Using aerial photography from both the students’ own neighborhood and from around the world, we presented students with an appealing, novel setting for mathematical exploration. The allure of these images kept students grounded in the context of the problem as they worked with the concept of map scale and proportional reasoning to analyze the relationship between the image size and the actual ground size of familiar objects. Our focus was on proportional reasoning with map scale, a critical part of Element One, Standard 1 of the National Geography Standards (NGS 1994).

Map scale expresses the relationship between the size of an object found on a map and its actual size on the ground. It can be expressed as a verbal scale (1 inch on the map represents 1 mile on the ground); as a representative ratio 1:63,360, or 1/36,360; or as a graphic scale (or scale bar), in which a designated length on a map is labeled with its associated ground distance (see fig. 1).

Map activities that involve proportional reasoning often provide students with a map and an accompanying scale that they use to find actual ground distances compared with distances on the map. There are indications that by the age of five years, children are able to relate, or scale up, the position of an object on a map of a familiar area to its corresponding location in that familiar area (Vasilyeva and Huttenlocher 2004).

However, Weinberg (2001) reports that only 23 percent of middle school students \((n = 387)\) could properly calculate the distance between two cities on a map that were 2 cm apart, when 5 cm represented 9 miles. This performance gap between students’ early intuition of distances and their later difficulties with calculations using map scale may result from reliance on an overly procedural approach using proportions, with too little attention directed to conceptually understanding why a solution works.

Principles and Standards for School Mathematics (2000) states that “Facility with proportionality involves much more than setting two ratios equal and solving for a missing term. It involves recognizing quantities that are related proportionally and using numbers, tables, graphs, and equations
Proportion, and Google™ Earth
to think about the quantities and their relationship” (p. 217).

Research on proportional reasoning (Miller and Fey 2000; Rachlin et al. 2006; Weinberg 2002) has called on teachers to focus students’ attention on the context of the problem so that they can develop a better understanding of the relationship between values in a proportion.

During 2008–2009 we worked with twenty-seven algebra 1/data analysis students in an urban school outside Washington, D.C. Our extracurricular program, the Geomatics Academy, was designed to help strengthen students’ mathematical reasoning. Our approach has been to pair mathematical concepts from the algebra 1/data analysis curriculum with interesting applications from geography. We used map scale and distance to introduce the more advanced topics of Cartesian coordinates and linear equations; however, the topic of proportionality is broadly applicable to a wide range of ages and is especially appropriate in the middle school classroom.

Our activity presents a constructivist approach to understanding map scale and proportional reasoning. We first engaged students with a series of authentic aerial images of their own familiar surroundings shown over a range of map scales. Each aerial photograph was the equivalent of a map, but without a scale. As students compared images, they recognized a need to express the map scale for each image.

Later, when students were asked to find the ground distance between two objects seen in a photograph, many students expressed the relationship of the image size to the ground size of the football field. They solved the problem by either constructing a proportion of equivalent ratios or measuring the distance through their invented “football-field rulers” whose units were the images’ map scale.

**AERIAL PHOTOGRAPHY**

Aerial photographs were first taken by Gaspar Felix Tournachon from a hot-air balloon in 1858 (Lillesand, Kiefer, and Chipman 2004, p. 59). By 1920, Sherman Fairchild had advanced the technology of aerial photography so that it became possible to survey entire cities. Today, some cities in the United States get remapped as often as four times each year.

Students are often fascinated by satellite imagery. Such images can bring a new dimension to mathematics instruction and generate enthusiasm for learning. Images from Google™ Earth are particularly well suited to investigating problems that involve relative distance and size.

To prepare for the first activity, we printed and displayed five large aerial photographs, in order of increasing map scale. Our first poster showed the entire Baltimore, Maryland–Washington, D.C. region; the second showed only the students’ school and two adjacent streets. The posters were printed in different sizes to dissociate the concept of map scale from the physical size of the paper.

Students oriented themselves to the images gradually and identified major features from a distance. Volunteers then approached the posters and pointed out minor features to the class. For example, students identified Baltimore; Washington, D.C.; and the Chesapeake Bay on the smallest-scale poster. On the largest-scale poster, they identified their school and playing fields and tried to identify the cars in the parking lot.

