

The Importance of Liquidity in Index Futures Pricing: Modelling and Empirical Evidence

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Abstract

Several empirical studies report that asset liquidity has a significant impact on asset prices. For example, Amihud and Mendelson (1986), Silber (1991), Kadlec and McConnell (1994), and Brennan and Subrahmanyam (1996) report that stock prices are c.p. the lower, the lower stock liquidity is. For bond markets similar evidence is reported by Sarig and Warga (1989), Amihud and Mendelson (1991), Boudoukh and Whitelaw (1991), Warga (1992), Kamara (1994), and Kempf and Uhrig-Homburg (2000).

In this paper, the impact of liquidity on index futures prices is studied theoretically and empirically. I derive a theoretical two-factor pricing model where the index futures price depends on the index level and the illiquidity of the future. The two factors are the index level and the costs of illiquidity proxied by the bid-ask spread in the empirical study. The model provides an analytical solution for imperfectly liquid index futures. The fair price of an imperfectly liquid future is determined by the cost-of-carry price and a discount function due to illiquidity. The price discount depends on the maturity of the contract and its bid-ask spread.

The model is tested in the sample and out of the sample using data on the German futures market. I show that the model explains futures prices significantly better than the cost-of-carry model and the two-factor model of Ramaswamy and Sundaresan (1985) do. The superiority of the liquidity model increases with the maturity of the contract. All models explain the prices of short contracts pretty well, but only the liquidity model provides a good explanation for the prices of index futures with long time to maturity. The pricing error of the liquidity model is much smaller than the error of both benchmark models. My main conclusion of the empirical study is that only index risk matters for short contracts, whereas index and liquidity risks matter for long contracts.

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1. Introduction

Several empirical studies report that asset liquidity has a significant impact on asset prices. For example, Amihud and Mendelson (1986), Silber (1991), Kadlec and McConnell (1994), and Brennan and Subrahmanyam (1996) report that stock prices are c.p. the lower, the lower stock liquidity is. For bond markets similar evidence is reported by Sarig and Warga (1989), Amihud and Mendelson (1991), Boudoukh and Whitelaw (1991), Warga (1992), Kamara (1994), and Kempf and Uhrig-Homburg (2000).

In this paper, the impact of liquidity on index futures prices is studied theoretically and empirically. I derive a theoretical two-factor pricing model where the index futures price depends on the index level and the liquidity of the future. Then I show that this liquidity based pricing model explains the prices of German index futures significantly better than the one-factor cost-of-carry model and the two-factor stochastic interest rate model by Ramaswamy and Sundaresan (1985). The superiority of my model is most pronounced for contracts with long maturity whereas short contracts are priced pretty well by all models. This leads to my conclusion that only index risk matters for (liquid) short contracts, whereas index and liquidity risks matter for (illiquid¹) long contracts.

2. Theoretical Futures Pricing Model

In this section I price futures in the presence of liquidity risk. The model is based on an approach of Grinblatt (1995) which was further developed by Kempf and Uhrig-Homburg (2000). The idea of the model is as follows: Illiquid assets must offer a higher yield than liquid assets to compensate investors for the disadvantage due to lower liquidity. One might think, for example, of higher bid-ask spreads when trading the asset. The yield spread offsets the costs of illiquidity so that the risk-adjusted expected net returns of all liquid and illiquid assets equal the short rate. The core assumption of both models is that the consequences of illiquidity are reflected solely in the yield spread, which is modelled as a second stochastic state variable.² Given this assumption one can use

¹ I will use the terms “illiquid” and “imperfectly liquid” interchangeably.

² In Grinblatt (1995) and Kempf and Uhrig-Homburg (2000), the instantaneous interest rate and the yield spread are the state variables, because they use the model to price fixed income securities. Whereas Grinblatt focuses on swap spreads in US market, Kempf and Uhrig-Homburg analyze the German bond

standard no-arbitrage pricing technique to value derivative securities. In the remainder of the section I show how to price futures based on this idea.

