Finitely Almost Split Sequences for Comodules

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Abstract.

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1. Introduction

We first show that the almost split sequences whose existence is guaranteed in [CKQ] are direct limits of almost split sequences. We then show that for countable-dimensional coalgebras, certain exact sequences exist which satisfy a condition weaker than being almost split. Under additional assumptions these sequences are almost split.

1.1. Notation. Let Γ denote a coalgebra. We can write Γ = ∪Γi as the directed union of finite-dimensional subcoalgebras Γi indexed by the set I. Further

MΓ denotes the category of right Γ-comodules
MΓf denotes the category of finite-dimensional right Γ-comodules
MΓqf denotes the category of quasifinite right Γ-comodules
MΓqc denotes the category of quasifinitely copresented right Γ-comodules
R denotes the dual convolution algebra Homk(Γ, k)
M denotes the category of right R-modules

We will also use the obvious left-handed variants of these notations.

2. local theory of almost split sequences

2.1. Almost split sequences as direct limits. Using the characterization of injectives in MΓ (e.g. [Gr]) as direct summands of direct sums of Γ, we note the well-known

Lemma 1. Let I ∈ MΓ be injective; then I□ΓΛ is injective in MΛ for any subcoalgebra Λ ⊂ Γ.

Let Λ be a subcoalgebra of Γ. We let TrΛ denote the transpose in MΛ (and as always TrΓ=TrΓ).

The next Lemma is probably folklore.

Lemma 2. Let 0 → A → B → C → 0 be an exact sequence of Γ-comodules. Then cf(B) ⊂ cf(C) ∧ cf(A).
Proof. By [Sw 9.0.0] (and using notation from that book), \((\text{cf}(A)^\perp \text{cf}(C)^\perp)^\perp = \text{cf}(A) \wedge \text{cf}(C)\). Clearly \(\text{cf}(A)^\perp \text{cf}(C)^\perp \subset \text{ann}_R(B)\) (= the annihilator of \(B\) in \(R\)). Thus (with \(\perp\) contextually in \(R\) or \(\Gamma\))

\(\text{(cf}(A)^\perp \text{cf}(C)^\perp)^\perp \supset \text{ann}_R(B)^\perp\)

The proof is completed by noting that \(\text{ann}_R(B)^\perp \supset \text{cf}(B)\). To see this suppose to the contrary that there are elements \(y \in \text{cf}(B)\) and \(r \in \text{ann}_R(B)\) with \(< r, y > \neq 0\).

There exists a basis \(\{\gamma_i\}\) of \(\text{cf}(B)\) dual to \(r\) in the sense that \(< r, \gamma_i > = 0\) if \(i > 0\) and \(< r, \gamma_0 > = 0\). By definition of coefficient space there exists \(b \in B\) such that \(\rho(b) = \sum b_i \otimes \gamma_i\) with \(b_0 \neq 0\). Therefore we obtain the contradiction \(r \rightarrow b \neq 0\) \(\square\)

Lemma 3. Let \(M \in \mathcal{M}_{\text{qc}}^D\) be finite-dimensional. Then there is a finite-dimensional subcoalgebra \(\Lambda_M \subset \Gamma\) such that for every finite-dimensional subcoalgebra \(\Lambda \supset \Lambda_M\),

\[\Lambda \boxtimes \text{Tr} M \cong \text{Tr}_\Lambda M.\]

Proof. Let \(0 \rightarrow M \rightarrow I_0 \rightarrow I_1\) be a minimal injective copresentation of \(M\) in \(\mathcal{M}_{\text{qc}}^D\) so that \(I_0 = I(M)\) and \(I_1 = I(I(M)/M)\). Then for every subcoalgebra \(\Lambda \subset \Gamma\)

\[0 \rightarrow M \rightarrow I_0 \square \Lambda \rightarrow I_1 \square \Lambda\]

is an injective copresentation for \(M\) in \(\mathcal{M}_{\text{qc}}^\Lambda\).

We show next that for large enough \(\Lambda\), this copresentation is minimal.