Royce enthusiastically announced, “Hey, that’s my house.” Tyrone observed that the school was included in all the posters, but that some posters were more blown up than others.” Other students described this quality as being zoomed in, or they talked about the size of the picture. However, several students disliked the term size, because it was easily confused with the physical dimensions of the posters. Once the students identified the concept of being zoomed in, the teacher introduced the term scale.

Addressing the students’ observation that the scale differed among the posters, the teacher explained that geographers refer to maps as large scale or small scale and then asked which posters could be identified as such. Kiara felt that the image of the entire Mid-Atlantic coast should be called a large-scale poster, since it was able to show such a large area. Latisha countered that it made more sense to refer to the photo of the school as the large-scale poster, since...
everything looked comparatively large. Two reasonable viewpoints had been expressed, leaving the class with a lack of consensus regarding the terms small scale and large scale.

THE CONCEPT OF MAP SCALE
To define large scale, students first needed to develop the meaning of map scale as a ratio, and then reason how to order the magnitudes of the map scales they found. Groups were assigned to study a poster and asked to find objects in the images whose real-life size they knew. Students pointed out baseball diamonds, basketball courts, and football fields, identifiable in all but the smallest-scale poster. Many could state the official length of a football field (100 yards, plus 10 yards for each end zone); we provided a key with the metric lengths of common ground objects (fig. 2).

Each group was asked to find and measure the image length of a football field (end line to end line) on their respective poster. Students were fascinated by how many football fields they could identify in some of the smallest-scale posters. Notably, these football fields were all the same size within a single poster, no matter where they looked. These sizes varied from too small to measure on the smallest-scale poster, to 281 mm on the largest-scale poster. Figure 3 shows images analogous to those used by our students.

Students were asked to consider how the image length of the football field compared with the true length of a football field. Each group recorded both the image length and the ground length of the field on the chalkboard below their poster, forming a representative fraction:

\[
\frac{\text{image length of football field}}{\text{ground length of football field}} = \frac{\text{image length of football field (mm)}}{110,000 \text{ mm}}
\]

Expressed with common denominators, the students quickly identified which fraction was largest. They realized that the five posters were ordered by increasing values of the representative fractions that they had just determined.

\[
\begin{align*}
\text{too small} & : \frac{2}{110,000} < \frac{8}{110,000} < \frac{14}{110,000} < \frac{281}{110,000} \\
\end{align*}
\]

The students were then asked to convert their representative fraction to a unit fraction, with the numerator equal to 1. Jasmine explained that she could reduce her fraction of 8/110,000 by dividing both numerator and denominator by 8, resulting in 1/13,750. Tyrone questioned his result of converting 14/110,000 to 1/7857.14, and...
his group agreed to round the resulting denominator to the nearest whole number, which became $\frac{1}{7857}$. Each group wrote its resulting representative unit fraction under the original fraction and took turns explaining what $1 \text{ mm}$ represented on the group poster. The teacher confirmed that the unit fraction that each group had found was the actual map scale for the group’s poster.

This was the original representative fraction:

\[
\frac{110,000}{2} < \frac{110,000}{8} < \frac{110,000}{14} < \frac{281}{110,000} < \frac{1}{110,000}
\]

This was the corresponding unit fraction:

\[
\frac{300,000}{1} < \frac{1}{55,000} < \frac{1}{13,750} < \frac{1}{7857} < \frac{1}{391}
\]

Students made comparisons of scales from poster to poster and noted that the smaller the scale (given as a representative unit fraction), the smaller the objects within the image. A few students struggled to compare the numerical values of the unit fractions, because smaller fractions had larger numbers in the denominator. Ramon reminded them that if you were to cut a pizza into 10 slices, each slice (1/10 of the pizza) would be larger than if you were to cut it into 100,000 pieces, with each slice equal to 1/100,000 of the pizza.

This exercise can be extended by having students repeat this activity using a tennis court instead of a football field to calculate their map scale. Students would discover that the resulting map scale for their poster would be the same, within measurement error, as was found using the football field.