Assume that the price of index futures depends on two stochastic factors, the index level $S(t)$ and the costs of illiquidity $x(t)$. The evolution of the factors is given by the following differential equations

$$(1) \quad \frac{dS}{S} = mdt + ndz$$

$$(2) \quad dx = a(g - x)dt + w\sqrt{x}dw,$$

where m , n , a , g and w are constants. z and w are standard Wiener processes. Furthermore, assume that the instantaneous correlation between dz and dw is zero.³

(1) is a standard model for the evolution of stock prices and indices. (2) models the costs of illiquidity as a continuous cash outflow. This is a rough modelling of the consequences of illiquidity, because it neglects that the total costs of illiquidity depends on how often the asset is traded before maturity. However, the purpose of this paper is not to determine the costs of illiquidity, but to calculate the consequences of illiquidity on futures prices. Therefore, it seems to be a reasonable first approach to model not illiquidity per se, but to model the aggregate financial consequences of illiquidity. This is done in (2) which assumes that the costs of illiquidity varies over time randomly, possible due to changing bid-ask spreads, varying trading frequencies, and so on. According to (2) negative costs of illiquidity cannot occur and the costs of illiquidity are fluctuating randomly around some mean level g .

Assume that only futures contracts are imperfectly liquid⁴ and that all consequences of futures illiquidity are reflected in the state variable $x(t)$. Then one can price index futures using a standard

³ market. The main difference between their models is that Grinblatt uses a non-market-fitting model whereas Kempf and Uhrig-Homburg fit their model to the market segment of liquid bonds. This assumption allows me to find an analytical solution to the pricing model. One could easily relax this assumption, but then the model has to be solved numerically.

no-arbitrage-technique. Since the price F of a future depends on time to maturity $T - t$, on the index level S and on the costs of illiquidity x , the fundamental pricing equation is given as⁵

$$(3) \quad F_t + F_S r S + \frac{1}{2} F_{SS} \sigma^2 S^2 + F_x [a \ln g - x \ln - l w x] + \frac{1}{2} F_{xx} w^2 x = x F$$

with the initial condition $F(T, T) = S(T)$. Subscripts denote partial derivatives and r is the risk-free interest rate which is assumed to be constant. Since the costs of illiquidity is not a traded asset, the market price of liquidity risk, l , shows up in (3).

The pricing equation (3) shows one important difference to other futures pricing models. It states that the expected risk-adjusted return has to be positive, although no capital input is required to build up a futures position.⁶ The economic reasoning is that a positive expected risk-adjusted gross return has to compensate for the costs of illiquidity, x . Therefore, the right hand side of (3) is $x F$. This assures that the expected risk-adjusted net return of a futures contract, i.e. the return corrected for costs of illiquidity, is zero. (3) can be solved by separating the variables.⁷ The solution F^L of the liquidity model is given as

$$(4) \quad F^L = F^{CC}(S, t, T) C(x, t, T)$$

where:

$$(5) \quad F^{CC}(S, t, T) := S(t) e^{r(T-t)}$$

$$(6) \quad C(x, t, T) := D(t, T) e^{-E(t, T)x(t)}$$

⁴ This means that the index is taken as exogenous and perfectly liquid. It allows to study the consequences of different futures liquidity on futures prices.

⁵ The impact of dividends is neglected here. This assumption is used since the empirical study focuses on the German index futures. They are based on the DAX performance index which is corrected for dividends. Therefore, the holder of a DAX future has no dividend disadvantage as compared with the holder of DAX stocks. Consequently, dividends have no impact on the price relation between futures and index.

⁶ Margin payments can be neglected since they earn interest.

⁷ Due to assumption of uncorrelated Wiener processes, no cross products occur in eq. (3) and the solution has the multiplicative structure $f(S, t, T)g(x, t, T)$.

$$(7) \quad D(t, T) := \left[\frac{2\sqrt{(a+lw)^2 + 2w^2} e^{(a+lw + \sqrt{(a+lw)^2 + 2w^2}) \frac{T-t}{2}}}{\left((a+lw + \sqrt{(a+lw)^2 + 2w^2}) \left(e^{\sqrt{(a+lw)^2 + 2w^2} (T-t)} - 1 \right) + 2\sqrt{(a+lw)^2 + 2w^2} \right)} \right]^{\frac{2ag}{w^2}}$$

$$(8) \quad E(t, T) := \frac{2 \left[e^{\sqrt{(a+lw)^2 + 2w^2} (T-t)} - 1 \right]}{\left((a+lw + \sqrt{(a+lw)^2 + 2w^2}) \left(e^{\sqrt{(a+lw)^2 + 2w^2} (T-t)} - 1 \right) + 2\sqrt{(a+lw)^2 + 2w^2} \right)}$$

