congruence theorems (ASA, SSS, SAA), the congruence of vertical angles, the existence of right angles, the possibility of angle and segment bisection, and many other propositions of Euclid, Book I. The details appear in Hilbert [21]. As a final example we give Hilbert's proof of the exterior angle theorem.

**Theorem B-17**

An exterior angle of a triangle is greater than either of the interior angles not adjacent to it.

**Proof.** In Figure B-5, choose \( D \) on \( BC \) extended so that \( AB = CD \). First, we shall show that \( \angle ACD \neq \angle BAC \). If \( \angle ACD \cong \angle BAC \) were true then \( \triangle BAC \cong \triangle DCA \) by Theorem B-16. But then \( \angle CAD \) would be congruent to the supplement of \( \angle ACD \) and hence congruent to the supplement of \( \angle BAC \). By III, 4, this in turn would imply that \( D \) lies on \( BA \), in violation of I, 2. Thus, \( \angle ACD \cong \angle BAC \) is impossible.

Next suppose \( \angle ACD < \angle BAC \) (Fig. B-6). Draw ray \( AB' \) on the \( B \) side of \( AC \) such that \( \angle B'AC \cong \angle ACD \). Then \( AB' \) passes through the interior of \( \angle BAC \) and so meets \( BC \) (Theorem III-17 in Section III-3) in a point, \( B' \). But then the exterior angle \( \angle ACD \) of \( \triangle B'AC \) is congruent to the interior angle \( \angle B'AC \). This was shown to be impossible in the first part of the proof.