EXTENDING MAP SCALE TO CREATE PROPORTIONS

Now that students had calculated and developed an understanding for the map scale of their poster, the next step was to apply this fraction to determine the real-life, or ground distance, between two objects on their poster. The students measured the distances from their school to Baltimore (approximately 50 km), and from the front door of their school to the home of one student, Royce, who lived less than 2 km away. Our discussion focused on their multiple methods to calculate the distance from school to home.

The majority of students used their understanding of the development of map scale (image length to ground length of a football field) to construct an equivalent ratio of image distance to ground distance, where the ground distance was expressed as an unknown $x$. Setting these two equivalent ratios equal to form a proportion, the students chose to cross-multiply and solve for the unknown $x$.

Knowing the ground length of a football field, other students constructed a ruler with image-size football fields as units and measured the number of image-size football fields found between objects. Multiplying the number of image-size fields by the ground length of a field, they found the ground distance between objects.

Students chose to calculate distance from school to home in a variety of ways. Some used their map scale of $1/13,750$ to reason multiplicatively that if 1 mm on the map equaled 13,750 mm on the ground, then 1 cm on the map had to equal 13,750 cm on the ground. These students recognized that the map scale was unitless. Using a ruler, they found the measured image distance from the school to Royce’s house on their poster to be 12 cm, then set up a proportion using the map scale:

\[
\text{map scale} = \frac{\text{image length}}{\text{ground distance}}
\]

\[
\frac{1}{13,750} = \frac{12 \text{ cm}}{x}
\]

They had already reasoned that 1 cm (on the map) = 13,750 cm (on the ground), so the 12 cm from the school to Royce’s house on the map...
represented a true distance of 12 cm \times 13,750 = 165,000 cm. Kiara explained that you can convert 165,000 cm to kilometers by “first dividing by 100 to get you to meters, and then dividing by 1000 to get you to kilometers,” resulting in the rounded value of 1.7 km.

Another group of students with the larger-scale (1/7857) poster used a variation of the previous method. Similar to the first group, they also used a ruler to measure the image distance between the school and Royce’s house, which was 204 mm on their poster. However, instead of using the unit fraction map scale, they felt more comfortable expressing the map scale using their original measurements, thereby preserving the context of the problem in their ratio expression of image length divided by ground length of a football field (14 mm/110,000 mm). Recognizing that the ratio between measured image distance and ground distance was the same for any distance (length of football field, distance between home and school), these students constructed an expression for a proportion that they solved through cross-multiplication.

\[
\frac{\text{image length of field}}{\text{ground length of field}} = \frac{\text{image distance to house}}{\text{ground distance to house}}
\]

There were several variations of this proportion. One member of the group chose to construct the alternative, but equivalent, proportion as this:

\[
\frac{\text{image distance to house}}{\text{image length of field}} = \frac{\text{ground distance to house}}{\text{ground length of field}}
\]

\[
\frac{204 \text{ mm}}{14 \text{ mm}} = \frac{x}{110,000 \text{ mm}}
\]

Weinberg (2002) explains that this expression relates to a size-change strategy, where the ratio 204/14 = 14.6 stretches the given amount (the ground length of a football field) by a factor of 14.6.

This stretching factor can be similarly viewed by yet another group’s method. Looking at the 1/7857 scale poster, this group marked the image length of their school’s football field repeatedly side by side on a piece of paper, making a ruler whose unit was their image’s football field length. They found that Royce’s house was about 15 football-field lengths away from the school and reasoned that if 1 football field were 110 m, then 15 fields would be 15 \times 110 = 1650 m, or 1.7 km.

After the groups worked to calculate the distance from Royce’s house to school, students shared their various methods with the entire class. They noted similarities between expressions of proportions, whether noting an equivalent rearrangement of the order of quantities or expressing the map scale as a unit fraction versus its original context-driven form. Kiara connected the cross-multiplication method to the football-field ruler, noting that the ratio

\[
\frac{\text{image distance to house}}{\text{image length of field}} = \frac{204 \text{ mm}}{14 \text{ mm}} = 14.6
\]

was relatively close to 15, the number of image-scale football fields that the football-field-ruler group had measured. Tyrone made an interesting point that the image distance that we measured was different for each poster, but as long as you used the map scale that went with the poster, you could set up a proportion, or create a football-field ruler to find the real distance on the ground. Objects and image lengths within a poster were proportional to its map scale.

