Abstracts

(in alphabetical order by speakers' last name)

E. Allgower

Turning Points and Bifurcations for Homotopies of Analytic Maps

Abstract:(Joint with Stefan-Gicu Cruceanu and Simon Tavener) Homotopy methods for finding all zero points of polynomial mappings have attracted a great deal of recent interest. Numerical continuation methods have made the computation of the solutions practical. Particular attention has been paid to means of avoiding the intersection or bifurcation of homotopy paths. By introducing appropriate perturbations into the homotopies, turning points and bifurcation points in the homotopy paths can generally be prevented, making the paths leading from starting solutions to end solutions proceed monotonically. We show that for analytic maps, simple turning points in homotopy paths are necessarily simple bifurcation points and the relationships between tangents and curvatures at the bifurcations can be determined. This then makes it possible to trace homotopy paths monotonically. Numerical examples are given in which all of the solutions of nonlinear elliptic boundary value problems having polynomial nonlinearities are approximated.

M. Beltrametti

A view on extending morphisms from ample divisors

Abstract: (Joint work with P. Ionescu) Let X be a smooth projective variety and let $Y \subset X$ be a smooth ample divisor on X. It is a natural classical question to try to understand how the structure of Y determines the one of X.

More precisely, given a surjective morphism $p = p_D : Y \to Z$ associated to a linear system $|D|$, we look for a linear system $|\overline{D}|$ on X defining a regular map $\overline{p} = p_{\overline{D}} : X \to W$ onto a projective variety W, such that the following diagram

(1)
$$\begin{array}{ccc} Y & \hookrightarrow & X \\ {\scriptstyle p}\downarrow & & \downarrow{\scriptstyle \overline{p}} \\ Z & \xrightarrow{\alpha} & W \end{array}$$

commutes. If the morphism $\alpha : Z \to \alpha(Z)$ is finite we say that $\overline{p}$ is a *lifting* of p. If $\overline{p}_{|Y} = p$, that is if $\alpha : Z \xrightarrow{\cong} \alpha(Z) \subset W$ is an isomorphism onto its image, we say that $\overline{p}$ is a *strict lifting* of p, or that p is *extendable* to $\overline{p}$. Note that this is always the case whenever the restriction map $H^0(X, \mathcal{O}_X(\overline{D})) \to H^0(Y, \mathcal{O}_Y(D))$ is surjective. Note also that this further condition is a posteriori satisfied in our setting.

Assume that the morphism p has a lifting $\overline{p}$. Up to taking the Remmert–Stein factorization, we can always assume that p has connected fibers. Therefore, by using the ampleness of Y in X, it is a standard fact that the following hold:

(1) If $\dim Y - \dim Z \geq 1$, then $\alpha : Z \xrightarrow{\cong} W$ (hence in particular p is extendable);
(2) If p and $\overline{p}$ are birational, then $\alpha : Z \xrightarrow{\cong} \alpha(Z)$.

Thus we conclude that *the only possible case when the lifting* $\overline{p}$ *is not strict is when* p *is a birational morphism and* $\dim X - \dim W = 1$. Note that this situation really occurs. E.g., consider a conic bundle $\overline{p} : X \to W$ with X embedded in a projective space $\mathbb{P}^N$.

For N large enough, we can choose by Bertini's theorem a hyperplane H in $\mathbb{P}^N$ such that the restriction Y of H to X is a smooth ample divisor. Then we get a commutative diagram

$$\begin{array}{ccc} Y & \hookrightarrow & X \\ {\scriptstyle p}\downarrow & \searrow & \downarrow{\scriptstyle \overline{p}} \\ Z & \xrightarrow{\alpha} & W \end{array}$$

where p and α are given by the Remmert-Stein factorization of the restriction $\overline{p}_{|Y} : Y \to W$. Since the fibers of $\overline{p}_{|Y}$ are not connected, the morphism α is finite (of degree two).

If the morphism p is extendable, our aim is to describe X by using the structure morphism $\overline{p}$. The occurrence that p is not extendable forces X to satisfy geometric costraints which, in turn, make X special enough to be completely classified.

There is a wide literature on this problem and a lot of results, most of them will be discussed throughout these notes. Our main purpose is to give a unified, alternative and shorter proof of several results on the topic, mainly based on Mori's theory and deformation theory of rational curves, by means of a good understanding of the behaviour of "lines in projective manifolds".

In this context, a quite natural illustrative motivation is the following simple question (in the sequel we will give a precise and more general statement which specializes to the following question).

Question. *Let X be an n-dimensional manifold embedded in a projective space $\mathbb{P}_{\mathbb{C}}$. Assume that the hyperplane section, H, of X is a linear $\mathbb{P}^d$-bundle over a smooth variety Z. Does it follow that the bundle projection $p : H \to Z$ extends to X giving a bundle projection $\overline{p} : X \to Z$?*

As soon as $n \geq 4$, the positive answer to that question relies on some non trivial result from Mori's theory and deformations of rational curves. It turns out that the key-fact is the condition $H \cdot f = 1$, where f is a line in a fiber $\mathbb{P}^d$ of $p : H \to Z$, i.e., f is a linear $\mathbb{P}^1$ with respect to the embedding of X in $\mathbb{P}_{\mathbb{C}}$ given by H.