Let \(\Lambda_M = \text{cf}(M) \wedge \text{corad}(\text{cf}(M))\) and let \(\Lambda\) denote a finite-dimensional subcoalgebra of \(\Gamma\) containing \(\Lambda_M\). Let \(M'\) be the submodule of \(I_0\) containing \(M\) such that

\[M'/M = \text{soc}(I_0 \square \Lambda) / M = \text{soc}(I_0 \square \Lambda) = I_0 \square \Lambda\]

Note that \(M'\) is an extension of \(M\) by simple comodules whose coefficients lie in \(\text{corad}(\text{cf}(M))\); hence \(M' \subset I_0 \square \Lambda\). Since \(M\) and \(\Lambda\) are finite-dimensional and \(M\) is quasifinitely copresented, we deduce that \(M'\) is finite-dimensional.

Since \(\text{cf}(M) \subset \Lambda\), it clear firstly that

\[\text{soc}(I_0 \square \Lambda) = \text{soc}(I_0) = \text{soc}(M)\]

The definitions of \(\Lambda_M \subset \Lambda\) and \(M'\) imply secondly that

\[\text{soc}(I_0 \square \Lambda) / M = M'/M = \text{soc}(I_0 \square \Lambda) = I_0 \square \Lambda\]

Thus we have a minimal copresentation

\[0 \rightarrow M \rightarrow I_0 \square \Lambda \rightarrow I_1 \square \Lambda\]

for \(M\) in \(\mathcal{M}_{\text{qc}}^\Lambda\). Thus we obtain

\[0 \rightarrow \text{Tr}_\Lambda M \rightarrow h_{-\Lambda}(M, I_1 \square \Lambda) \rightarrow h_{-\Lambda}(M, I_0 \square \Lambda)\]

On the other hand, the defining copresentation

\[0 \rightarrow \text{Tr} M \rightarrow h_{-\Gamma}(M, I_1) \rightarrow h_{-\Gamma}(M, I_0)\]

can be cotensored with \(\Lambda\). In view of \([\text{Tak}, 1.14(?)\], which says that \(\Lambda \square h_{-\Gamma}(M, \Gamma) \cong h_{-\Lambda}(M, \Lambda)\), the result is established. \(\square\)

We see next that the transpose commutes with certain direct limits: Let \(\text{tr}_{DA}\) and \(\text{Tr}_\Lambda\) denote the transposes in the categories \(DA\) and \(\mathcal{M}_{\text{qc}}^\Lambda\) (\(\Lambda\) finite-dimensional), respectively. Since cotensoring commutes with direct limits and is left exact, it follows from the preceding lemma that
Proposition 1. Suppose that $M \in \mathcal{M}^\Gamma_{qc}$ indecomposable and not injective. Then

$$\lim_{\mathcal{L}} \text{Tr}_\Lambda M = \text{Tr} M$$

for all finite-dimensional indecomposable $M \in \mathcal{M}^\Gamma_{fd}$ for some direct system $\mathcal{L}$ of finite dimensional subcoalgebras of $\Gamma$ depending on $M$.

The preceding result gives a way of constructing indecomposables which are unions of chains of indecomposable subcomodules.

We show next that the almost split sequences (e.g. those whose existence is provided by [CKQ]) are direct limits of almost split sequences over finite dimensional subcoalgebras.

Proposition 2. Suppose that $C \in \mathcal{M}^\Gamma_{qc}$ is finite-dimensional. Let

$$\delta : 0 \rightarrow A \rightarrow B \rightarrow C \rightarrow 0$$

be an almost split sequence in $\mathcal{M}^\Gamma$. Then $\delta$ is the direct limit of almost split sequences $(\delta \Box \Lambda)_{\Lambda \in \mathcal{L}}$ for some direct system $\mathcal{L}$ of finite dimensional subcoalgebras of $\Gamma$.

Proof. Let $\Lambda_C$ be any finite-dimensional coalgebra of $\Gamma$ containing both the coefficient space of $C$, and a finite dimensional subspace of $B$ mapping onto $C$. It is clear that the sequence $\delta \Box \Lambda : 0 \rightarrow A \Box \Lambda \rightarrow B \Box \Lambda \rightarrow C \Box \Lambda = C \rightarrow 0$ is exact (onto $C$) in $\mathcal{M}^\Lambda$ for any finite-dimensional subcoalgebra $\Lambda$ provided $\Lambda_C \subseteq \Lambda$. It is straightforward to check that $\delta \Box \Lambda$ is right almost split.

