Investigating the Puzzling conditional Market Risk-Return Relationship

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Abstract

This paper investigates the conflicting results documented by the existing empirical literature on the relationship between the expected market risk premium and conditional market variance. We show that the previous tests are biased because they use the realized market risk premium as a proxy for the expected market risk premium without accounting for the negative portion of the market risk premium distribution. The empirical evidence based on a new test, allowing up and down-market volatility to have different impacts on the market risk premium, indicates a consistent and significant risk-return relationship.

KEY WORDS : Market Risk Premium, Market Volatility, EGARCH.

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Abstract: *This paper investigates the conflicting results documented by the existing empirical literature on the relationship between the expected market risk premium and conditional market variance. We show that the previous tests are biased because they use the realized market risk premium as a proxy for the expected market risk premium without accounting for the negative portion of the market risk premium distribution. The empirical evidence based on a new test, allowing up and down-market volatility to have different impacts on the market risk premium, indicates a consistent and significant risk-return relationship.*

Many papers have examined the intertemporal relationship between expected returns and conditional volatility of the market. Theoretically, if investors are risk-averse, the capital asset pricing model (CAPM) developed by Sharpe (1964) and Lintner (1965) implies a positive, linear relationship between the expected market risk premium and conditional market variance. However, the empirical findings on this relationship are controversial. Bollerslev, Engle and Wooldridge (1988) and Harvey (1989) find a significant positive relationship between the expected market risk premium and conditional volatility of the market. French, Schwert and Stambaugh (1987) show that this relationship is positive, but insignificant. Baillie and DeGennaro (1990) find no evidence for a statistically significant relationship between the market risk premium and conditional variance or standard deviation and conclude that others measures of risk are more important than the variance of returns; Whereas, Campbell (1987) and Glosten, Jagannathan and Runkle (1993) find a significantly negative relationship. Other papers have also examined the relationship between expected returns and conditional volatility of the market, but most of them reject the cross-sectional and the intertemporal implications of the CAPM.[1] Recently, Scruggs (1998) investigates these conflicting results and shows that the negative relationship between the expected market risk premium and conditional market variance reported in previous studies is due to the omission of an interest rate-state variable in the conditional market risk premium equation. By estimating an ad hoc model, which incorporates the interest rate in both conditional mean and variance equations, Scruggs (1998) restores the positive and significant relationship between the market risk premium and conditional market variance.

However, we can argue that including the interest rate in both conditional mean and variance in Scruggs' equations is not theoretically justified. This is because no theoretical argument to this time justifies the incorporation of an interest rate variable into the conditional market risk premium equation. Furthermore, the presence of the interest rate in both the conditional market risk premium and the conditional market variance equations results in multicolinearity problems with the result that the estimates of both equations do not have a valid interpretation. Moreover, when the interest rate is not included in the conditional mean and variance equations, the relationship remains weak or negative (model 2a, p. 587). We think that the puzzle related to the single-factor market risk-return relationship remains unresolved at this time.

The theoretical positive relationship between the market risk premium and conditional market variance supported by asset pricing models requires the positivity of the market risk in all states. Since all previous studies use realized returns to test the relationship between the expected market risk premium and conditional market variance, the implicit assumption being made is that the realized market risk premium is an unbiased estimate of the expected market risk premium. However, since the realized market risk premium may be negative in some states, we believe that previous studies don't explicitly test the relationship between the market risk premium and conditional market variance.

The purpose of this paper is to reexamine the puzzling single-factor relationship between the expected market risk premium and conditional market variance found in previous studies. Unlike these studies, this paper presents a new test which recognizes the impact of using realized market risk premium as a proxy for expected market risk premium. This test allows up and down-market volatility to have distinct impacts on the market risk premium. We show that the estimates of the single factor market risk-return relationship may be biased downwards due to the use of the realized market risk premium as a proxy for the expected market risk premium. Therefore, not accounting for states where the realized market risk premium is negative leads to an aggregation bias resulting from the compensation effects of the positive and the negative market risk premium. We find evidence of a positive (negative) and significant relationship between the market risk premium and conditional market variance in bull (bear) market context. This result is important since it (i) explains the negative or the weak relationship between volatility and expected return reported in some studies and (ii) restores the pertinence of the variance as a measure of risk .

