line are called supplementary. An angle which is congruent to one of its supplementary angles is called a right angle.

The interior of an angle (or of a triangle) is defined in part d of Section III-3.

III,4. Let $\overline{\alpha}(h,k)$ be an angle [in a plane $\alpha$] and $a'$ a line [in a plane $\alpha'$] and let a definite side of $a'$ [in $\alpha'$] be given. Let $h'$ be a ray on the line $a'$ that emanates from the point $O'$. Then there exists [in the plane $\alpha'$] one and only one ray $k'$ such that the angle $\overline{\alpha}(h,k)$ is congruent or equal to the angle $\overline{\alpha}(h',k')$ and at the same time all interior points of the angle $\overline{\alpha}(h',k')$ lie on the given side of $a'$. Symbolically, $\overline{\alpha}(h,k) \cong \overline{\alpha}(h',k')$. Every angle is congruent to itself, i.e., $\overline{\alpha}(h,k) \cong \overline{\alpha}(h,k)$ is always true.

Following standard practice, if A and B are points (other than O) on rays h and k, respectively, we shall usually write $\overline{\alpha}AB$ or $\overline{\alpha}BOA$ rather than $\overline{\alpha}(h,k)$.

III,5. If for two triangles $\triangle ABC$ and $\triangle A'B'C'$ the congruences

$$\overline{AB} \cong \overline{A'B'}, \quad \overline{AC} \cong \overline{A'C'}, \quad \overline{BA}C \cong \overline{B'A'C'}$$

hold, then the congruence $\overline{\alpha}ABC \cong \overline{\alpha}A'B'C'$ is also satisfied.

Note: from an interchange of the letters B and C in III,5, it follows that under the same hypotheses $\overline{\alpha}ACB \cong \overline{\alpha}A'B'C$ holds also.

From now on, we shall write $\overline{AB} = \overline{CD}$ to indicate $\overline{AB} \cong \overline{CD}$. It is a consequence of Hilbert's postulates that real numbers (lengths) can be assigned to segments in such a way that segments are congruent if and only if they have the same length, and so that if the distance between two points A and B is defined to be the length of AB, then the usual properties of a distance function hold. (We shall not go into the details here.) In this text, we shall use $\overline{AB}$ to denote the distance from A to B. Thus $\overline{AB} = \overline{CD}$ indicates both congruence of segments and (equivalently) equality of distance.

Theorem B-13

The point $B'$ whose existence is asserted in Axiom III,1 is unique (for a given side of $A'$).

Proof. Suppose there were two (distinct) such points, $B'$ and $B''$. Choose any point C not on $A'B'$, and apply III,5 to $\triangle A'B'C'$ and $\triangle A'B''C'$ to show that $\overline{\alpha}A'CB' \cong \overline{\alpha}A'C'B''$. Then deduce a contradiction to the uniqueness asserted in III,4.