PHY 171
Homework 6 solutions
(Due by beginning of class on Thursday, February 16, 2012)

Submit neat work, with answers or solutions clearly marked by the question number. Unstapled, untidy work will be charged a handling fee of 20% penalty. Writing only an answer without showing the steps used to get to that answer will fetch very few points, even if the answer is correct. Late homework will not be accepted.

1. A rod has a length of 10.000 cm at 20.000°C and a length of 10.015 cm at the boiling point of water.

(a) What is the length of the rod at the freezing point of water?

Solution: Given

$$L_o = 10.000 \text{ cm} \quad \text{at } T = 20.000°C$$
$$L_f = 10.015 \text{ cm} \quad \text{at } T = 100.000°C$$

We need first to find α for this substance. Using $\Delta L = \alpha\, L_o\big(\Delta T\big)$, and knowing that $\Delta L = L_f - L_o = 0.015$ cm, we get

$$0.015 = \big(10.000\big)\alpha\big(100° - 20°\big) \qquad \Rightarrow \qquad \alpha = 1.875 \times 10^{-5}/°C$$

Now, we can carry out the required calculations.

Length at freezing point of water (0°C) is

$$L_{FP} = L_o - \Delta L = 10.000 - 10.000\big(1.875 \times 10^{-5}\big)(20 - 0) = \boxed{\mathbf{9.996\,cm}}$$

(b) At what temperature would the length of the rod be 10.009 cm?

Solution: In this part, we are asked to find the temperature T at which the length of the rod will be 10.009 cm. Again, using $\Delta L = \alpha\, L_o\big(\Delta T\big)$, we get

$$10.009 - 10.000 = 10.000\big(1.875 \times 10^{-5}\big)(T - 20)$$

from which we get

$$\frac{0.009}{10.000\big(1.875 \times 10^{-5}\big)} = T - 20$$

This gives

$$T = 20 + \frac{0.009}{10.000\big(1.875 \times 10^{-5}\big)} = 68°\text{C}$$

Therefore, the temperature at which the length of the rod is 10.009 cm is $\boxed{\mathbf{68°C}}$

2. A concrete highway is built of slabs 12.0 m long. How wide should the expansion cracks between the slabs be (at 20.0°C) to prevent buckling if the range of temperature is −30.0°C to +50.0°C?

Solution: From value announced in class, $\alpha_{\text{concrete}} = 12 \times 10^{-6}/°C$

Since we are asked to find the length of the cracks at 20°C (which is presumably the temperature at which the highway was constructed), we care only about the range of temperature from 20°C to 50°C. That is, between −30.0°C to 20.0°C, the highway will contract, so we don't need to consider temperatures below 20°C. So

$$\Delta L = \alpha_{\text{concrete}}\, L_o\big(\Delta T\big) = \big(12 \times 10^{-6}\big)12.0\big(50 - 20\big) = 0.00432\,\text{m}$$

Therefore, we need to leave expansion cracks between the slabs of length **0.432 cm**

3. One way to keep the contents of a garage from becoming too cold on a night when a severe subfreezing temperature is forecast is to put a tub of water in the garage. Suppose the mass of the water is 125 kg, and its initial temperature is 20.0°C.

(a) Calculate how much thermal energy the water would transfer to its surroundings in order to freeze completely.

Solution: Given, mass of water is $m_w = 125$ kg, initial temperature is $T_i = 20$°C.

To freeze completely, water must go from 20°C to 0°C, then freeze at 0°C.

$$\text{Thermal energy required} = \underbrace{125\, c_{\text{water}}\,\big(20 - 0\big)}_{\text{water from }20°\text{C to }0°\text{C}} + \underbrace{125\, L_{\text{water}}}_{\text{water to ice at }0°\text{C}}$$

$$= 125\,\Big(4186\Big)20 + 125\,\Big(3.33 \times 10^5\Big) = 51,715,000\,\text{J}$$

Therefore, the water transfers $\mathbf{5.2 \times 10^7\ J}$ of energy to its surroundings.

(b) What is the lowest possible temperature of the water and its surroundings until that happens? Explain clearly.

Solution: Until all the water freezes, the lowest temperature attained will be **0°C**

This is because heat drawn out of the system first goes into completely freezing the water at its freezing point, before there can be any change in temperature.

4. A 150 gram copper bowl contains 220 grams of water, both at 20.0°C. A very hot 300 gram copper cylinder is dropped into the water, causing it to boil, with 5 grams being converted to steam. The final temperature of the system is 100°C. What is the original temperature of the cylinder?

   Solution: In this problem, water and the copper bowl will ***gain*** heat energy, while the copper cylinder will ***lose*** heat energy.