**THE WORLD IMAGERY CHALLENGE**

Our next goal was to have students apply map scale within a proportion to explore new situations. For the activity titled the World Imagery Challenge, we asked students questions about a series of nineteen aerial photographs of famous places, interesting landforms, and patterned structures from around the world that had been printed using a wide variety of scales. Some of these questions required mathematics to answer, others were simply qualitative. Each image was printed from Google Earth on 8 1/2 in. \times 11 in. paper. In some cases the aerial photograph was supplemented with additional photographs of the object taken from the ground.

The aerial photographs were labeled A through S and hung on the walls of a long hallway. (See fig. 4 for some of the supplied images.) Students were given a set of questions, a ruler, and a calculator and were encouraged to move freely from image to image, solving problems in any order they wished. Solving the imagery puzzles generated enthusiastic interactions among the students as they eagerly shared their discoveries.

The aerial photographs fell into two categories, showing either (1) a recognizable landmark, such as the U.S. Capitol or the Eiffel Tower; or (2) a visually striking image, such as the dune fields of the Namib Desert or crops grown with pivot irrigation. The scale on each image was indicated in the lower left-hand corner, either through a scale bar produced automatically by Google Earth or
Fig. 4 Students viewed these images to try to discern what they could be.

(a) The U.S. Capitol, Washington, D.C.
Sources: © 2009 Google; Image © 2009 Sanborn; Image District of Columbia (DC GIS)

(b) Namib Desert, Namibia
Sources: © 2009 Google; Image © 2009 Digital Globe; © 2009 Cnes/SPOT Image

(c) Fields with pivot irrigation
Sources: © 2009 Google; Image © 2009 TerraMetrics

(d) Dallas/Fort Worth International Airport, Texas
Sources: © 2009 Google; Image USDA Farm Agency

(e) The Vatican, Rome, Italy
Sources: © 2009 Google; Image © 2009 DigitalGlobe

(f) Taj Mahal, Agra, India
Sources: © 2009 Google; Image © 2009 DigitalGlobe

(g) Victoria Falls, Zimbabwe
Sources: © 2009 Google; Image © 2009 DigitalGlobe

(h) Thames River, London, England
Sources: © 2009 Google; Image © 2009 Bluesky
through a representative fraction handwritten by the teacher. In the case of the scale bar, we provided no additional instruction. Instead, students recognized that the role of the scale bar was analogous to our use of the football fields; from the bar, they calculated the representative fraction. In most cases, the chosen scale and the unfamiliar bird’s-eye view rendered even the most familiar places, such as the Statue of Liberty, into interesting visual puzzles that needed to be solved. Knowing the scale of the image could make all the difference in recognizing the subject of the photo and being able to answer the associated questions.

The image in figure 4d, showing a complex geometric pattern of green and gray, and unrecognizable at first, proved to be especially captivating. Students were asked to answer: What is this place? What is it used for?

Shauntay used the scale bar and realized that the poster was printed at a much smaller scale than the other hallway posters and thus the image subject was actually quite large. This prompted her to look closely at the center of the photo, where a densely textured area turned out to be a large number of airplanes parked around terminals at the sprawling Dallas/Fort Worth International Airport. She excitedly announced that it is an airport. Similarly, another image (fig. 4e) showed an abstract pattern of dots that fit within the scale bar of only 60 m, guiding students to recognize that the dots were people in line at the Vatican to see the Sistine Chapel.