The remainder of this paper is organized as follows. The next Section presents theoretical models and empirical procedures used for tests. Section II discusses methodology. Section III describes the data and presents empirical results and finally Section IV concludes the paper.

I. Models and Procedures

This section describes the theoretical models and empirical procedures that we use to test the different specifications of the single-factor relationship between the market risk premium and conditional market variance. Previous tests on this issue made the implicit assumption that the realized market risk premium is an unbiased estimate of the expected market risk premium. The major contribution of this paper is to show the impact of this assumption on testing the risk-return relationship since the positive risk-return relationship postulated by most asset pricing models is based on the expected returns.

A. The Theoretical Model

Following the majority of previous studies on the intertemporal relationship between the market risk premium and conditional market variance, we consider a conditional version of the Sharpe-Lintner CAPM in which expected market risk premium is related to the conditional volatility of the market according to the following equation:

$$E_{t-1}[r_{m,t}] = \lambda_m \sigma^2_{m,t} \tag{1}$$

where $E_{t-1}[.]$ denotes the expectation operator conditional on information available at time t-1; $r_{m,t}$ is the market risk premium; $\sigma^2_{m,t}$ is the conditional market variance; and λ_m is the price of risk. Since λ_m is also the coefficient of relative risk aversion, and therefore should be positive, equation (1) implies a positive and linear relation between the expected market risk premium and conditional market variance.

B. The Empirical Procedures

First, we present traditional tests of the conditional single-factor relationship between the market risk premium and conditional market variance. Second, we conduct our empirical tests using up and down market information decomposition which recognize the impact of using realized market risk premium to proxy for expected market risk premium.

B.1. Traditional Tests

In order to test the relationship between the market risk premium ($r_{m,t}$) and conditional market variance ($\sigma^2_{m,t}$), we use the following model as derived from the previous studies [2]:

Model 1:

$$r_{m,t} = \lambda_0 + \lambda_m \hat{\sigma}^2_{m,t} + \varepsilon_{m,t} \quad (2)$$

The investors' risk-aversion hypothesis implies that the λ_m coefficient (the price of risk) is positive. Equation (2) is tested, among others, by Campbell (1987), French, Schwert and Stambaugh (1987), Glosten, Jagannathan and Runkle (1993) and Scruggs (1998).

B.2. Tests with Up and Down-Market Information Decomposition

Equation (1) indicates that the conditional expected market risk premium is a positive and linear function of the conditional market variance. When the expected market risk increases, the anticipated returns should adjust accordingly in order to compensate the investors for the additional risk. This relationship has crucial implications for the empirical tests because an increase of the conditional market variance must be associated with an increase in the risk premium anticipated by investors. This theoretical association led researchers to directly test for a positive relationship. However, as these tests use the realized market risk premium as a proxy for the expected market risk premium, we argue that they do not explicitly test the conditional single-factor market risk-return relationship. In fact, the positivity of the expected market risk premium in all states is a necessary condition for equation (1), however, there are states where the realized market risk premium is negative (Table 1 shows that the market risk premium is negative in 44% of the cases). Therefore, the traditional tests of the conditional single-factor market risk-return relationship should account for the negative portion of the market risk premium distribution.

Below, we present the methodology that we are using to obtain the empirical model that recognizes the impact of using realized market risk premium as a proxy for the expected market risk premium and allows up and down-market periods volatility to have different effects on the market risk premium. Our approach is inspired by the study of Pettengill, Sundaram and Mathur (1995) on the CAPM. According to these authors (p. 103) "since these tests use realized returns instead of expected returns, we argue that the validity of the SLB (Sharpe-Lintner-Black) model is not directly examined. Indeed, recognition of a second critical relationship between the predicted market returns and the risk-free return suggests that previous tests of the relationship between beta and the returns must be modified. The need to modify previous tests results from the model's requirement that a portion of the market return distribution be below the risk-free rate".