The direct system $\mathcal{L}$ of subcoalgebras is taken to be the finite-dimensional subcoalgebras that contain $\text{cf}(M \wedge \text{corad}(\text{cf}(M))) + \Lambda_C$ as in the proof of the Lemma above with $M = DC$ (note: $\text{cf}(C) = \text{cf}(DC)$). The condition on $\Lambda \in \mathcal{L}$ guarantees by the Lemma that $A \Box \Lambda = \text{Tr}_\Lambda D(C)$, and $\text{Tr}_\Lambda D(C)$ is indecomposable by [CKQ, 3.2]. This is the left-hand term of the right almost split sequence $\delta \Box \Lambda$, so by [?] $\delta \Box \Lambda$ is in fact an almost split sequence in $\mathcal{M}^\Lambda$. □

3. Finitely almost split sequences

3.1. The Mittag-Leffler condition. In an abelian category, the direct limit (filtered colimit) is an exact functor but inverse limits are left exact but not always right exact. The Mittag-Leffler condition [Gro] (see also [Ha, p.119]) guarantees that inverse limits of certain inverse systems of exact sequences are exact. It has the following special case: Let

$$d_i : 0 \rightarrow M_i \rightarrow E_i \rightarrow N_i \rightarrow 0$$

be an exact sequence of inverse limits of vector spaces where the index set has a countable cofinal subset. If each $M_i$ is finite-dimensional, then the Mittag-Leffler condition holds; hence

$$\lim_{\leftarrow} d_i = d : 0 \rightarrow M \rightarrow E \rightarrow N \rightarrow 0$$

is exact.

Assume for the moment that $\Gamma$ is a finite-dimensional, so that $\mathcal{M}^\Gamma_{fd} = \text{mod } R$. Let tr denote the usual transpose on $\text{mod } R$, as in the representation theory of finite dimensional algebras (defined using projective resolutions, see [ARS, §IV]).
It is easy to see from duality that for finite-dimensional comodules that $D \text{Tr} D = \text{tr}$, so we can say $D \text{tr} = \text{Tr} D$ and $\text{tr} D = D \text{Tr}$.

**3.2. The prorational category**. Define $D : \mathcal{M}_q^\Gamma \to \mathcal{M}_q^\Gamma$ to be the functor $D$ with range? being the full subcategory of $\mathcal{M}_R$ having objects $DM, M \in \mathcal{M}_qf$. We show below that $\mathcal{D}M^\Gamma$ is the dual category of the category of quasifinite comodules.

If $\rho : M \to M \otimes \Gamma$ is the structure map of $M$, then by restricting to $DM \otimes R \subset D(M \otimes R)$ and abusing notation, we obtain the map

$$D \rho : DM \otimes R \to DM$$

It is straightforward to check that this coincides with the rational right $R$-module structure on $DM$ which arises from the left $\Gamma$-$\text{comodule structure}$ on $DM$, which in turn comes from $\rho$.

Note that the finite dimensional objects in $\mathcal{D}M^\Gamma$ are precisely the finite dimensional rational right $R$-modules.

In the following Lemma, the tensor product is dual to the cotensor product.

**Lemma 4.** Let $M \in \mathcal{M}_q^\Gamma$ and $N \in \mathcal{M}_q^\Gamma$ and assume that $M \Box_\Gamma N$ is finite dimensional. Then $D(M \Box_\Gamma N)$ is isomorphic to $DM \otimes DR \otimes DN$.

**Proof.** Let $\rho : M \to M \otimes \Gamma$ and $\lambda : N \to \Gamma \otimes N$ be the structure maps of $M$ and $N$ respectively. Then as is noted above, $DM$ is a left comodule and is a (rational) right $R$-module. Similarly $DN$ is a left $R$-module.