The price of an illiquid futures is determined by the futures price given it were perfectly liquid, i.e. the price of the cost-of-carry model, F^{CC} , and an illiquidity correction function, $C(x, t, T)$. This function depends only on time to maturity of the future and the stochastic costs of illiquidity. The solution (6) - (8) of the function $C(x, t, T)$ can be found in Cox, Ingersoll and Ross (1985). The function $C(x, t, T)$ has the following properties. It is smaller than one for $t < T$, i.e. the price of an illiquid futures is smaller than the cost-of-carry price. The price discount is the larger, the longer the time to maturity of the futures contract and the higher the costs of illiquidity.

Thus, the model is consistent with underpricing⁸ of index futures markets which is well documented in the literature, mainly for European index futures. For example, see Yadav and Pope (1990) and Strickland and Xu (1993) for the British market, Berglund and Kabir (1995) for the Dutch market, Puttonen and Martikainen (1991) and Puttonen (1993) for the Finish market, Gruenbichler and Callahan (1994), Bamberg and Roeder (1995), Buehler and Kempf (1995), and Kempf (1999) for the German market and Stulz, Wasserfallen and Stucki (1990) for the Swiss market. For US markets underpricing of futures is reported for the initial years of trading⁹ while mixed results concerning the average mispricing are found later.¹⁰

⁸ In this paper, the terms underpricing and overpricing always relate to the cost-of-carry model. A future is underpriced (overpriced) when its price is below (above) the fair futures price implied by the cost-of-carry model.

⁹ See, e.g., Cornell and French (1983), Figlewski (1984), and Merrick (1987).

¹⁰ An excellent overview is Sutcliffe (1997), Chapter 5.1.

The model is also consistent with the empirical observation in futures markets that the mispricing increases with time to maturity.¹¹ Buehler and Kempf (1995) and Kempf (1999) show that the underpricing of futures contracts is the larger, the longer time to maturity is. Yadav and Pope (1990) find a similar result, but only for the first part of the research period.

In the remainder of the paper, I will test whether this model is able to explain observed futures prices better than benchmark models. Since there are no competing pricing models which explain index futures prices by index level and liquidity, I have to choose benchmark models which explain futures prices by other factors. My first benchmark model is the cost-of-carry model which explains index futures prices as a function of the index level only. Its pricing equation is given by (5). My second benchmark model is the two-factor model of Ramaswamy and Sundaresan (1985) which assumes that the futures price depends on the stochastic index level and on stochastic interest rates. Comparing the results of my model with the results of the Ramaswamy-Sundaresan model provides insights on the relative importance of liquidity risk and interest-rate risk.

3. Empirical Evidence

3.1 Data

I test the pricing models using data on the German stock index DAX, on DAX futures and on interest rates from the money market.¹² The organization of the stock and futures markets in Germany is described in Buehler and Kempf (1995), for example.

The study analyzes the four futures contracts maturing in 1996 (March, June, September, December). Each contract is analyzed from its first trading day until maturity, i.e. over a period of about nine months. I do not restrict my attention to the nearby contracts because the liquidity of a contract depends on whether it is the short contract (maturity 0 - 3 months), the mid contract (maturity 3 - 6 months) or the long contract (maturity 6 - 9 months).¹³

¹¹ See Sutcliffe (1997), Chapter 5.2.1, for a survey on the empirical evidence.

¹² The data are supplied by the Deutsche Finanzdatenbank in Karlsruhe and Mannheim.

¹³ The average bid-ask spread is about 0.6 index points in short contracts, 1.3 index points in mid contracts and 1.8 index points in long contracts.

The raw data consists of all time stamped quotations and transactions data of the futures market, the index midquote on a minute-by-minute basis and the interest rates with different maturities (overnight, 1-month, 3-month, 6-month, 12-month money) on a daily basis.