A reasonable inference about this critical relationship is that returns associated to high volatility are less than returns associated to low volatility when the market risk premium is negative (down market period). To infer this, assume that the economy is represented by two states of the nature, Ω_1 and Ω_2, characterizing up and down-market periods, respectively. These two states of nature are assumed to be independent with the respective occurrence probabilities $p_1 = p(\Omega_1)$ and $p_2 = (1-p_1) = p(\Omega_2)$. In this case, we obtain the following relationship:

$$E_{t-1}[r_{m,t}] = p_1 E_{t-1}[r_{m,t} \mid \Omega_1] + p_2 E_{t-1}[r_{m,t} \mid \Omega_2] \tag{3}$$

We can then express the conditional market variance as:

$$\sigma^2_{m,t} = p_1^2 \, \sigma^2_{m1,t} + p_2^2 \, \sigma^2_{m2,t} \tag{4}$$

where $\sigma^2_{m1,t}$ and $\sigma^2_{m2,t}$ are conditional market variances associated respectively to up and down-market periods. Using equation (4), equation (1) can be re-written as:

$$E_{t-1}[r_{m,t}] = \left(p_1^2 \lambda_m\right) \sigma^2_{m1,t} + \left(p_2^2 \lambda_m\right) \sigma^2_{m2,t} \tag{5}$$

A testable version of the relation (5) is therefore:

<u>Model 2:</u>

$$r_{m,t} = \lambda_0 + \lambda_{m1} \, \delta \hat{\sigma}^2_{m,t} + \lambda_{m2} (1-\delta) \, \hat{\sigma}^2_{m,t} + \varepsilon_{m,t} \tag{6}$$

where $\delta = 1$ if the market-risk premium is positive (up market) and $\delta = 0$ if the market risk premium is negative (down market). Knowing that λ_{m1} (λ_{m2}) is estimated during up (down) market periods, the expected sign of this coefficient is positive (negative).

II. Methodology

A. The Empirical Model of the Conditional Variance

The estimation of the models previously described requires modeling the conditional volatility of the market risk premium. The ARCH models pioneered by Engle (1982) permit the measurement and prediction of the time-varying conditional volatility. Following empirical evidences on the behavior of market volatility over time, we assume that the conditional market volatility is time-varying and follows an exponential generalized autoregressive conditional heteroskedastic (EGARCH) process, as developed by Nelson (1991). The use of EGARCH model is motivated by the fact that it captures the volatility clustering which is a characteristic of high-frequency asset returns (see Mandelbrot (1963) and Fama (1965)). Also, its formulation is well-suited to accommodate asymmetric effects in the evolution of volatility process. Since most studies find that one period is enough to capture the characteristics of most financial data series, we have considered the EGARCH (1,1) model.[3] According to this model, the conditional market variance depends on the amplitude as well as the past innovation sign. The conditional market variance is given by:

$$\begin{aligned} &\ln(\hat{\sigma}^2_{m,t}) = \omega + \alpha g(e_{t-1}) + \beta \ln(\hat{\sigma}^2_{m,t-1}) \\ &e_t = \varepsilon_{m,t} / \hat{\sigma}_{m,t} \qquad e_t \sim N(0,1) \\ &g(e_{t-1}) = \theta e_{t-1} + \left(|e_{t-1}| - \sqrt{2/\pi} \right) \end{aligned} \tag{7}$$

What distinguishes the EGARCH model is that it is suited to accommodate asymmetric effects since it incorporates the news response function $g(e_{t-1})$ with coefficient α and allows measurement of the sign and past innovation amplitude. The coefficient θ measures the asymmetry of the response of the conditional market variance to signs of past return shocks. A negative (positive) coefficient θ implies that negative (positive) return shocks have more impact on the conditional volatility than positive (negative) return shocks of the same magnitude. When $\theta = 0$, the news response function $g(e_{t-1})$ is then symmetric and depends only on lagged return shocks. According to Engle and Ng (1993, p. 1753) "The EGARCH model differs from the standard GARCH model in two main respects: (1) the EGARCH model allows good news and bad news to have different impacts on volatility, while the standard GARCH model does not, and (2) the EGARCH model allows big news to have a greater impact on volatility than the standard GARCH model".

