l))\geq\lambda\geq\rho u'(c_2(r_h)).$$

On the other hand, if the incentive constraint is not binding then

$$(\pi_l+\pi_h)u'(c_1(r_l))+\pi_l\rho u'(\rho c_1(r_l))=\rho u'(c_2(r_h))(2\pi_l+\pi_h).$$

In either case,

$$\hat{c}_2(r_h)=\max\left\{\rho\hat{c}_1(r_h),\phi(\hat{c}_1(r_h))\right\},$$

as claimed.

The second part of the proposition is concerned with showing that the consumption allocation benchmark can be approximated. Let $x=\hat{x}$. There are two cases to consider.

Case 1. If the incentive constraint is binding in the state r_h, then the bond holders and depositors receive a constant fraction of the total value of the bank's assets in each state. Let β denote the fraction going to the bond holders. Then from the first-period budget constraint, we can calculate that

$$\hat{x}+y=2+\frac{\beta}{\rho}\left\{\pi_l\gamma r_l h(\hat{x})+\frac{\pi_h r_h}{\rho}h(\hat{x})+\rho y\right\}$$

or

$$y(1-\beta)=2-\hat{x}+\frac{\beta}{\rho}\left\{\pi_l\gamma r_l+\frac{\pi_h r_h}{\rho}\right\}h(\hat{x}).$$

So for each value of β we can calculate a feasible value of y. Then set

$$\begin{aligned}c_1(r_h) &= \frac{c_2(r_h)}{\rho}=d=\frac{(1-\beta)}{2}\left\{\frac{r_h h(\hat{x})}{\rho}+\rho y\right\},\\ c_1(r_l) &= \frac{c_2(r_l)}{\rho}=\frac{d}{2d+b}\left\{\gamma r_l h(\hat{x})+\rho y\right\}\end{aligned}$$

and

$$b = \beta \left\{ \frac{r_h h(\hat{x})}{\rho} + \rho y \right\}$$

and we have determined a feasible set of choices for each value of β. Now let $\beta \to 1$ and observe that $(2d + b)$ equals $r_h h(\hat{x})/\rho + \rho y$ so

$$\begin{aligned} \frac{c_1(r_l)}{c_1(r_h)} &= \frac{\gamma r_l h(\hat{x}) + \rho y}{2d + b} \\ &= \frac{\gamma r_l h(\hat{x}) + \rho y}{r_h h(\hat{x})/\rho + \rho y} \end{aligned}$$

converges to 1 as $y \to \infty$. Then the budget constraint ensures that $\{c_1(r_i), c_2(r_i)\} \to \{\hat{c}_1(r_i), \hat{c}_2(r_i)\}$ as $b \to \infty$.

Case 2. In the case where the incentive constraint is not binding in state $R = r_h$, put $x = \hat{x}$, $d = \hat{c}_1(r_h)$ and choose b arbitrarily. Then, as before, the first-period budget constraint tells us that

$$y = 2 - x + \frac{1}{\rho} \left\{ \pi_l \frac{b}{2d + b} \left(\gamma r_l h(\hat{x}) + \rho y \right) + \pi_h b \right\}$$

or

$$y = \left(1 - \frac{b}{2d + b} \right)^{-1} \left\{ (2 - x) + \frac{1}{\rho} \left(\pi_l \frac{b}{2d + b} \gamma r_l h(\hat{x}) + \pi_h b \right) \right\}.$$

This shows that y is uniquely determined by our choice of x, b, and d. Then the consumption allocation is determined by the budget constraints at dates 1 and 2:

$$\begin{aligned} c_1(r_h) &= d \\ c_2(r_h) &= r_h h(x) + \rho^2 y - \rho(b + d) \\ c_1(r_l) &= c_2(r_l)/\rho = \frac{d}{2d + b} \left(\gamma r_l h(x) + \rho y \right). \end{aligned}$$

