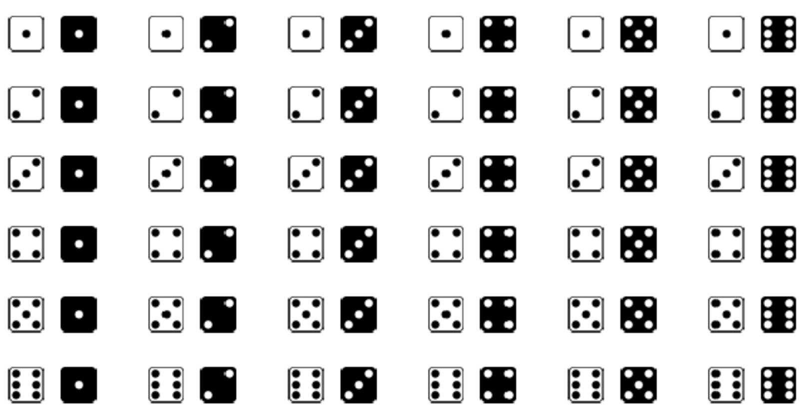
Module 5: Probability and Randomness
Practice exercises

PART 1: Introduction to probability

EXAMPLE 1: Classify each of the following statements as an example of exact (theoretical) probability, relative frequency interpretation of probability, or personal (subjective) probability.

a. On the basis of prior counts, a quality control engineer says that there is a 0.005 probability that a vacuum hose is defective.
b. The probability that the Cubs will win the World Series this year is 60%.
c. The probability of selecting five card of the same suit (a flush) from a standard deck of cards is 0.001981
d. The chance that a randomly selected person in the United States is between 15 and 24 years old is 0.14.
e. The probability that General Electric's stock price will rise today is 75%.

EXAMPLE 2: ROLL A PAIR OF DICE.



a) What is the probability of getting a sum of 7 on a roll of two dice?
b) What is the probability of getting a sum of “3” on a roll of two dice?
c) What’s the probability of getting one “3” in the roll of two dice?
d) What’s getting a sum of 2 or 3 on a roll of two dice?

PART 2: Mutually exclusive events and additive rule

EXAMPLE 1: Determine which of the following events is mutually exclusive (disjoint)

a. A person having a high school diploma and a person having a college diploma.
    i. *Mutually exclusive* *NOT Mutually exclusive*
b. Applicant with less than a bachelor’s degree and an applicant with a graduate degree
    Mutually exclusive *NOT Mutually exclusive*

EXAMPLE 2: AIRLINE PASSENGERS: The number of passengers that cannot be boarded because there are no more seats can be represented by the following probability distribution:

Number of passengers	0	1	2	3	4 or more
probability	0.820	0.113	0.057	0.009	0.001

a. Does this table represent a well-defined probability distribution?
b. Compute the probability that 2 or more passengers are unable to board the plane b/c of overbooking.

EXAMPLE 3: SOFTWARE ENGINEERS Among software developers of a certain firm, 70% know C/C++, 60% know Java and 45% know both languages.

a. Are the events "knowing Java" and "knowing C/C++" mutually exclusive?
b. What portion of programmers does not know Java?
c. What portion of programmers knows either Java or C++?
d. What portion of programmers does not know either Java or C/C++?
e. What portion of programmers knows C/C++ but not Java

PART 3: Independent events and multiplicative rule

EXAMPLE 1: Determine which events are independent and which are dependent.

a. Drawing a card from a deck, not replacing it and then drawing a second card

Independent *Dependent*

b. Getting a salary raise and buying a new car

Independent *Dependent*

c. Having a large shoe size and having a high IQ

Independent *Dependent*

EXAMPLE 2: In January 2007 Apple was ranked in fifth place for total dollar volume and unit share of laptop sales, with 10 percent of the U.S. laptop sales in that month. If we assume that the probability that someone bought an Apple laptop in the past month is 10%, what's the probability that out of 10 people that want to buy a laptop, at least one will buy an Apple laptop?

EXAMPLE 3: A computer program is tested by 3 independent tests. When there is an error, three tests T1, T2, and T3 will discover it with probabilities 0.2, 0.3, 0.5 respectively.

a. What is the probability that an error in the program will be found by all three tests?
b. What is the probability that an error in the program will be found by the first test T1 only?
c. What is the probability that it will be found by at least one test?