B. Maximum Likelihood Methodology

The models are estimated using the maximum likelihood method, whose estimates are obtained by searching for values of parameters that maximize the likelihood function (L), calculated from the products of all conditional densities of the prediction errors.

$$lnL = \sum_{t=1}^{N} \frac{1}{2}\left[-\ln(2\pi) - \ln(\hat{\sigma}^2_{m,t}) - \frac{\varepsilon_{m,t}}{\hat{\sigma}^2_{m,t}} \right] \tag{8}$$

The likelihood function is maximized by using the dual Quasi-Newton algorithm. The starting values for the regression parameters are obtained by using the ordinary least squares estimates.

III. Empirical Results

A. Data Description

We use the value-weighted monthly returns of all traded stocks in the Toronto Stock Exchange as a proxy for the market returns. The three-month Treasury-bill returns are used as risk-free returns. These data are from the TSE-Western file and cover the period from March 1950 to December 1995.

Table 1 presents the different characteristics of returns. Panel A reports the descriptive statistics. The results show that the realized risk premium is positive in only 56% of cases. The risk premium time series presents negative skewness (-0.59). Besides, the p-value associated with the Shapiro-Wilk (1965) test is 0.018 that indicates the rejection of the risk premium normality hypothesis at 5% level. Panel B reports the autocorrelation coefficients of orders 1, 2, 3, 4, 6, 9 and 12. The results indicate weak autocorrelation coefficients for the market risk premium while the autocorrelation coefficients are very strong for the risk-free returns. The amplitudes of these autocorrelation coefficients are always higher than two standard deviations. Finally, Panel C presents the McLeod and Li (1983) parametric portmanteau test (Q^2). This test is based on the squares of residuals and cover shifts of order 1, 2, 3, 4, 6, 9 and 12 of the autocorrelation function. These tests clearly indicate the presence of heteroskedasticity in the series of returns and justify the use of EGARCH models.

Insert Table 1

B. The Results

Table 2 presents results from Model 1 estimation with a variance modeled using an EGARCH (1,1) process as described by equation (8). Model 1 is replicated in order to facilitate comparison to the Model 2. It is also, comparable with models estimated by Glosten et *al*. (1993) and Scruggs (1998) in the US context. The results show that the estimate of the conditional variance coefficient (λ_m) in the market risk premium equation is negative (-0.157), but insignificant. This result confirms some findings in the US context and is compatible with the conclusion of Baillie and DeGennaro (1990, p.211) “that traditional two-parameter models relating portfolio means to variances are inappropriate and indicate the need for research into other measure of risk”.

Insert Table 2

Figures 1 and 2 plot Canadian monthly market risk premium and residuals from the predicted values of market risk premium based on both the structural and time-series parts of Model 1, respectively. Similarities between these two figures indicate that Model 1 fails to capture the market risk premium changes. This is confirmed by the very low coefficient of determination obtained (0.04%).

Insert Figures 1&2

Like Scruggs (1998) and contrary to Glosten et *al*. (1993), the parameters in the variance equation are significant. Estimates of the coefficient of lagged conditional volatility (β) is significantly positive at 0.1% level and is associated with a half-life of 2.97 months.[4]

Comparing this result with that found by Scruggs (1998) ($h = 6.43$ months) reveals that shocks are more persistent on the American market than on the Canadian market. The impact of past return shocks measured by α is significant at 0.1% level. These two parameters (α and β) indicate a strong *hysteresis* of shocks. Results highlight also the asymmetric effect in the evolution of conditional variance. The θ coefficient (-0.386) is significantly negative ($t = -2.751$) indicating that negative return shocks have more impact on conditional volatility than positive returns shocks. This is explained by the leverage effect first reported by Black (1976) and theorized by Christie (1982).

