Geometric Thinking and Learning—van Hiele Levels

1. Geometric learning has discontinuities; there are jumps in the process.
   - There are five identifiable levels of geometric thinking (although the jumps in learning from the third to the fourth to the fifth level are less obvious).
   - Each level is characterized both by its **objects of thought** (content) and its **forms of reasoning**.
   - The objects of thought which are implicit at one level become the explicit focus of attention at the next level.
   - Students are likely to use different levels of geometric thinking depending on the concept or topic.

2. The levels are hierarchical and sequential; students must progress through them in order.
   - For the most part, geometric development is not the result of maturation or of experience with other content.
   - Movement from one level to another is largely a function of appropriate geometric experience with geometric objects representing specific content.
   - A student cannot function adequately at one level without having had experiences which allow intuitive functioning at the preceding level.
   - If one level is not mastered before instruction moves on, good students may appear to function at the higher level but only algorithmically. They may get right answers but not for good reasons.

3. At each level, it is important to focus on students' language and reasoning patterns.
   - Each level has its own language which may or may not exactly correspond to appropriate language at higher levels.
   - Each level has its own reasoning patterns and its own set of relations between terms which may or may not be appropriate at higher levels.
   - Instruction must be within the students' "zone of proximal development." If it is at a higher level than students' language and thought processes, they will not benefit.

4. The van Hieles did not stop with a characterization of levels of development of geometric thought. They also proposed a sequence of instruction for any level:

   Inquiry—Conversation, activity, and observation focused on the objects being studied at this level. The outcome should be specific observations, questions, and an emerging vocabulary for talking about geometry.

   Directed Orientation—Exploration using carefully sequenced materials for short tasks which gradually reveal the structures characteristic at this level.

   Explication—Discussion by students of their emerging understanding with minimal participation by the teacher.

   Free Orientation—Exploration of more complex and open-ended tasks.

   Integration—Reviews and summaries, formulated by students and the teacher, of what has been learned with (perhaps) some questions to be pursued at a higher level.

Van Hiele Levels of Geometric Development

First Level (Level 0)	**Visualization/Recognition**
Content	***Geometric objects***, including simple shapes (such as triangles, squares, rectangles, line designs, parallelograms, rhombi, polygons, or circles) and patterns constructed from these shapes
Form of Reasoning	Visual/tactile/sensory manipulation, observation and comparison of whole objects in a global (gestalt) manner
Outcome	Students recognize, name, and compare shapes and patterns.
Second Level (Level 1)	**Description/Analysis**
Content	***Properties*** of classes of geometric shapes and patterns
Form of Reasoning	Empirical description through observation, manipulation, construction and measurement of the parts and attributes of geometric shapes and patterns
Outcome	Students can characterize classes of geometric objects by describing their necessary properties.
Third Level (Level 2)	**Informal Deduction**
Content	***Relationships*** among the properties of classes of geometric objects
Form of Reasoning	Logical connection and ordering of (previously discovered) properties into short deductive chains (although fundamental properties still may be established empirically or intuitively)
Outcome	Students can form abstract definitions, distinguish between necessary and sufficient conditions, understand and sometimes invent logical arguments.
Fourth Level (Level 3)	**Formal Deduction**
Content	A ***formal system*** of relationships among properties of objects which usually includes an underlying system of logic, undefined terms, definitions, postulates, theorems and their proofs
Form of Reasoning	Formal logical analysis of relationships among propositions (statements about properties of objects) in the geometric system
Outcome	Students learn the system's traditional organization; classical results and their logical necessity; solve problems; and construct proofs.
Fifth Level (Level 4)	**Rigor**
Content	***Various*** (two or more) formal geometric systems
Form of Reasoning	Rigorous analysis and comparison of the axiomatic foundations of various organizations of geometry (e.g., the independence of the parallel postulate) or various geometries (e.g., Euclidean, non-Euclidean, affine, projective, etc.)
Outcome	Students can reason formally about different axiomatic systems.