As $b \to \infty$ we have $y \to \infty$ and $b/(2d + b) \to 1$. From the first-period budget constraint, we have

$$\begin{aligned} \frac{y}{b} &= \left(1 - \frac{\pi_l b}{2d + b} \right)^{-1} \left\{ \frac{(2 - x)}{b} + \frac{1}{\rho} \left(\pi_l \frac{1}{2d + b} \gamma r_l h(\hat{x}) + \pi_h \right) \right\} \\ &\to (1 - \pi_l)^{-1} \left\{ \frac{1}{\rho} \left(\pi_h \right) \right\} = \frac{1}{\rho}. \end{aligned}$$

Thus, from the definition of the consumption allocation above,

$$\begin{aligned}\frac{c_1(r_l)}{c_1(r_h)} &= \frac{1}{2d+b}\left(\gamma r_l h(x) + \rho y\right) \\ &\to 1.\end{aligned}$$

Thus, in the limit as $b \to \infty$, the consumption of early consumers is equalized between the two states, and then the budget constraint implies that $c_2(r_h) \to \hat{c}_2(r_h)$. Thus, $\{c_1(r_i), c_2(r_i)\} \to \{\hat{c}_1(r_i), \hat{c}_2(r_i)\}$ as $b \to \infty$ and this in turn implies that for sufficiently large values of b the consumption level $c_2(r_h)$ defined above is non-negative and satisfies the incentive constraint.

This completes the proof that $\{\hat{c}_1(r_i), \hat{c}_2(r_i)\}$ can be approximated by some feasible choice of $(\{c_1(r_i), c_2(r_i)\}, b, d, x, y)$ in the original problem (16). ■

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Figure 1
The Efficient Allocation in a Closed Economy