Figures 3 and 4 plot EGARCH (1,1)-M estimates of the conditional market variance and the monthly-expected risk premium estimated from Model 1, respectively. It is clear from visual inspection of Figure 3 (confirmed by the statistical tests) that the market risk premium is not i. i. d through time. The plot of Figure 3 is very similar to plot of conditional market volatility presented in Scruggs (1998) and shows several distinct periods of volatility clustering. Figure 4 reveals that the predicted market risk premium does not exhibit the same patterns as the realized market risk premium (Figure 1) and exhibits a weak variation, even if it is represented to the tenth of the scale of Figure 1. Moreover, the predicted risk premium is positive throughout the sample period and consequently fails to fit the negative risk premium reported in Table 1 (44% of the cases). This induces a great dispersion of errors, particularly in the down-market periods, which results in a non-constant error variance (heteroskedasticity) and non-significant estimates of Model 1.

Insert Figures 3&4

In summary, our results confirm those obtained for the American market. This is not surprising knowing the findings of Eun and Shim (1989) that shocks in the American market are quickly disseminated to the rest of the world. Moreover, Theodossiou and Lee (1993) show that the conditional volatility in the Canadian market is imported from outside, particularly from the US market. Using EGARH process to model market volatility, we find a negative and insignificant relationship between the Canadian market risk premium and conditional market variance.

Table 3 presents the results of estimating Model 2. As we discussed previously, we transformed Model 1 in order to allow for different reactions (in sign and amplitude) depending on up and down-market periods. We modelled the volatility using an EGARCH (1,1) process. As expected, the coefficient λ_{m1} (16.64) is significantly positive at the 0.1% level (t = 11.04). Thus, during up-market periods, increases in volatility results in a rise of the market risk premium. The coefficient λ_{m2} (-16.01) is significantly negative at the 0.1% level (t = -10.95). This implies a negative relationship between the market risk premium and conditional variance during the down-market periods. Thus, an increase of the market conditional variance results in an increase of losses during the down market periods. These results go against those of traditional tests. Indeed, the amplitudes of the price of risk for up and down-market periods are very close in absolute value but in opposite sign. Consequently, we argue that aggregating these two-segmented conditional relations results in a mispecification which may explain the weakness and the absence of consistency (both economically and statistically) of traditional tests of the conditional market risk-return relationship.

Insert Table 3

The estimation of the conditional variance equation shows the presence of heteroskedasticity. The coefficient for past volatility shocks (α) and past conditional variance (β) are statistically significant, indicating that the volatility of the Canadian market risk premium is predictable using past information. The asymmetry of response parameter (θ) is statistically insignificant. This shows that unexpected change of the market risk premium has a symmetric impact on volatility when the asymmetrical effect is taken into account in the mean equation (Model 2).

Figures 5 and 6 plot EGARCH (1,1)-M estimates of conditional market variance and monthly-expected risk premium estimated from Model 2, respectively. The conditional volatility plot exhibits extreme volatility as shown in Figure 5. Figure 6 reveals that Model 2 also predicts down market patterns and gives a plot that exhibits a better fit of realized market risk premium. This is confirmed by the appreciable increase of the likelihood function and the coefficient of determination of Model 2 compared to Model 1 (49.41% versus 0.04%).

Insert Figures 5&6

IV. Conclusion

This paper investigates the relation between the market risk premium and conditional market variance and attempts to resolve the conflicting results reported by previous studies. Results from the traditional test reveal a negative and insignificant relation between the market risk premium and conditional market variance and confirm weak relations reported in the US market. However, we show that these traditional tests employed by previous studies are biased because they aggregate the risk premium associated with up and down-market periods when they use of realized market risk premium as a proxy for expected market risk premium.

We also conduct a new test which recognizes the impact of using realized market risk premium to proxy for expected market risk premium and allows up and down market volatility to have different effects on the market risk premium. The empirical results indicate a strong relationship between the market risk premium and conditional market variance whatever the sign of the market risk premium is. We obtain a positive (negative) and significant relationship between the market risk premium and conditional market variance in bull (bear) market context. These results restore not only the importance of the variance as a measure of risk, but also the cross-sectional and the intertemporal implications of the market-based CAPM.

ENDNOTES

[1] This includes, among others, Theodossiou and Lee (1995) who found no relation between expected returns and conditional volatility in ten national stock markets including Canada and U.S.A.