For the questions requiring calculations, students began to vary their approach on the basis of which technique would answer a particular question most efficiently. For the image of the Taj Mahal (fig. 4f), Darren and Kiara transferred the scale bar to a separate piece of paper, laid the paper against the Taj Mahal, and found it to be exactly two scale bars in length. They reasoned that the length of the Taj Mahal must be twice the length that the scale bar represents. The image of Victoria Falls on the Zambezi River (fig. 4g) was spread out across several taped-together pieces of paper and was over 1 m in length; however, the scale bar was only about 1 cm long. Conklin transferred the scale bar to a piece of paper and slowly counted the number of scale-bar lengths. Meanwhile, another
student walked up to the poster, measured the scale bar with his ruler, and then measured the length of the falls with a meterstick. He created a proportion equation from these values and had completed his calculations before Conklin had finished his counting. The proportional equations were necessary again in cases where the scale bar was larger than the object to be measured, such as the width of the Thames River in London (fig. 4h), or where the teacher obscured the scale bar and gave a representative fraction instead.

ADAPTING THE ACTIVITIES

Our activities can be easily adjusted to fit a variety of situations, but it is important to keep the students’ age in mind when choosing the scale of the imagery. Younger students will have a smaller range of experiences and be familiar with a smaller neighborhood around their homes. An aerial photograph of the school grounds makes an excellent large-scale image for all age groups, since this area will be familiar to all the students and because it best shows off the aerial photography. With luck, it will be possible for students to identify cars in the parking lot, bleachers near the athletic fields, and other familiar landmarks. If some of the students live close enough to walk to school, their proximity presents an excellent opportunity to use imagery that covers both the school and surrounding neighborhoods.

Unfortunately, once the scale of the imagery is reduced to include a larger region, it quickly exceeds students’ experience. Although teachers may be able to identify major streets in the region, younger students may only have the vaguest understanding of where their homes are located in relation to downtown, for example. It is possible to ease this difficulty by identifying the school as an initial point of reference in the smaller-scale images. From there, students will often be able to recognize major landmarks and will be able to relate the distances between these landmarks and their school to the amount of time they may have spent on a bus or in a car while visiting these locations.

We suggest limiting the size of the smallest-scale image to a region that students might know. For example, include familiar destinations such as a nearby city, an amusement park, or a beach or lake that can be easily identified in the images. For our smallest-scale image, we were able to cover a region 60 miles across that included Baltimore and Washington, D.C. Our students live just outside the Washington, D.C. boundary and had recently taken a field trip to Baltimore. However, once they identified Washington, D.C.; Baltimore; and the Chesapeake Bay, they had difficulty pointing out any other landmarks. Therefore, although it is possible to include a composite satellite image of the entire United States as one of the scale posters, we find it is difficult for students to relate their experiences to such a vast region. Furthermore, it will be impossible to measure football fields using map scales smaller than 1/100,000.

It is also possible to adjust the activities by simplifying the numbers used in the calculations. For example, relatively little error is introduced to the scale calculations by rounding the length of a football field up to 110,000 mm. However, it is more difficult to print posters that are exactly 1/10,000 scale or some other nice round number. First, although it is possible to precisely control the image scale in Google Earth (Google maps does not offer the same amount of precision), it is difficult to control the final printing size of the document. Second, even if you are able to print a map to a precise scale, the students’ measurements will be subject to measurement error. Thus, a student may measure the length of a football field as 10 mm despite all your efforts to produce a 1:10,000 scale image with 11 mm football fields. As a result, we strongly recommend using calculators in these exercises.

CONCLUDING REMARKS

Aerial imagery has a great capacity to engage and maintain student interest while providing a contextual setting to strengthen their ability to reason proportionally. Free, on-demand, high-resolution, large-scale aerial photography provides both a bird’s-eye view of our world and a new perspective on our own community. We challenged our students to master two concepts:

1. Map scale from geography
2. Proportional reasoning from mathematics

Taught together, across disciplines, these investigations enabled students to explore their world and understand the embedded mathematics.

One great strength of this activity was the experience of developing the map scale (physically measuring the image-size football field and relating it to its known size) later helped students understand the relationship between values as they constructed proportions or created football-field rulers to calculate distance between objects on the images. Students delighted in solving puzzles and recognizing landmarks as they experimented with multiple methods to find a solution.
(Ed. note: To find links to these and other images used in the World Imagery Challenge, visit the project’s Web site at http://www.towson.edu/math/WIC.)

REFERENCES

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