   - Within a phase (e.g., while water is still a liquid, or for changes in temperature of the copper cylinder or bowl), we can find the heat lost or gained from $mc(\Delta T)$, where m is the mass, c is the specific heat, and ΔT is the change in temperature. From Table 17.2 (p. 519), the specific heat of copper is $c_{\text{Cu}} = 385$ J/(kg.°C), whereas the specific heat of water is, as usual, $c_{\text{w}} = 4186$ J/(kg.°C).
   - If there is a phase change, the heat lost or gained can be determined from mL, where m is the mass, and L is the latent heat. The latent heat of vaporization (water to steam) is given in Table 17.3 (p. 521) as $L_{\text{w}} = 22.6 \times 10^5$ J/kg.
   - We will assume that all numbers supplied have at least 2 significant digits (since the mass of the copper cylinder is stated as 300 g, this is not clear), and supply one more digit than significant in the answer.

   Since the water boils, with 5 g being converted into steam, we can find the energy gained by water by adding the energy required to take all 220 g of the water from 20°C to 100°C, and the energy required to convert 5 g of water to steam at 100°C. So

$$\begin{aligned}\text{Energy gained by water } &= \underbrace{(0.220 \text{ kg})\, c_{\text{water}} \left(100 - 20\right)}_{\text{all water from } 20^\circ\text{C to } 100^\circ\text{C}} + \underbrace{(5 \times 10^{-3} \text{ kg}) L_{\text{w}}}_{\text{5g of water to steam}} \\ &= \Big(0.220\Big)\ 4186\ \Big(80\Big) + \Big(5 \times 10^{-3}\Big)\ 22.6 \times 10^5 = 84,973.6\,\text{J}\end{aligned}$$

   Since the bowl remains solid throughout, the energy transferred to it can be found easily.

$$\text{Energy gained by bowl} = (0.150 \text{ kg})\ c_{\text{Cu}} \left(100 - 20\right) = 0.150 \Big(385\Big)\ 80 = 4620\,\text{J}$$

   The total energy lost by the cylinder is equal to the sum of the energy gained by the water and the bowl (assuming no energy was lost to the environment). This allows us to find the initial temperature T_i of the cylinder.

$$\underbrace{\Big(0.300 \text{ kg}\Big)\ 385 \Big(T_i - 100\Big)}_{\text{Energy lost by cylinder}} = 84973.6 + 4620$$

   Upon carrying out the calculation, we will find that the initial temperature of the cylinder is $T_i = \boxed{\textbf{876°C}}$

5. A cube of ice is taken from the freezer at $-8.5°C$ and placed in a 75 gram aluminum calorimeter filled with 300.0 grams of water at room temperature of 20.0°C. The final situation is observed to be all water at 17°C. What was the mass of the ice cube?

   Solution: We will use the same procedure as in the problem above. We will work in SI, specific heat values are taken from Table 17.2, latent heat values from Table 17.3 (L_{water} is the latent heat of fusion of water, that is, the heat absorbed by 1 kg of ice to melt at 0°C).

   The heat lost by water and aluminum is easy to calculate using $m\,c\,\Delta T$ for each, similar to the previous problem. Both start at 20°C and end at 17°C.

   The heat gained by ice must be calculated in three steps, by adding the heat required to take ice from $-8.5°C$ to 0°C, melting the ice at 0°C, and then raising the temperature of the melted ice (which is now water) from 0°C to the final equilibrium value of 17°C.

   Let the required mass of ice be m.

$$\text{Heat gained by ice} = \text{Heat lost by water and aluminum}$$

$$\underbrace{m\,c_{\text{ice}}\left[0-(-8.5)\right]}_{\text{ice from } -8.5°\text{C to } 0°\text{C}} + \underbrace{m\,L_{\text{water}}}_{\text{ice melts at } 0°\text{C}} + \underbrace{m\,c_{\text{water}}\left(17-0\right)}_{\text{water from } 0°\text{C to } 17°\text{C}} = \underbrace{0.300\,c_{\text{water}}\left(20-17\right)}_{\text{water from } 20°\text{C to } 17°\text{C}} + \underbrace{0.075\,c_{\text{Al}}\left(20-17\right)}_{\text{Al from } 20°\text{C to } 17°\text{C}}$$

$$\Rightarrow \quad m\left(2090\right)8.5 + m\left(3.33\times10^{5}\right) + m\left(4186\right)17 = 0.300\left(4186\right)3 + 0.075\left(900\right)3$$

$$\Rightarrow \quad m\left(421,927\right) = 3969.9$$

   Solving for m, we find the mass of the ice cube to be $\boxed{\mathbf{9.4\ \times 10^{-3}\ kg\ or\ 9.4\ g}}$