[2] We don't test a constrained version of equation (3) ($\lambda_0 = 0$) as in Merton (1980) and Harvey (1989). Scruggs (1998, p.589) shows that constraining the regression line to pass by the origin may result in an overestimation of the coefficient λ_m.

[3] See Bollerslev, Chou and Kroner (1992) for a survey.

[4] The half-life (h) of a market shock is given by the following expression: $h = \ln(0.5)/\ln(\beta)$, see Nelson (1991).

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Table 1

Descriptive Statistics

This table summarizes statistics regarding the market return (R_m), the risk-free rate of return (R_f) and the market risk premium ($r_m = R_m - R_f$) from March 1950 to December 1995 (550 observations). The market return corresponds to the return of the Canadian value-weighted index. Returns on Canadian three-month Treasury-bill proxy for the risk-free rate of return. Panel A presents the usual descriptive statistics of these three variables (R_m, r_m and R_f). The last column (S-W test) provides the p-value associated with the Shapiro-Wilk (1965) normality test. The p-value indicates the error probability of rejecting the null of normality when it is true. Panel B presents the autocorrelation coefficients of orders 1, 2, 3, 4, 6, 9 and 12 for these three series. Panel C presents the parametric portmanteau (Q^2) tests of McLeod and Li (1983). This test is based on squared residuals and cover shifts of orders 1, 2, 3, 4, 6, 9 and 12 of the autocorrelation function.

Panel A: Univariate statistics

	Mean (x 100)	Standard Deviation (x 100)	Median (x 100)	Skewness	Kurtose	Positive Values	S-W test (p-value)
R_m	0.9593	4.3350	1.0900	-0.4752	2.8855	61%	0.0973
r_m	0.4425	4.3444	0.5301	-0.5852	3.0675	56%	0.0188
R_f	0.5168	0.3345	0.4777	0.8730	1.1047	100%	0.0001

Panel B: Autocorrelation coefficients (ρ_j)

	ρ_1	ρ_2	ρ_3	ρ_4	ρ_6	ρ_9	ρ_{12}
R_m	0.0708	-0.0554	0.0637	0.0075	0.0074	0.0276	0.0069
r_m	0.0820	-0.0423	0.0750	0.0195	0.0169	0.0313	0.0059
R_f	0.9127[a]	0.8874[a]	0.8514[a]	0.8334[a]	0.7985[a]	0.7880[a]	0.7289[a]

Panel C: Parametric portmanteau tests

	$Q^2(1)$	$Q^2(2)$	$Q^2(3)$	$Q^2(4)$	$Q^2(6)$	$Q^2(9)$	$Q^2(12)$
R_m	3.5913[b]	12.4645[c]	12.8692[c]	17.6097[c]	21.8078[c]	32.1836[c]	33.076[c]
r_m	3.4563[b]	10.1908[c]	10.5118[b]	15.0729[c]	19.3903[c]	28.9424[c]	29.7908[c]
R_f	262.585[c]	450.515[c]	556.439[c]	626.387[c]	754.877[c]	1024.9[c]	1188.4[c]

Table 2

Traditional Test of Conditional Models of the Market Risk Premium

This table presents the results on the conditional model of the relation between the market risk premium ($r_{m,t}$) and conditional market variance ($\sigma^2_{m,t}$) using Canadian monthly data for the period of 1950:3 to 1995:12 (550 observations). Parameters are estimated within the following EGARCH (1,1) – M system

$$r_{m,t} = \lambda_0 + \lambda_m \hat{\sigma}^2_{m,t} + \varepsilon_{m,t}$$

$$\varepsilon_{m,t} = \hat{\sigma}_{m,t}\, e_t \qquad e_t \sim N(0,1)$$

$$\ln(\hat{\sigma}^2_{m,t}) = \omega + \alpha g(e_{t-1}) + \beta \ln(\hat{\sigma}^2_{m,t-1})$$

$$g(e_{t-1}) = \theta e_t + \left(|e_t| - \sqrt{2/\pi}\right)$$

	λ_0 (x100)	λ_m	ω	α	β	θ	ln L	R^2
Coefficient	0.5448	-0.1570	-1.3188	0.3169	0.7915	-0.3857	968.62	0.0004
t-test	1.350	-0.069	-2.713	4.248	10.372	-2.751		
p-value	0.1772	0.9446	0.0067	0.0001	0.0001	0.0059		