To reduce the noise in the data I aggregate them to get daily observations. This aggregation is done for each contract separately and is described for the contract maturing in December 1996 in detail below. This contract is traded from 15th March 1996 until 19th December 1996. I use the DAX index midquote from the computerized stock exchange IBIS which is available on a minute-by-minute basis from 8.30 a.m. until 5.00 p.m., i.e. there are 510 observations per day. To each index level I assign the contemporaneous midquote of the futures contract and the interest rate with matching maturity. This matching interest rate is obtained by linearly interpolating the two interbank rates which match the maturity of the futures contract best. Based on these data I calculate an intraday time series of simultaneous futures prices, index levels and theoretical futures prices derived from the cost-of-carry model based on eq. (5).¹⁴ Then I use the arithmetic mean of the futures prices of the day t as the price variable $F(t, T)$. In the same way daily observations of $S(t)$ and $F^{CC}(t, T)$ are obtained. The mean relative bid-ask spread of the futures contract at day t is used as proxy of the costs of illiquidity at day t .¹⁵

This aggregation leads to separate time series for each contract. Since the liquidity of futures contracts depend heavily on whether they are the short (0 - 3 months), the mid (3 - 6 months) or the long (6 - 9 months) contract,¹⁶ I construct three time series. The first one consists of all daily observations of the four contracts when they are the short contracts. The second series includes all observations of the mid contracts and the final one all observations of the long contracts. Each series consists of 253 daily observations. The following study is based on these three series.

¹⁴ Dividends are not taken into account since the DAX is a performance index corrected for dividend payments. See Buehler and Kempf (1995).

¹⁵ For each contract there are about 4800 spreads per day.

¹⁶ Liquidity is highest in the short contracts and smallest in the long contracts. The average bid-ask spread is between 0.6 and 0.7 index points in short contracts, between 1.2 and 1.4 in the mid contracts, and between 1.6 and 2.0 in the long contracts. The differences are highly significant.

3.2 In-Sample-Test

The fair price of the cost-of-carry model is given by (5). The observed futures prices, F , differs from F^{CC} significantly at the 1%-level. The signed mispricing is -2.45 index points for short contracts, -6.11 for mid contracts, and -10.11 for long contracts. This result is consistent with earlier studies on the German futures markets: DAX futures are mainly underpriced.¹⁷ There is mean underpricing in 184 (out of 253) days of the short contracts, 245 (out of 253) days of the mid contracts and in all days of the long contracts. Whether this underpricing is due to interest-rate risk or due to liquidity risk is tested next.

First, I have to estimate the unknown parameters a , g , w , and l of the liquidity model. I determine them implicitly using a two-step grid search. As eq. (4) - (8) show, it is impossible to estimate all four parameters separately, but only three parameters. I choose the parameters w , $b_1 := ag$, and $b_2 := a + lw$ in such a way that they minimize the sum of the absolute differences between observed futures prices, F , and theoretical futures prices of the liquidity model, F^L . For the in-sample test I fit the parameters to all observations. This is done for all three futures series separately.

The two-factor model of Ramaswamy and Sundaresan (1985) is implemented in the same way. Again, I use a two-step grid search to find those three parameters which minimize the sum of the absolute differences between observed futures prices and theoretical futures prices F^I of their model. The overnight interbank rate is used as proxy of the short rate.¹⁸

To judge the quality of the models I determine the average absolute difference between observed futures prices and futures prices predicted by the models. The results for all three models are reported in Table 1.

¹⁷ See, e.g., Gruenbichler and Callahan (1994), Bamberg and Roeder (1995), Buehler and Kempf (1995), and Kempf (1999).

¹⁸ The analysis was redone using one-month interest rates as proxy. There are no notable differences in the results.

Table 1: Absolute pricing errors of all models in the sample

	short contracts	mid contracts	long contracts
Cost-of-carry model: $ F - F^{CC} $	2,82	6,18	10,11
Interest-rate model: $ F - F^I $	2,53	3,71	3,59
Liquidity model: $ F - F^L $	2,37	3,10	2,17
$ F - F^{CC} - F - F^L $	0,44**	3,08**	7,94**
t-Statistics	6,22	15,25	41,26
$ F - F^I - F - F^L $	0,16**	0,61**	1,41**
t-Statistics	3,30	7,19	7,58