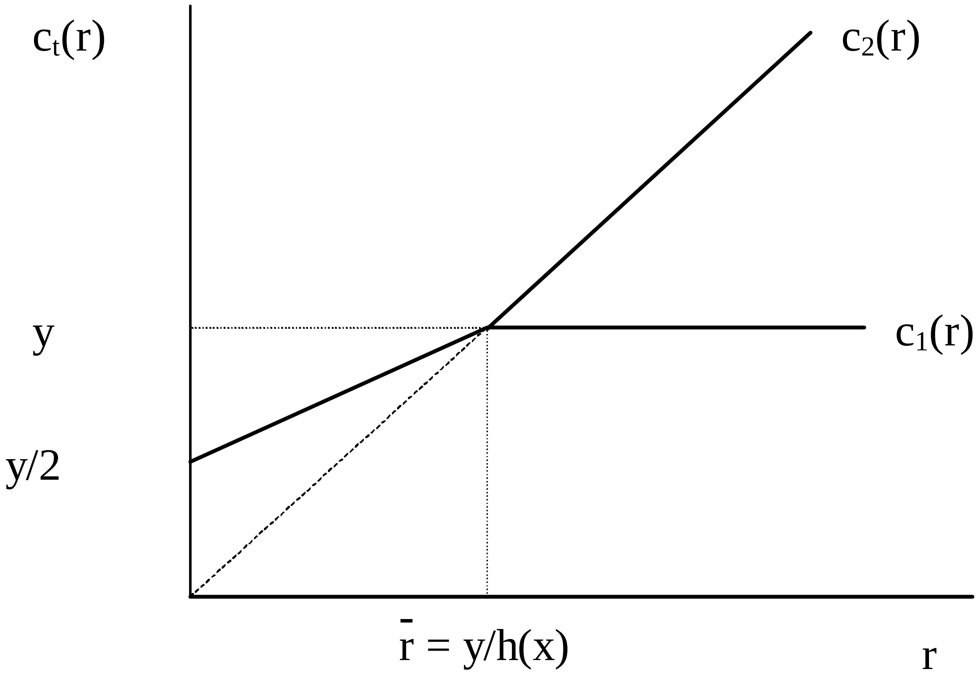

Figure 2
Banking Equilibrium without a Central Bank in a Closed Economy

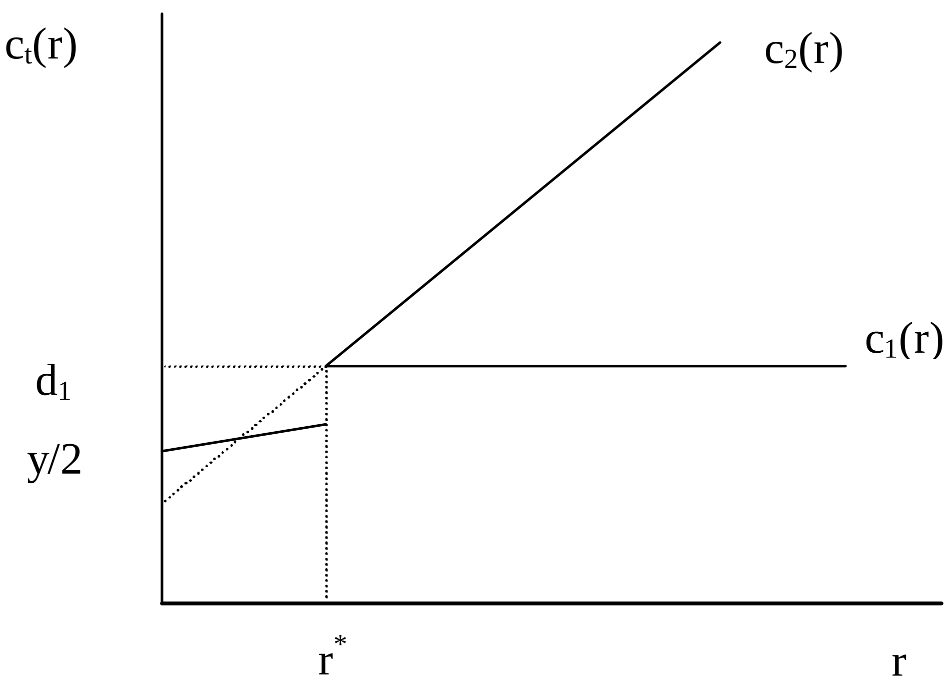

Figure 3
The Price Level in a Closed Economy

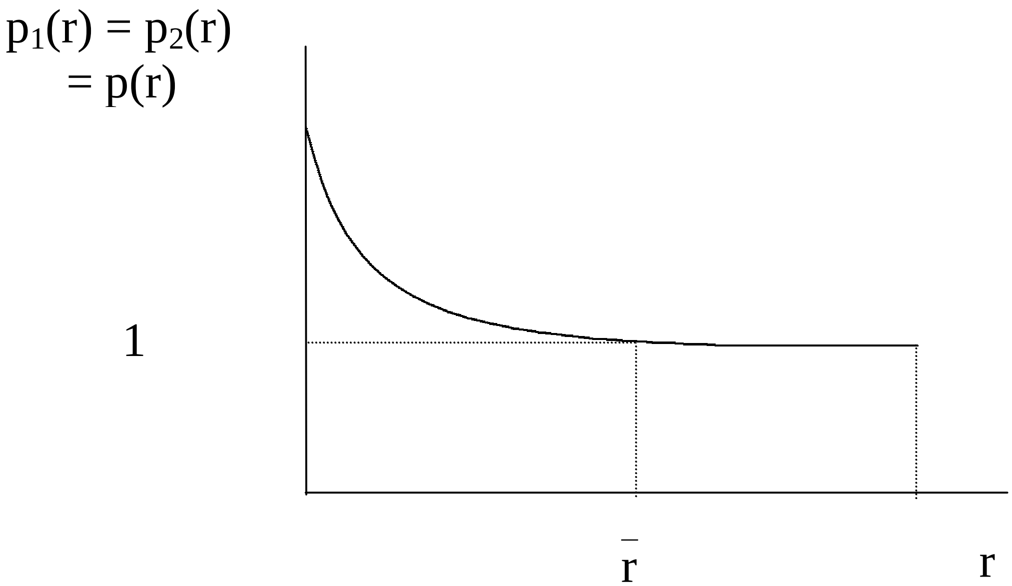

Figure 4
The Effect of Varying $\bar{B}$ on the Price Level