Table 3

Estimation of the Conditional Relation Between the Market Risk Premium and Conditional Market Variance at Up and Down-Market Periods

This table presents the results on the conditional relation between the market-risk premium ($r_{m,t}$) and conditional market variance in up ($\delta\sigma^2_{m,t}$) and down ($(1-\delta)\sigma^2_{m,t}$) market periods (Model 2) using Canadian monthly data for the period of 1950:3 to 1995:12 (550 observations). The dummy variable δ [$\delta = 1$ if $r_{m,t} \geq 0$, and $\delta = 0$ if $r_{m,t} < 0$] is used to separate the up and down-market patterns. Parameters are estimated within the following EGARCH (1,1) – M system

$$r_{m,t} = \lambda_0 + \lambda_{m1}\, \delta\, \hat{\sigma}^2_{m,t} + \lambda_{m2}\, (1-\delta)\, \hat{\sigma}^2_{m,t} + \varepsilon_{m,t}$$

$$\ln(\hat{\sigma}^2_{m,t}) = \omega + \alpha\, g(e_{t-1}) + \beta \ln(\hat{\sigma}^2_{m,t-1})$$

$$g(e_{t-1}) = \theta e_t + \left(\left|e_t\right| - \sqrt{2/\pi}\right)$$

$$\varepsilon_{m,t} = \hat{\sigma}_{m,t}\, e_t \qquad e_t \sim N(0,1)$$

	λ_0 (x100)	λ_{m1}	λ_{m2}	ω	α	β	θ	ln L	R^2
Coefficient	0.1042	16.6398	-16.0125	-2.8886	0.4503	0.5888	0.0257	1175.47	0.4941
t-test	0.386	11.040	-10.951	-3.036	6.742	4.406	0.200		
p-value	0.6998	0.0001	0.0001	0.0024	0.0001	0.0001	0.8414		

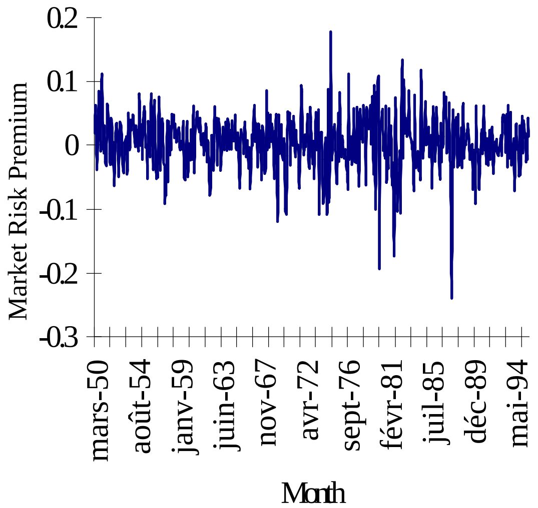


Figure 1: Canadian market risk premium

The figure plots the Canadian monthly market risk premium ($r_{m,t}$) for the sample period of 1950:3 to 1995:12.

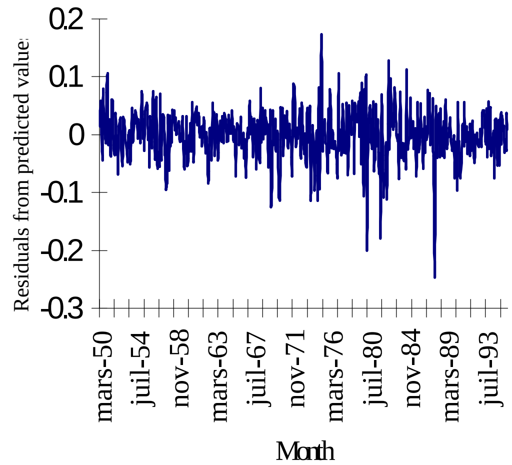


Figure 2: Residuals of Model 1

The figure plots residuals from predicted values of market risk premium, which is the difference between the market risk premium ($r_{m,t}$) and its predicted value $E_{t-1}[r_{m,t}]$from estimation of Model 1.