EXAMPLE 4: A small business performs a service and then bills its customers. From past experience, 90% of the customers pay their bills within two weeks.

a. What is the probability that a randomly selected customer will not pay within two weeks.
b. The business has billed two customers this week. What is the probability that neither of them will pay within two weeks?
c. The business has billed four customers this week. What's the probability that at least **one** customer will pay within two weeks?

EXAMPLE 5: PROCESS QUALITY CONTROL (CHALLENGE!)

A computer company produces poor quality chips, and each shipment contains 5% defective chips. A car manufacturer buys a shipment of such chips.

a. Each car uses 4 of these chips selected independently, what is the probability that all 4 are defective (D)?
b. Suppose you continue installing chips until you found a defective. What is the probability of selecting a bad chip on the first try?
c. What is the probability of selecting a bad chip at the second try, but not before? Is this the same probability computed above?
d. What is the probability of selecting a bad chip at the fourth try, but not before?
e. What is the probability of selecting a defective chip out of 4 chips? Is this the same probability computed above?
f. What is the probability of selecting at least a defective chip out of 4 chips? Is this the same question as before?

Part 4: Random variables and probability distributions

EXAMPLE 1: REMOTE CONNECTION

The probability of being able to log on to a certain computer from a remote terminal is 0.7. Let X be the number of attempts that must be made before gaining access to the computer. Suppose your remote connection tries up to 5 times. The random process is described by the following table:

Attempts X	1	2	3	4	5
Probability	0.7	0.21	0.063	0.0189	0.0081

a) Is this a valid probability distribution? Explain.
b) Draw the probability histogram
c) Compute the probability that the computer connect remotely at the first or second attempt
d) Compute the average number of attempts to login to the remote terminal.

EXAMPLE 2: INSURANCE POLICIES

Insurance companies make money on random processes. Let's consider a simplified case. All customers buy the same policy for $500 per year. There is a 10% chance that customers submit a claim in any given year and the claim is always for $1500. This is summarized in the probability table below:

Claim Paid?	Probability	X: Amount gained by insurance
Yes	0.10	-$1,000
No	0.90	+$500

How much can the company expect to make per customer?

EXAMPLE 3: NUMBER OF CARS OF AMERICAN HOUSEHOLDS

Census data give us information about the number of cars owned by American households. Let the random variable X be the number of cars (including SUVs and light trucks) owned by an randomly selected American household, the probability model is

X: number of cars	**0**	**1**	**2**	**3**	**4**	**5 or more**
Probability	0.09	0.36	0.35	0.13	0.05	0.02

a) Verify that this is a legitimate discrete distribution. Display the distribution in a probability histogram.
b) A housing company builds houses with two-car garages. What percent of households have more cars than the garage can hold?
c) Compute the average number of cars per American households (use the value X=6 for the outcome "5 or more").

EXAMPLE 4: Every day the number of network blackouts has the following distribution

Number of network blackouts	**0**	**1**	**2**
Probability	0.7	0.2	0.1

A small internet trading company estimates that each network blackout results in a $500 loss. Compute the average daily loss due to blackouts.

EXAMPLE 5: A game is fair if the mean net gain equals 0. This means that on average players neither win nor lose. In the game Keno, there are 80 balls, numbered 1 to 80. On each play, the casino chooses 20 balls at random. Suppose you bet $1 on 17 in each Keno play. When you win, the casino gives you your dollar back and 2 dollars more. When you lose, the casino keeps your dollar. So the bet pays 3 to 1.

Outcome	**X: Player's Payoff**	**Probability**
Win: 17 is among the 20 balls	$3	20/80 = 0.25
Loss: 17 is not among the 20 balls	$-1	60/80 = 0.75

Is the bet fair?

EXAMPLE 6: An American roulette wheel has 38 slots with the numbers 0, 00, (in green) and 1 through 36 (evenly distributed with the colors red and black). The winning number is drawn between 1 and 36. The roulette table where bets are played is pictured in the figure below.

There are many possible bets that players can place. Let's consider the following three bets:

Straight bet: player bets on a single number in (1, 2,…,36) – the bet pays 35 to 1.

Color bet: player bets either on red or black – bet pays 1 to 1 and always loses if 0 or 00 comes up

4-number bet: player bets on four numbers in a square, for instance on the top square {1,2,4,5} – the bet pays 8 to 1.

a. Which bet has the highest mean payoff? (*HINT: define the random variable X=bet payoff and create the probability table of X for each bet. The mean of X is the mean payoff)*
b. Which bet is associated to the highest risk, in terms of largest variation of wins and losses?