The cotensor $M \Box_\Gamma N$ is defined by the usual equalizer

$$M \Box_\Gamma N \to M \otimes N \xrightarrow{\rho \otimes 1} M \otimes \Gamma \otimes N.$$

Dualizing, we have the coequalizer

$$D(M \otimes \Gamma \otimes N) \xrightarrow{D(\rho \otimes 1)} D(M \otimes N) \xrightarrow{\rho \otimes \lambda} D(M \Box_\Gamma N) \to 0.$$

By hypothesis $M \Box_\Gamma N$ is finite dimensional, so the density of $D(M) \otimes D(N)$ in $D(M \otimes N)$ implies that the restriction $DM \otimes DN \to D(M \Box_\Gamma N)$ is onto. The kernel of this map is $\ker p \cap (DM \otimes DN)$. Thus we have the coequalizer

$$DM \otimes DG \otimes DN \xrightarrow{D(\rho \otimes 1)} DM \otimes DN \xrightarrow{\rho \otimes \lambda} D(M \Box_\Gamma N).$$

This finishes the proof of the Lemma.

If $\Gamma$ is of countable dimension, we shall write $\Gamma = \cup \Gamma_i$ as the ascending union of a chain of finite-dimensional subcoalgebras $\Gamma_i$.

Let $R_i$ denote the finite-dimensional algebra $DG_i$; it is immediate that $R_i$ is isomorphic to $DG_i / \Gamma_i^\perp$, where $\Gamma_i^\perp$ is the ideal of functionals in $R$ vanishing on $\Gamma_i$. The Lemma shows that

**Proposition 3.** Let $\Gamma$ be of countable dimension. Let $M_i$ denote $M \Box_\Gamma \Gamma_i$, $M \in \mathcal{M}_qf$. Then

$$D(M_i) \cong DM \otimes R_i \cong DM / (ME_i^\perp).$$

It is now elementary that $DM \cong \lim \to D(M_i)$. 

Definition 1. We say that $DM \in D\mathcal{M}_\Gamma$ is coquasifinite if $\text{Hom}_R(DM, F)$ is finite-dimensional for all finite-dimensional $F \in D\mathcal{M}_\Gamma$.

The dual notion “quasifinite” was introduced in [Tak].

Lemma 5. (a) $D:\mathcal{M}_qf^\Gamma \to D\mathcal{M}_\Gamma$ is a duality.
(b) $D\mathcal{M}$ is coquasifinite for all $M \in \mathcal{M}_qf^\Gamma$.

Proof. We check that $D$ is full, faithful and dense. The functor $D$ is dense by definition. Suppose $g \in \text{Hom}_R(DN, DM)$. We set $f = \lim\rightarrow Dg_i$ where $g_i = g \otimes R_i$. It is easy to check that $Df = g$, so that $D$ is full. Similarly $D$ is faithful.

A functor $D':D\mathcal{M}_qf^\Gamma \to \mathcal{M}_qf^\Gamma$ defined by setting $D'M = M$, giving an obvious correspondence on objects. Suppose $f \in \text{Hom}_R(DN, DM)$. We put $D'f = \lim\rightarrow Df_i$ where $f_i = f \otimes R_i$. The functor $D'$ is inverse equivalence for $D$.

(b) follows from the duality. □

3.3. finitely almost split sequences.

Definition 2. We say that a nonsplit exact sequence of objects $0 \to A \to B \to C \to 0$ in $R\mathcal{M}$ is right finitely almost split if
1. $C$ is indecomposable
2. every morphism $X \to C$ with $X \in \mathcal{M}_{fd}^\Gamma$, which is not a split epimorphism, lifts to $B$.

Dually, the sequence is said to be left finitely almost split if $A$ is indecomposable if every morphism $A \to X$ with $X \in \mathcal{M}_{fd}^\Gamma$, which is not a split monomorphism, extends to $B$.

These definitions extend the definition of finitely almost split sequences given for “local nests of quivers” given in [GM], where they are called “special sequences”. Further extending this work we have

Theorem 1. Let $\Gamma$ be a countable-dimensional coalgebra. If $C \in \mathcal{M}_{fd}^\Gamma$ is non-projective and indecomposable, then there exists a right finitely almost split sequence $d : 0 \to A \to B \to C \to 0$ in $\mathcal{M}_{qf}^\Gamma$. If $DC$ is quasifinitely copresented, then $d$ is an almost split sequence in $\mathcal{M}_{qf}^\Gamma$.

Proof. We can write $\Gamma = \cup \Gamma_i$ as the ascending union of a chain of finite-dimensional subcoalgebras $\Gamma_i$ where we may assume that $C \cap \Gamma_0 = C_i$.