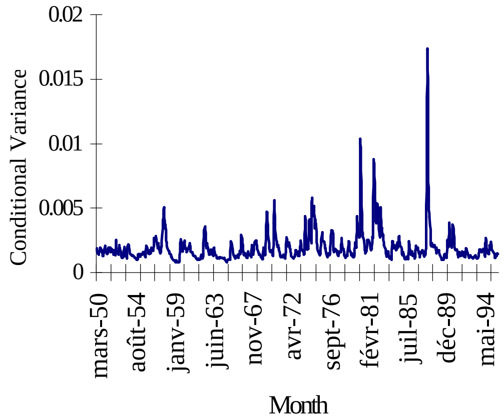


Figure 3: Conditional Variance of the Market Risk Premium for Model 1

The figure plots the EGARCH (1,1)-M estimates of the conditional market variance for Canadian monthly data for the sample period of 1950:3 to 1995:12.

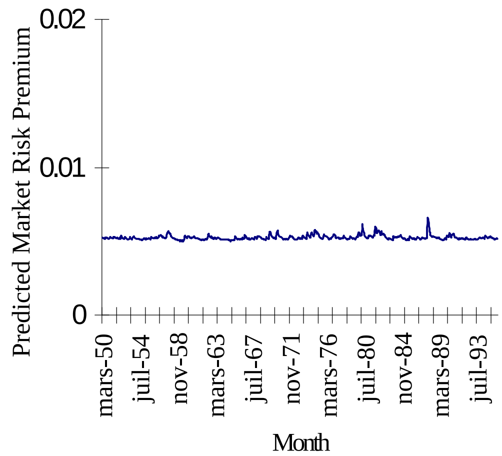
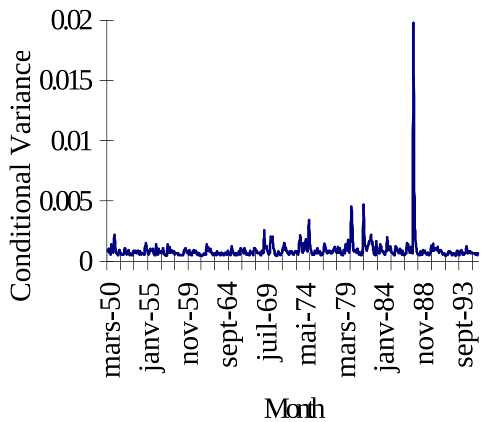


Figure 4: Predicted Market Risk Premium for Model 1

The figure plots predicted values of market risk premium, using Model 1 for Canadian monthly data for the sample period of 1950:3 to 1995:12.



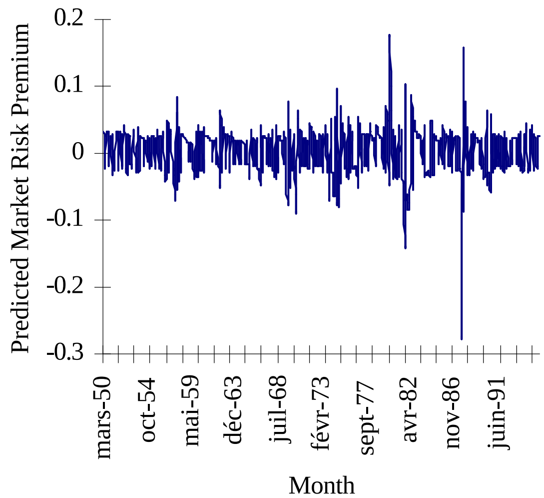


Figure 5: Conditional Variance of Market Risk Premium for Model 2

The figure plots the EGARCH (1,1)-M estimates for the conditional market variance for Canadian monthly data for the sample period of 1950:3 to 1995:12.

Figure 6: Predicted Market Risk Premium for Model 2

The figure plots the predicted values of market risk premium, using Model 2 for Canadian monthly data for the sample period of 1950:3 to 1995:12.