** significant at the 1%-level

The table shows four main results. First, the cost of carry model is a fairly good pricing model for DAX futures, but only for the short contracts. The relative mispricing is about 0.2% in the short contract. In the mid and long contracts the mispricing is much larger than in the short contract. Second, the interest-rate model of Ramaswamy and Sundaresan (1985) outperforms the cost-of-carry model for all contracts¹⁹, but its pricing error increases with time to maturity. In this respect, the results are similar to the results of the cost-of-carry model. Third, the liquidity model outperforms both benchmark models significantly. Finally, the liquidity model is able to explain the prices of all futures contracts pretty well. This is the main difference to both benchmark models. Both benchmark models are fairly good for short contracts, but their fit becomes worse for mid and long contracts. Thus, the superiority of the liquidity model (which takes care of liquidity effects) over the benchmark models (which assume perfect liquidity) is most pronounced in the mid and long contracts which are less liquid than short contracts. This indicates that liquidity matters for prices of long contracts much more than for prices of short contracts.

¹⁹ This result is consistent with the empirical evidence in Caikici/Chatterjee (1991).

3.3 Out-of-Sample-Test

Finally, I will test the models out of sample. I estimate constant parameters of the model using data of the first three months of the research period. The estimation procedure is the same as in Section 3.2. The estimated parameters are then used to explain the futures prices of the next nine month. The validity of the three models is again judged based on the mean absolute pricing error. The results are shown in Table 2.

Table 2: Absolute pricing errors of all models out of the sample

	short contracts	mid contracts	long contracts
Cost-of-carry model: $ F - F^{CC} $	3,35	7,23	11,10
Interest-rate model: $ F - F^I $	2,92	3,92	9,98
Liquidity model: $ F - F^L $	2,96	4,14	2,73
$ F - F^{CC} - F - F^L $	0,39**	3,08**	8,36**
t-Statistics	8,61	23,13	50,83
$ F - F^I - F - F^L $	-0,04	-0,22	7,25**
t-Statistics	-0,47	-1,09	18,26

** significant at the 1%-level

The out-of-sample pricing errors in Table 2 are larger than the in-sample errors shown in Table 1. This is not surprising since the models are fit to data of the contract maturity in March 96 and the estimated parameters are used to explain the prices of the futures maturing in June, September and December 1996.²⁰

The most interesting result of Table 2 is the fact that the liquidity model explains prices of long futures contracts significantly better than both benchmark models do. This can be explained by the fact that futures with long time to maturity are the least liquid ones. Therefore, it is important to consider their liquidity as a factor determining their prices.

²⁰ Since there are no free parameters in the cost-of-carry model, no data of the contract maturing in March 96 are used to calculate the results of the cost-of-carry model in Table 3. The values shown are the average absolute mispricing of the contracts maturing in June, September and December 1996.

The results of the out-of-sample tests can be briefly summarized as follows: For short contracts, all three pricing models are about equally good, i.e. for short contracts only index risk matters. For mid contracts, the two-factor models are much better than the cost-of-carry model, but the results do not allow to judge whether liquidity or interest risk is the second risk factor which matters. For long contracts, however, only the liquidity model provides good results, i.e. index risk and liquidity risk are the two factors driving the prices of long futures contracts.

4 Summary

This paper addresses the question of whether liquidity influences the prices of index futures. I presented a two-factor model to price index futures. The two factors are the index level and the costs of illiquidity proxied by the bid-ask spread. The model provides an analytical solution. The fair price of an imperfectly liquid future is determined by the cost-of-carry price and a discount function due to illiquidity. The price discount depends on the maturity of the contract and its bid-ask spread.

The model was tested using data on the German futures market. I show that the model explains futures prices significantly better than the cost-of-carry model does. This is true in the sample as well as out of the sample. The second benchmark, the two-factor model of Ramaswamy and Sundaresan (1985), is more challenging. In the sample, the liquidity model fits the prices of all futures significantly better than the Ramaswamy-Sundaresan model does which assumes that the interest-rate is the second stochastic factor. Out of sample, I find no significant differences between the two models for short and mid contracts. For long contracts, however, the liquidity model is much better than the approach of Ramaswamy and Sundaresan (1985). The absolute pricing error of their model is about four times higher than the absolute pricing error of the liquidity model. This result suggests that it is the liquidity risk which matters for contracts with long time to maturity.

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