It is easy to check that $C$ is not projective in $\mathcal{M}_{fd}^{\Lambda_i}$. By results for finite-dimensional algebras, there exists an almost split sequence $d_i : 0 \to A_i \to B_i \to C \to 0$ in $\mathcal{M}_{fd}^{\Gamma_i}$ with $A_i = \text{Dtr}_i C = \text{Tr}_i DC$. We obtain exact sequences indexed by $\mathbb{N}$, along with maps $d_i \to d_{i+1}$ obtained by diagram chasing [Mark?] using the fact that $d_{i+1}$ is almost split. Furthermore, these maps are monomorphisms on $A_i$ and $B_i$, and are the identity on the right hand term $C$ (as in [ARS, V, 3.2.2(b)]). Thus we obtain a direct system of exact sequences $(d_i)_{i \in \mathbb{N}}$ with monomorphic maps, each being the identity on $C$. 
The direct limit is exact sequence
\[ d : 0 \to A \to B \to C \to 0. \]
We show that \( d \) is left finitely almost split. The sequence is not split, for otherwise a splitting map \( C \to B \) would have its image in \( B_i \) for some \( i \) (since \( C \) is finite-dimensional). This would then be a splitting of \( d_i \), a contradiction.

Let \( X \in \mathcal{M}_{fd}^{\Gamma} \), so that \( X \in \mathcal{M}_{i}^{\Gamma} \) for some \( i \). Then any morphism \( X \to C \), which is not a split epimorphism, lifts to \( B_i \). Composing with the map \( B_i \to B \), we see that we have the required lifting. This shows that \( d \) is right finitely almost split.

Now assume as in the statement that \( D \mathcal{C} \) is quasifinitely copresented. To show that \( d \) is right almost split in \( \mathcal{D} \mathcal{M}^{\Gamma} \), it suffices to show that \( \text{Hom}(X, d) \) is exact for all \( X \in \mathcal{M}_{qf}^{\Gamma} \). Let \( X_i = X \square \Gamma_i \), which is a finite-dimensional comodule (since \( X \) is quasifinite). By the Mittag-Leffler condition, it suffices to show \( \text{Hom}(X_i, A) \) is finite-dimensional for all \( i \). This holds because \( A = \lim_{\to} A_i = \lim_{\to} \text{Tr}_i DC = \text{Tr} DC \) (by Lemma?) and an application of [CKQ, Lemma 3.2] which asserts that \( \text{Tr} DC \) is quasifinite.

By duality, \( DC \) in indecomposable and noninjective. By [CKQ, 3.2] and Lemma 2.7 above, \( A = \text{Tr} DC \) is also indecomposable noninjective. By [?], \( d \) is an almost split sequence.

This completes the proof of the Theorem. \( \square \)

The argument dualizes straightforwardly: \( \text{(Put in proof?)} \)

**Theorem 2.** Let \( \Gamma \) be a countable-dimensional coalgebra. Let \( A \in \mathcal{M}_{fd}^{\Gamma} \). If \( A \) is noninjective and indecomposable, then there exists a left finitely almost split sequence
\[ d : 0 \to A \to B \to C \to 0 \]
in \( \mathcal{DM}^{\Gamma} \). If \( A \) is quasifinitely copresented, then \( d \) is an almost split sequence in \( \mathcal{DM}^{\Gamma} \) with \( C \) indecomposable nonprojective.

**Uniqueness for fass’s?** [GM] proof???

**Remark 1.** The uniqueness of almost split sequences gives a way of showing that the category of finite-dimensional comodules does not have almost split sequences. Let \( C \) be a finite-dimensional comodule such that \( \text{Tr} DC \) is infinite-dimensional. Therefore, there is no almost split sequence in \( \mathcal{M}_{fd}^{\Gamma} \) ending in \( C \). Dually, if \( \text{Tr} A \) is infinite-dimensional, then there is no almost split sequence starting with \( A \) in \( \mathcal{M}_{fd}^{\Gamma} \). An example of this type is given below.