It should be noted that if Y is a smooth ample divisor on a projective n-dimensional manifold X, $n \geq 4$, and (Y, L), $\pi : Y \to Z$, is a scroll in the adjunction theoretic sense, then a standard argument shows that $(X, \mathcal{L})$ has a scroll structure which agrees with π as soon as the line bundle $\mathcal{L}$ on X which extends L is ample. Note that $\mathcal{L} \cdot \ell = L \cdot \ell = 1$ for a line ℓ in the general fiber of π. E.g., this is the case when $L = \mathcal{O}_Y(Y)$ is the normal bundle of Y in X.

M. A. de Cataldo

Filtrations in cohomology and geometry

Abstract: I will report on joint work with L. Migliorini on a new characterization of the perverse filtration on the cohomology of a constructible complex on a quasi projective variety using hyperplane sections.

L. Goettsche

Holomorphic Euler characteristic of line bundles on moduli spaces of sheaves on surfaces

Abstract: Abstract: Let (X, H) be a polarized algebraic surface. Let $M = M_X^H(c_1, c_2)$ be the moduli spaces of H-semistable rank 2 sheaves on X with Chern classes c_1, c_2. We the holomorphic Euler characteristics of "Donaldson" line bundles on M, which can be viewed as a refinement of Donaldson invariants.

These invariants are subject to wallcrossing when H varies, and we determine a generating function for the wallcrossing in terms of elliptic functions.

This result will be used to determine generating functions for these invariants for rational surfaces, thus proving some cases of Le Potiers strange duality conjecture. This conjecture gives a duality between spaces of sections of line bundles on different moduli spaces of sheaves on the same surface.

T. Y. Li

HOM4PS-2.0 and HOM4PS-2.0para

Abstract: **HOM4PS-2.0** is a software package in FORTRAN 90 which implements the polyhedral homotopy continuation method for solving polynomial systems. It updates its original version HOM4PS in three key aspects: (1) New method for finding mixed cells, (2) Combining the polyhedral and linear homotopies in one step, (3) New way of dealing with the curve jumping. Numerical results show that this revision leads to a spectacular speed-up, ranging up to the 50's, over its original version on all bench mark systems, especially for large ones. It surpasses established packages in this category, such as PHCpack and PHoM, in speed by huge margins.

HOM4PS-2.0para is the parallel version of HOM4PS-2.0. Excellent scalability in the numerical results shows that the parallelization of the homotopy method always provides a great amount of extra computing resources to help solve polynomial systems of larger size which would be very difficult to deal with otherwise.

In this talk, these two packages will be elaborated and numerical results will be presented.

C. Peterson

Encodings and decodings of algebro-geometric data

Abstract: In several contexts within algebraic geometry we see transformations of information in one setting to a very different form. An early example one encounters is found in the construction of an ideal sheaf as a degeneracy locus of sections of a vector bundle in which much of the cohomology of the vector bundle gets transferred to the cohomology of a scheme. Another example is in the Hilbert Series of a module encoding the Hilbert function. In this talk, we will consider several somewhat more subtle encodings of algebro-geometric data including aspects of the numerical data that arises in the context of numerical algebraic geometry.

B. Shiffman

Number variance for simultaneous zeros of random sections of holomorphic line bundles

Abstract: I will give a universal asymptotic formula for the variance of the number of simultaneous zeros in a fixed domain U of m random holomorphic sections of the powers of a holomorphic line bundle over a compact Kaehler manifold of dimension m. This formula show that the standard deviation of the number of zeros in U grows like $N^{m/2-1/4}$, which is of lower order than the expected number, which grows like N^m. The methods involve the off-diagonal asymptotics of the Szego kernel and a "pluri-bipotential" for the variance current. This talk involves joint work with Steve Zelditch.

F. O. Schreyer

Betti numbers of Graded Moduls

Abstract: It can be very difficult to analyze for a given system of polynomials equations qualitative properties, such as the geometry of the corresponding variety. The theory of syzygies offers a tool for looking at systems of equations, which might help to make their subtle properties visible. In a recent paper Boij and Söderberg introduced a series of conjectures, which characterize all possible syzygy numbers of Cohen-Macaulay modules up to rational multiples. In the talk I report on the proof of these conjectures.

J. Verschelde

Toolboxes and Blackboxes for Solving Polynomial Systems

Abstract: Numerically solving polynomial systems in the past meant computing approximations to all isolated solutions. Based on the blackbox solver of PHCpack, new tools were developed to compute a numerical irreducible decomposition. These toolboxes implement the algorithms which define the field of numerical algebraic geometry. Nowadays, solving a polynomial system numerically includes giving accurate and reliable descriptions of positive dimensional solution sets and their multiplicities.

C. Wampler

From isolated solutions to positive dimensions and back again: A brief history of numerical algebraic geometry

Abstract: This talk will follow the arc of numerical algebraic geometry as it emerged from earlier work on polynomial continuation, which sought only to find isolated solutions to systems of polynomial equations, to blossom into a method for finding and manipulating positive dimensional algebraic sets. Along the way, we will reflect on the ways that classical algebraic geometry has provided the theoretical underpinnings that guarantee that the methods of numerical algebraic geometry do find all solutions. While the main impetus for the subject is to solve systems of equations arising in engineering and science, examples of which will be presented, the tools have also recently been used to attack problems usually considered of greater interest to mathematicians, such as computing the genus of a curve.

Finally, we discuss how the methods developed for handling positive dimensional sets can be turned around to produce new, efficient algorithms for finding isolated solutions. It is fitting that this talk be presented at a conference honoring Andrew Sommese, as his contributions, and those of his students, have been vital to the development of the subject.

J. Wisniewski

Algebraic Geometry and Phylogenetics

Abstract: I will discuss new developments regarding algebraic models of binary symmetric phylogenetic trees relating them to varieties arising as group actions quotients.