**4. Examples**

The coalgebras in the next two examples are neither right nor left semiperfect, and finite-dimensional indecomposables have finite-dimensional transposes. The almost split sequences are almost split in the category of finite-dimensional comodules as predicted by [SN]. In these examples, the almost split sequence obtained is the same as one obtained over a finite-dimensional subcoalgebra (i.e. over a finite subquiver).
In contrast, the third example shows that a simple comodule can have an infinite-dimensional transpose. Therefore the category of finite-dimensional comodules does not have almost split sequences. Comparing with the result in [SN] for quivers of type $D_\infty$ illustrates why the orientation there is important.

1. Let $Q$ be the quiver of type $A_\infty^\infty$ with vertices labeled by the integers and arrows $a_i : i \to i + 1$, $i \in \mathbb{Z}$. We identify vertices with the corresponding simple in $\mathcal{M}^{kQ}$.

By the theory of Nakayama algebras, the isomorphism classes of finite-dimensional indecomposable comodules are given by the representations $V_i = (V, f)_{ij}$ defined by $V_i = k$ for $i \leq t \leq k$, zero otherwise ($i \leq j$): the linear maps are $f_t : V_i \to V_{i+1}$ given by $f_t = (1)$ for $i \leq t < j$ and zero otherwise. We compute injective hulls $I(V_{ij}) = I(j)$, $I(V_{ij})/V_{ij} = I(i - 1)$. Thus

$$0 \to \operatorname{Tr}((V_{ij}) \to I(i - 1)^* \to I(j - 1)^*$$

is a copresentation yielding $D\operatorname{Tr}(V_{ij}) = V_{i-1,j-1}$. The almost split sequences are

$$0 \to V_{i,j} \to V_{i-1,j} \oplus V_{i,j-1} \to V_{i-1,j-1} \to 0$$

with irreducible maps $V_{i,j} \to V_{i,j+1}$ and $V_{i-1,j} \to V_{i,j}$ being the obvious monomorphism into the first summand and epimorphism onto the second summand. For the map on the right, we use the natural epi and mono, and alternate signs.

2. Similarly, let $Q$ denote the quiver with one vertex and one loop. Then there is a unique finite-dimensional indecomposable right comodule $V_n$ of length dimension $n \geq 0$. It is straightforward to see that $D\operatorname{Tr}V_n = V_n$ and that the almost split sequences are given by

$$0 \to V_n \to V_{n-1} \oplus V_{n+1} \to V_n \to 0$$

(just as in [ARS, p. 141]).

3. Let $Q$ be the quiver of type $D_\infty$ with vertices labeled by positive integers and two special vertices $0, 0'$. The arrows are defined to be $a_i : i \to i + 1$, $i \in \mathbb{Z}^+$, and $a_0 : 0 \to 1$, $a_{0'} : 0' \to 1$. We identify vertices with the corresponding simple in $\mathcal{M}^{kQ}$.

We compute the transpose of the simple right noninjective comodule $1$.

$I(1)$ is given by the representation $V = (V, f)$ defined by $V_i = k$ for $t = 1, 0, 0'$ and zero otherwise, with linear maps are $f_t : V_{i+1} \to V_i$ given by $f_t = (1)$ for $t = 0, 0'$. Thus $I(1)/1$ is $0 \oplus 0'$, the direct sum of two simple injectives.

Next observe that $I(0 \oplus 0')^* = I(0)^* \oplus I(0')^*$ is given by the representation $V = (V, f)$ with by $V_t = k$ for $t = 0, 0'$, $V_t = k^2$ for $t > 0$, and zero otherwise. We leave the maps to the reader.

$I(1)^*$ is given by the representation $V = (V, f)$ with by $V_t = 0$ for $t = 0, 0'$, $V_t = k$ for $t > 0$, and zero otherwise.

Finally, $0 \to \operatorname{Tr}(1) \to I(0 \oplus 0')^* \to I(1)^*$ is a copresentation yielding $\operatorname{Tr}(1) = V$ where the representation $V = (V, f)$ of $Q_{\infty}$ is defined by $V_i = k$ for all vertices $i$, with linear maps are $f_t : V_{i+1} \to V_i$ given by $f_t = (1)$ for all $t$. We know that $V$ is an infinite-dimensional indecomposable left comodule; thus $\mathcal{M}_{\ell d}$ does not have an almost split sequence starting at $1$. The almost split sequence in $D\mathcal{M}^{kQ}$ given by the theorem is of the form

$$0 \to 1 \to B \to DV \to 0$$
References


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